Time-Frequency Decomposition of an Ultrashort Pulse: Wavelet Decomposition

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Abstract. *An efficient numerical algorithm is presented for the numerical modeling of the propagation of ultrashort pulses with arbitrary temporal and frequency characteristics through linear homogeneous dielectrics. The consequences of proper sampling of the spectral phase in pulse propagation and its influence on the efficiency of computation are discussed in detail. The numerical simulation presented here is capable of analyzing the pulse in the temporal-frequency domain. As an example, pulse propagation effects such as temporal and spectral shifts, pulse broadening effects, asymmetry and chirping in dispersive media are demonstrated for wavelet decomposition.*

Keywords

Refraction index, femtosecond pulse, chromatic dispersion, chirp, Fourier analysis, wavelet decomposition.

1. Introduction

Propagation of ultrashort optical pulses in a linear optical medium consisting of free space [1-5], dispersive media [6, 7], diffractive optical elements [8, 9], focusing elements[10-12] and apertures [13-16] has been extensively studied analytically, though only a few isolated attempts have been made on numerical simulation. Most studies are based analytical calculations assuming a plane wave or TEM_{00} Gaussian transverse profile and a Gaussian temporal profile for the pulse. The change in the spectral properties of the pulse on propagation of the pulse was investigated analytically by Sheppard and Gan [3] taking special forms of Gaussian pulsed beams. Agrawal [4] considered spatial broadening of diffracted pulses assuming Gaussian transverse and temporal profile. However, analytical methods have the limitations of not being able to handle arbitrary pulse profiles. Also, closed form solutions are often obtained after certain levels of approximations. This has motive a few studies based on the use of numerical simulation techniques in the analysis of pulse propagation. For example, Kaplan [5] introduced numerical evaluation by fast Fourier transform to analyze pulses of arbitrary temporal profile and investigated on-axis temporal evolution of the pulse in the far field. In view of the recent advance in ultrashort pulse propagation, a strong need is felt for developing a numerical formalism capable of performing such a complete analysis of the issues involved in pulse propagation.

Here we introduce a numerical simulation tool for propagation of ultrashort pulses of arbitrary shape through linear homogeneous media based on wave optical field representation [17] which enables an easy evaluation for the merit functions of the pulsed field. This allows us to analyze the pulse in the time-frequency domain at any arbitrary plane. With this tool, we investigate the spectral and temporal evolution of ultrashort pulses at any arbitrary propagation distance. The propagation of the pulse is achieved in terms of its spectral equivalent. Further, we introduce certain sampling rules for the spectral phase so that the phase information is sampled properly when we move from one spectral component to another in the spectral equivalent of the pulse. As a consequence, the algorithm becomes computationally efficient since we only considered a small number of spectral components for simulation of pulse propagation.

2. Ultrashort Laser Pulses

One very important feature of ultrashort laser pulses is the close relation between the pulse duration, τ, and the spectral bandwidth, Δw. This relation is manifested in the time-bandwidth product: $\tau \cdot \Delta w \geq 2\pi \cdot c_B$ where c_B is a constant that depends on the shape of the pulse and w is the angular frequency related to the frequency f, and wavelength, λ through

$$w = 2\pi f = \frac{2\pi c}{\lambda} . \tag{1}$$

If follows directly from (1) that the minimum achievable duration is limited by the spectrum of the pulse. In other words, in order to produce ultrashort pulses a very broad spectral bandwidth is needed. The shortest possible pulse, for a given spectrum, is known as the *transform-limited pulse duration*. It should be noted that (1) is not equality, i.e. the product can very well exceed $2\pi \cdot c_B$. If the product exceeds $2\pi \cdot c_B$ the pulse is no longer transform-limited and all frequency components that constitute the pulse do not coincide in time, i.e. the pulse exhibits frequency modulation is very often referred to as a *chirp*.

2.1 Mathematical Description of Laser Pulses

Ultrashort laser pulses are coherent bursts of electromagnetic radiation, confined in time and space. They are characterized by several parameters: temporal coherence, spatial coherence (i.e. focusing ability), contrast, power, etc. Here the description is concentrated on their temporal aspects.

In order to completely describe a laser pulse, the temporal profile, the spectral profile and the phase of the pulse have to be known. However, time and frequency are related through a Fourier transform and it is therefore sufficient to know only two of these parameters since the third can always be calculated from the other two.

2.1.1 Time-Domain Description

Since in this paper the main emphasis is on the temporal dependence, all spatial dependence is neglected, i.e., $E(x, y, z, t) = E(t)$, the electric field $E(t)$ is a real quantity and all measured quantities are real. However, the mathematical description is simplified if a complex representation is used:

$$\tilde{E}(t) = \tilde{A}(t) \cdot e^{-iw_0 t} \tag{2}$$

where $\tilde{A}(t)$ is the complex envelope, usually chosen such that the real physical field is twice the real part of the complex field, and w_0 is the carrier frequency, usually chosen to the centre of the spectrum. In this way the rapidly varying is separated from the slowly varying envelope $\tilde{A}(t)$. $\tilde{E}(t)$ can be further decomposed into:

$$\tilde{E}(t) = \left|\tilde{E}(t)\right| \cdot e^{i\varphi_0} \cdot e^{-i\varphi(t)} = \left|\tilde{E}(t)\right| \cdot e^{i\varphi_0} \cdot e^{-i(\Phi(t)-w_0)} \tag{3}$$

$\varphi(t)$ is often referred to as the temporal phase of the pulse and φ_0 the absolute phase, which relates the position of the carrier wave to the temporal envelope of the pulse (see Fig. 1). In $\Phi(t)$ the strong linear term due to the carrier frequency, wt is omitted. The absolute phase is important mainly for pulses consisting of only a few cycles and has recently attracted a great deal of attention. The absolute phase has, for instance, been shown to be very important when generating high-order harmonics with few-cycle pulses.

The instantaneous frequency $w(t)$ is given by the first derivative of the temporal phase

$$w(t) = \frac{d\varphi(t)}{dt} = \frac{d\Phi(t)}{dt} - w_0 \tag{4}$$

which means that a nonlinear temporal phase yields a time-dependent frequency modulation- the pulse is said to carry a chirp (illustrated in Fig. 2).

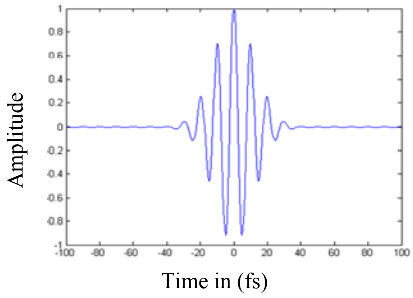


Fig. 1. The electric field of an ultra-short lasers pulse consisting of only a few cycles.

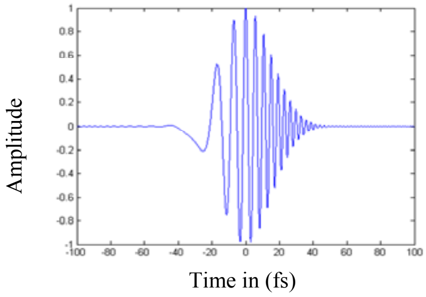


Fig. 2. The electric field of an ultrashort lasers pulse with a strong positive chirp. Note the frequency variation as a function of time; at the leading edge (to the left) the wavelength is longer than at the trailing edge.

2.1.2 Frequency-Domain Description

It is usually more convenient to represent the pulse in the frequency domain rather than in the time domain. The frequency representation is obtained from the time domain by a complex Fourier transform,

$$E(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} E(t) \cdot e^{iwt} dt \ . \tag{5}$$

Just as in the time domain, $\tilde{E}(w)$ can be written as

$$\tilde{E}(w) = \left|\tilde{E}(w)\right| e^{i\varphi(w)} \tag{6}$$

where $\varphi(w)$ now denotes the spectral phase. An inverse transform leads back to the time domain,

$$\tilde{E}(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \tilde{E}(w) \cdot e^{iwt} dw \tag{7}$$

From (7) it is clear that $\tilde{E}(t)$ can be seen as a superposition of monochromatic waves.

The square of the spectral amplitude, $|\tilde{E}(w)|^2$, represents the power spectrum, or spectral power density, which is the pulse parameter that is most easily accessible experimentally. It is commonly referred to as the spectrum of the pulse.

The spectral phase can, in the same manner as the temporal phase, be decomposed into different parts. A common procedure is to employ Taylor expansion

$$\Phi(w) = \Phi_0 + \sum_{n=1}^{\infty} \frac{1}{n!} a_n (w - w_0)^n \tag{8}$$

with $a_n = \left. \frac{d^n \varphi}{dw^n} \right|_{w=w_0}$.

It can be seen that the first two terms will not change the temporal profile of the pulse. A linear phase variation does not change the shape of the pulse, but only introduces a temporal shift of the entire pulse. Therefore, usually only the nonlinear part of the spectral phase is of interest. Any nonlinear addition to the phase will redistribute the frequency components and alter the temporal shape of the pulse.

3. Propagation of a Light Pulse in a Transparent Medium

What happens to a short optical pulse propagating in a transparent medium? Because of its wide spectral width and because of group velocity dispersion in transparent media, it undergoes a phase distortion inducing an increase of its duration. This happens with any optical element and needs to be properly corrected for in the course of experiments [18].

The frequency Fourier transform of a Gaussian pulse has already been given as

$$E(w) = \exp\left(\frac{-(w - w_0)^2}{4\Gamma} \right). \tag{9}$$

After the pulse has propagated a distance x, its spectrum is modified to

$$E(w,z) = E(w) \exp[\pm ik(w) z], \tag{10}$$

$$k(w) = \frac{n(w) \cdot w}{c}$$

where $k(w)$ is now a frequency-dependent propagation factor. In order to allow for a partial analytical calculation of the propagation effects, the propagation factor is rewritten using a Taylor expansion as a function of the angular frequency, assuming that $\Delta w << w_0$ (this condition is only weakly true for the shortest pulses). Applying the Taylor expansion to (11), the pulse spectrum becomes.

$$k(w) = k(w_0) + k'(w - w_0) + \frac{1}{2} k''(w - w_0)^2 + \dots \tag{11}$$

where

$$k' = \left(\frac{dk(w)}{dw} \right)_{w_0} \text{ and } k'' = \left(\frac{d^2 k(w)}{dw^2} \right)_{w_0}.$$

$$E(w,z) = \exp\left[-ik(w_0) z - ik' z (w - w_0) - \left(\frac{1}{4\Gamma} + \frac{i}{2} k'' \right) (w - w_0)^2 \right] \tag{12}$$

The time evolution of the electric field in the pulse is then derived from the calculation of the inverse Fourier transform of (13),

$$e(t,z) = \int_{-\infty}^{+\infty} E(w,z) \cdot e^{-iwt} dw \tag{13}$$

So that

$$e(t,z) = \sqrt{\frac{\Gamma(z)}{\pi}} \cdot \exp\left[iw_0 \left(t - \frac{z}{V_\Phi(w_0)} \right) \right] \cdot \exp\left[-\Gamma(z) \left(t - \frac{z}{V_g(w_0)} \right)^2 \right] \tag{14}$$

where

$$V_\Phi(w_0) = \left(\frac{w}{k} \right)_{w_0}, \quad V_g(w_0) = \left(\frac{dw}{dk} \right)_{w_0}, \quad 1/(\Gamma(z)) = 1/\Gamma + 2ik'' z. \tag{15}$$

In the first exponential term of (14), it can be observed that the phase of the central frequency w_0 is delayed by an amount z/V_Φ after propagation over a distance x. Because the phase is not a measurable quantity, this effect has no observable consequence. The phase velocity $V_\Phi(w_0)$ measures the propagation speed of the plane wave components of the pulse in the medium. These plane waves do not carry any information, because of their infinite duration.

The second term in (14) shows that, after propagation over a distance x, the pulse keeps a Gaussian envelope. This envelope is delayed by an amount z/V_g, V_g being the group velocity.

The second term in (14) also shows that the pulse envelope is distorted during its propagation because its form factor $\Gamma(z)$, defined as

$$1/(\Gamma(z)) = 1/\Gamma + 2ik'' z \tag{16}$$

depends on the angular frequency w through $k''(w)$

$$k'' = \left(\frac{d^2 k}{dw^2} \right)_{w_0} = \frac{d}{dw} \left(\frac{1}{V_g} \right)_{w_0}. \tag{17}$$

This term is called the "Group Velocity Dispersion".

The temporal width of the pulse at point z:

$$\Delta t = \Delta\tau_0 \sqrt{1 + 4 \cdot (\Gamma \cdot k'' z)^2} \tag{18}$$

with $k'' = \frac{\lambda^3}{2 \cdot \pi \cdot c^2} \frac{d^2 n}{d\lambda^2}$, $\Gamma = \frac{2 \log 2}{\Delta\tau_0^2}$.

3.1 Application in Silica

The index of silica is given by the following expression [19]

$$n^2(w) = 1 + \sum_{i=1}^{m} \frac{B_i w_i^2}{w_i^2 - w^2} \tag{19}$$

where w_i is the frequency of resonance and B_i is the amplitude of resonance

B_i	0.6961663	0.4079426	0.8974794
$\lambda_i(\mu m)$	0.0684043	0.1162414	9.896161

Tab. 1. Parameters for Bulk-fused silica.

In the case of optical fibres, the parameters w_i and B_i are obtained experimentally by fitting the measured dispersion curves to (19) with $m = 3$ and depend on the core constituents [20-21].

3.2 Parameter of Dispersion

One can also define the coefficient C_D controlling the frequency shifts such as

$$C_D = \frac{z}{D} \quad \text{with} \quad D = \frac{\Delta\tau_0^2}{z \cdot k''}. \tag{20}$$

D is called parameter of dispersion. This parameter measures the relative importance of chromatic dispersion. z is the length of the medium crossed by the pulse laser [19]:
If $z < D$, the group velocity dispersion is negligible. If $z > D$, it is necessary to take account of the dispersive effects.

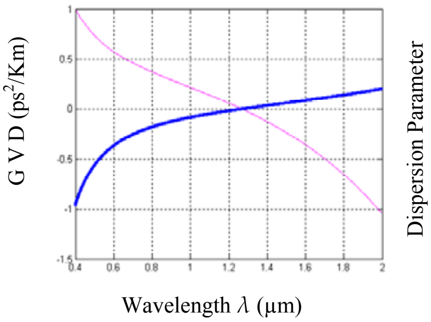


Fig. 3. Variation of group velocity dispersion k'' and dispersion parameter D (solid line) with wavelength for fused silica.

We consider dispersions of orders two. The pulse broadens on propagation as a result of group velocity dispersion (GVD). In summary, the propagation of a short optical pulse through transparent medium results in a delay of the pulse, a duration broadening and a frequency chirp.

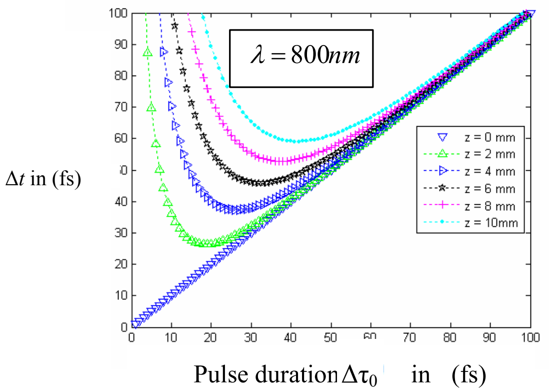


Fig. 4. Temporal broadening of the transform-limited pulse for different values of the propagation distance z.

3.3 Group Velocity Dispersion

The Group Velocity Dispersion (GVD) is defined as the propagation of different frequency components at different speeds through a dispersive medium. This is due to the wavelength-dependent index of refraction of the dispersive material. GVD causes variation in the temporal profile of the laser pulse, while the spectrum remains unaltered. A transform-limited pulse is also called short pulse or unchirped pulse. It is said that the initial short pulse will become positively chirped (or upchirped) after propagating through a medium with "normal" dispersion (e.g. silica glass). This corresponds to the situation when higher frequencies travel slower than lower frequencies (blue slower than red). The opposite situation, where the pulse travels through a medium with "anomalous" dispersion, leads to a negative chirp (or downchirp). Here the bluer frequencies propagate faster than the redder frequencies.

To the first place we limited only to the order two of the Taylor expansion of the phase. It is noticed that the analysis of Fourier remains valid only for durations of pulse which are higher than ≈ 60 fs.

In addition media we consider all higher order dispersion, which completely describes the physical processes involved in ultrashort dispersive pulse dynamics. The pulse broadens in time and becomes asymmetric. In addition, the off axis pulse becomes wider than the pulse on axis [20].

$$\begin{bmatrix} \Phi^{(2)} \\ \Phi^{(3)} \\ \Phi^{(4)} \\ \Phi^{(5)} \\ \Phi^{(6)} \end{bmatrix} = (-1)^n 2\pi z \left[\frac{\lambda}{2\pi c}\right]^n \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \\ 12 & 8 & 1 & 0 & 0 \\ 60 & 60 & 15 & 1 & 0 \\ 360 & 480 & 180 & 24 & 1 \end{bmatrix} \tag{21}$$

The various terms of the Taylor expansion to order n can be written in the shape of a matrix [**A**], which's we can express various terms A_{ij}

$$\Phi(w) = \Phi(w_0) + (w - w_0)\Phi^{(1)} + \sum_{i=2}^{p} \frac{1}{i!}(w - w_0)^i \Phi^{(i)}\Big|_{w=w_0} + \theta(w), \tag{22}$$

$$\Phi^{(p)} = (-1)^p \cdot 2\pi \cdot z \left[\frac{\lambda}{2\pi \cdot c}\right]^p \sum_{j=2}^{p} \lambda^{j-1} A(p-1, j-1) n^{(j)} \tag{23}$$

with $p > 2$.

Analytically known and experimentally observed propagation effects such as spectral shift, pulse broadening and asymmetry in dispersive media can be easily brought out in the simulation using formalism presented here. In addition, such studies can be extended to pulses of arbitrary temporal shape without any further algorithmic complexity by numerical simulation. Higher order dispersion effects can be handled easily in the numerical simulation unlike in the case of analytical calculation [22].

Discussion

The Fourier theorem is the most classical approach for describing the propagation of electromagnetic signals through dispersive media. In the case of signals characterized by a slow temporal varying envelope, the phase is usually approximated by the Taylor expansion in the neighborhood of the central frequency of the input pulse. For shorter pulses, the concept of group velocity is irrelevant and the envelope distortion is a function of the higher order terms. Ultrashort pulses less than 10 fs are now available. Their envelope harmonic content is so high that the Taylor expansion of the phase is now more possible. There is no other way than a numerical computation of the Fourier integral. However this method does not permit a straightforward physical understanding of the envelope propagation and principally does not picture the fact that this is the group velocity dispersion which generates the ultrashort pulses distortions. Such a situation claims for another type of decomposition involving both a time and frequency dependence of the components. Numerous bidimensional representations of acoustic and electromagnetic signals have already been suggested. We propose here a method derived from the Gabor transformation in order to decompose the signal into an infinite number of elementary components (wavelet) of the same duration (much longer than that of the original signal), each of them being centered at a frequency Ω belonging to the Fourier spectrum of the pulse.

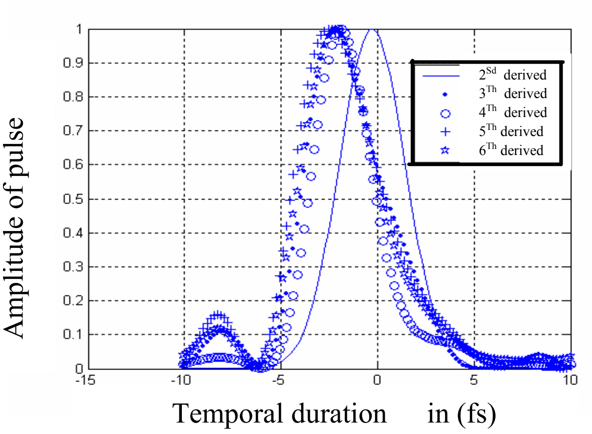


Fig. 5.a The pulse broadens on propagation as a result of group velocity dispersion (GVD) (λ=800 nm, z=4 mm).

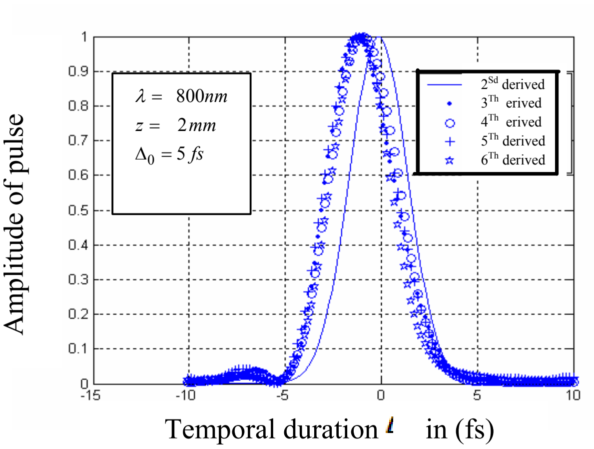


Fig. 5.b The pulse shape is no longer Gaussian and it becomes asymmetric due to higher order dispersion.

4. Time-Frequency Decomposition

4.1 Wavelet Theory

In 1983 geophysicist *Jean Morlet* proposed a revolutionary process, the analysis and the synthesis by the wavelet, which makes it possible to analyze signals effectively or combine very different phenomena of scales.

The wavelets are very particular elementary functions, these are the shortest vibrations and most elementary that one can consider. One can say that the wavelet east carries out a zooming on any interesting phenomenon of the signal which place on a small scale in the vicinity of the point considered [23].

4.2 Wavelet Techniques

Starting with a signal $e(t)$, in the plane $z = 0$, we define a wavelet centered at Ω by

$$\theta(\Omega) = E(w)\cdot\exp\left[-\frac{(w-\Omega)^2}{4\gamma}\right], \tag{24}$$

with $E(w) = \dfrac{E_0}{2\pi}\sqrt{\dfrac{\pi}{\Gamma}}\exp\left[\dfrac{(w-w_0)^2}{4\Gamma}\right]$.

We calculate the electric field associated with the wavelet $\theta(\Omega, z = 0)$.

$$\theta(t, z=0) = TF\{\theta(\Omega, z=0)\}, \tag{25}$$

$$\theta(t, z=0) = E_0\sqrt{\frac{\gamma}{\gamma+\Gamma}}\cdot\exp\left[\frac{-(w_0-\Omega)^2}{4(\gamma+\Gamma)}\right] \times\exp\left[-\frac{\gamma\Gamma}{\gamma+\Gamma}t^2\right]\cdot\exp\left[j\frac{\gamma w_0+\Gamma\Omega}{\gamma+\Gamma}t\right] \tag{26}$$

In time, the pulse is also Gaussian, of parameter $\dfrac{\gamma\Gamma}{\gamma+\Gamma}$.

The maximum of amplitude of the wavelet $\theta(t, z = 0)$ varies with Ω [22].

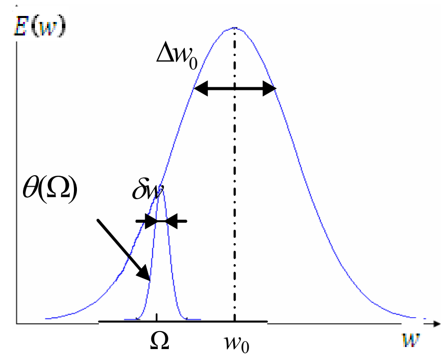


Fig. 6. Gaussian envelope decomposed into a number of wavelets.

The signal propagates in the positive x direction in a linear dispersive and transparent medium, which fills the half space $z > 0$ and whose refractive index is $n(w)$. After propagation, the wavelet $\theta(\Omega, x)$ may be written as

$$\theta(\Omega,z)=\frac{E_0}{2\sqrt{\pi\gamma}}E(w)\cdot\exp\left[-\frac{(w-\Omega)^2}{4\gamma}\right]\cdot\exp[j\Phi(w)]\cdot \quad (27)$$

As already mentioned, τ_{wavelet} is large enough to ensure that analyzing function has only non negligible values over a spectral range lying in the neighborhood of Ω in Fig. 6. Under these circumstances, we have

$$\begin{aligned}\Phi(w)&=\Phi(\Omega)+(w-\Omega)\left.\frac{d\Phi}{dw}\right|_{w=\Omega}+\\&+\frac{1}{2!}(w-\Omega)^2\left.\frac{d^2\Phi}{dw^2}\right|_{w=\Omega}+\ldots+\frac{1}{n!}(w-\Omega)^n\left.\frac{d^n\Phi}{dw^n}\right|_{w=\Omega}=\\&=\Omega+\theta w\end{aligned} \quad (28)$$

Neglecting the higher terms in (28)

$$\begin{aligned}\Phi(w)&=\Phi(\Omega)+(w-\Omega)\left.\frac{d\Phi}{dw}\right|_{w=\Omega}+\\&+\frac{1}{2!}(w-\Omega)^2\left.\frac{d^2\Phi}{dw^2}\right|_{w=\Omega}+\theta(w)\end{aligned} \quad (29)$$

$$\begin{aligned}\theta(\Omega,z)&=\frac{E_0}{2\sqrt{\pi\gamma}}\sqrt{\frac{\pi}{\Gamma}}\exp\left[-\frac{(w-w_0)^2}{4\Gamma}\right]\cdot\exp\left[-\frac{(w-\Omega)^2}{4\gamma}\right]\\&\cdot\exp\left[j\Phi^{(0)}+j(w-\Omega)\Phi^{(1)}+\frac{1}{2}j(w-\Omega)^2\Phi^{(2)}\right]\end{aligned} \quad (30)$$

We calculate the temporal electric field associated with the wavelet $\theta(\Omega,z)$.

$$\theta(t,z)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\theta(\Omega,z)\cdot\exp(jwt)dw, \quad (31)$$

$$\begin{aligned}\theta(t,z)&=\frac{1}{2\pi}\frac{E_0}{2\sqrt{\pi\gamma}}\sqrt{\frac{\pi}{\Gamma}}\exp\left[-\frac{(\Omega-w_0)^2}{4\Gamma}\right]\exp\left(j\Phi^{(0)}\right)\\&\times\exp\left[-\left(\frac{1}{4\Gamma}+\frac{1}{4\gamma}-\frac{1}{2}j\Phi^{(2)}\right)\Omega^2\right]\exp\left\{\left[\frac{(\Omega-w_0)}{2\Gamma}-j\Phi^{(1)}\right]\Omega\right\}\\&\times\int_{-\infty}^{+\infty}\left\{\exp\left[-\left(\frac{1}{4\Gamma}+\frac{1}{4\gamma}-\frac{1}{2}j\Phi^{(2)}\right)w^2\right]\cdot\exp\left[\left(\frac{1}{4\Gamma}+\frac{1}{4\gamma}-\frac{1}{2}j\Phi^{(2)}\right)2w\Omega\right]\right.\\&\left.\times\cdot\exp\left(-\frac{(\Omega-w_0)^2}{2\Gamma}-j\Phi^{(1)}\right)\cdot\exp(jwt)\right\}dw\end{aligned} \quad (32)$$

The amplitude of the incident Ω wavelet is given

$$\begin{aligned}\theta(t,z)&=\frac{E_0}{2\sqrt{\pi\gamma}}\sqrt{\frac{\Gamma(z)}{\Gamma}}\cdot\exp\left(j\Phi^{(0)}\right)\exp\left(-\Gamma(z)\left[t+\frac{z}{V_g(\Omega)}\right]^2\right)\\&\times\exp\left(-\frac{(\Omega-w_0)^2}{4\Gamma}\left[1-\frac{\Gamma(z)}{\Gamma}\right]\right)\cdot\exp\left[j\left(1-\frac{\Gamma(z)}{\Gamma}\right)\Omega+\frac{\Gamma(z)}{\Gamma}w_0\right]\\&\left(t+\frac{z}{V(\Omega)_g}\right)\end{aligned} \quad (33)$$

This wavelet is characterized by a Gaussian envelope. This decomposition is valid only for the values of Δw much larger than δw ($\Delta w \gg \delta w$).

The delay of group of the wavelet $[t + z/V_g(\Omega)]$ is characterized by a Gaussian envelope which is the temporal width.

The delay of group of the wavelet is inversely proportional to the velocity of group its envelope propagates without deformation [22].

4.3 Simulations

Parameters of the simulations

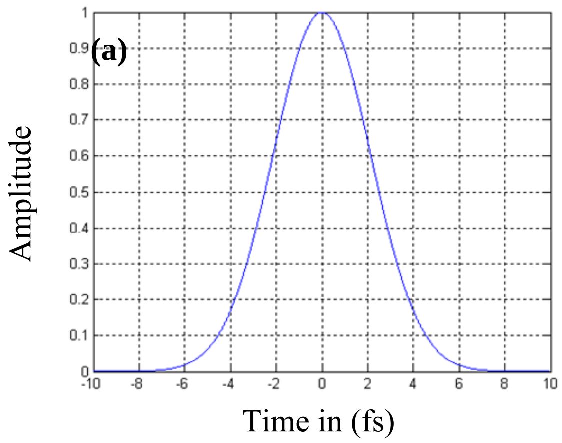
Initial pulse: $\Delta\tau_0 = 5$ fs

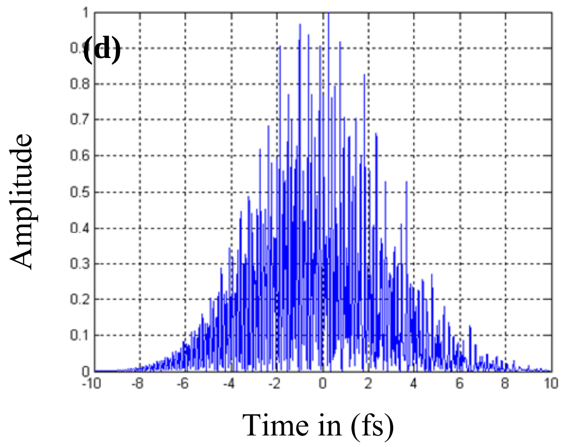
Wavelength : $\lambda = 800$ nm

Pulse of the wavelet: $\Delta\tau_{\text{wavelet}} = 1000$ fs

Length of the silica : $z = 5$ cm

To describe the propagation of the pulse, we only consider the propagation of the maximum of each wavelet in a three dimensional representation.





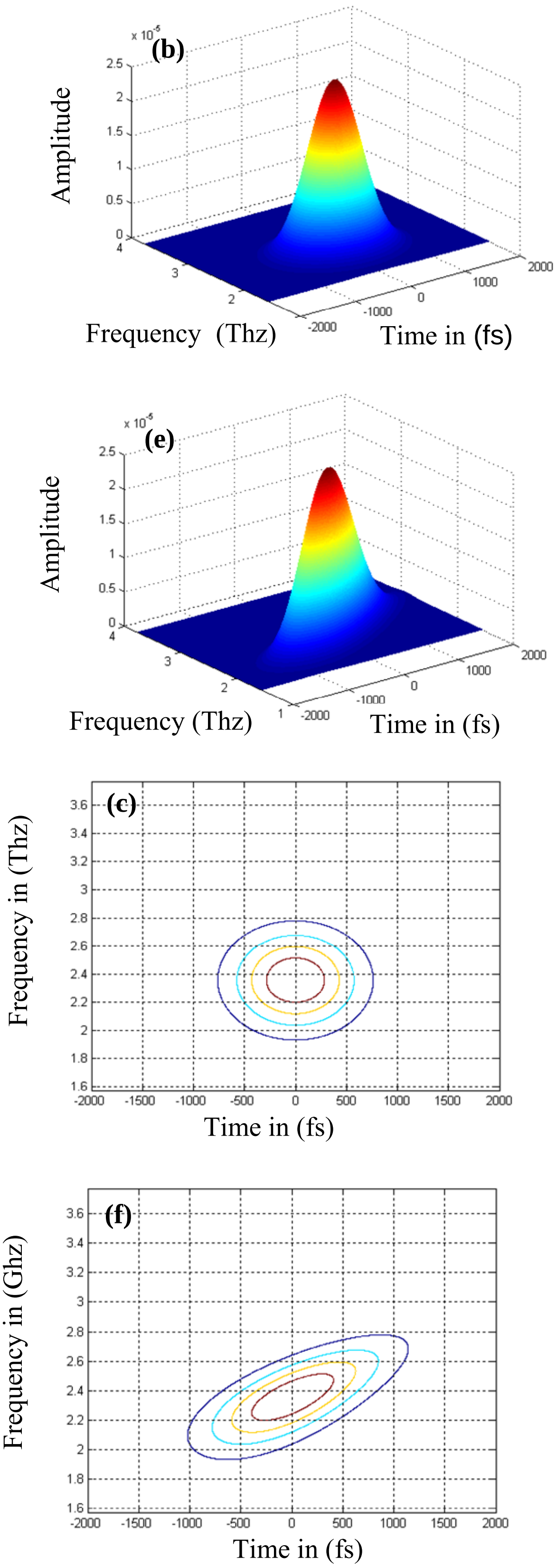


Fig. 7. (a) Initial pulse, (b) Initial wavelet, (c) Contour of the initial wavelet, (d) Pulse after propagation of 5 cm in the fused silica, (e) the wavelet representation, and (f) contour of the wavelet after propagation of 5 cm in the fused silica.

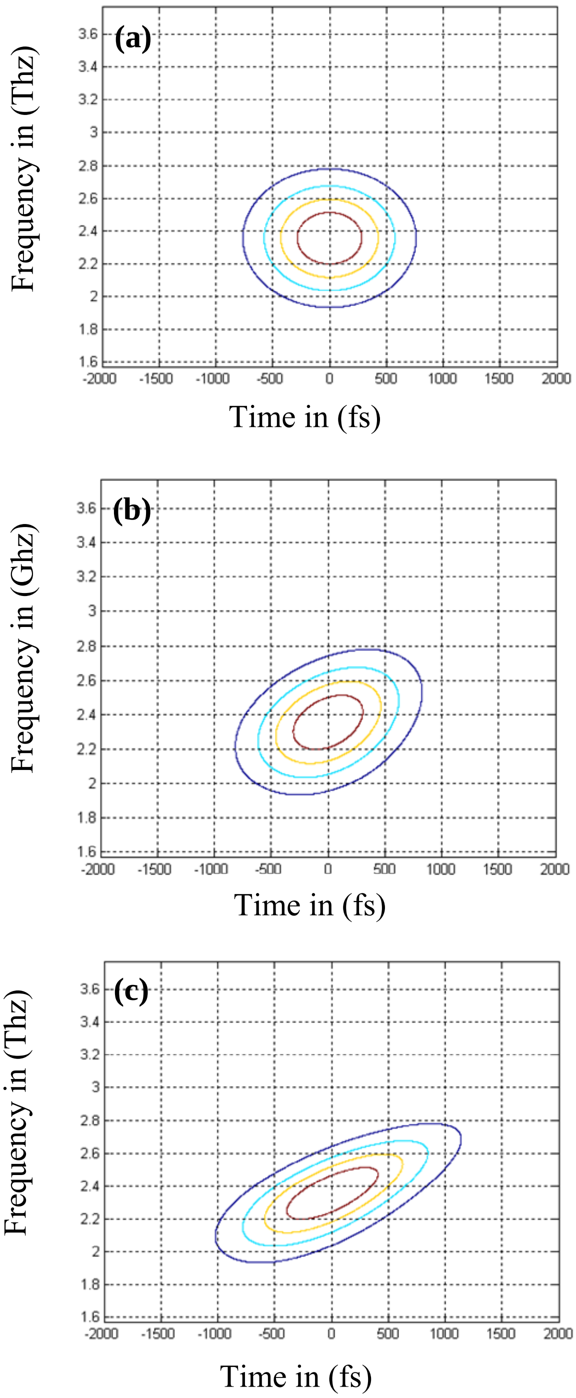


Fig. 8. (a) Contour of the initial wavelet, (b) contour of the wavelet after propagation of 2 cm in the fused silica, (c) contour of the wavelet after propagation of 5cm in the fused silica.

5. Conclusions

In conclusion, we have demonstrated here the possible decomposition of an ultrashort pulse into an infinite number of longer Fourier transform limited wavelets which propagate without any deformation through a dispersive medium. After propagation through the medium, the pulse may be visualized in a three dimensional representation by the locus of the wavelet maxima. This representation permits the evaluation of the broadening suffered by the pulse. For a transparent medium, the propagation of the Ω wavelet is described by the convolution of the incident Ω wavelet with a $\theta(\Omega)$ distribution centered at the group delay relative to Ω.

The application to absorbing media is relatively straightforward and will be presented in a further publication, as well as a generalization to nonlinear media. The time-frequency representation is peculiarly suitable to the latter case for which the refractive index is phenomenological time dependent.

Although this technique represents a vast improvement in our ability to describe such pulses, they require additional effort, both in the apparatus and in the extraction of the pulse intensity and phase from the experimental trace.

Acknowledgements

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