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BRNO UNIVERSITY OF TECHNOLOGY

FAKULTA STROJNÍHO INŽENÝRSTVÍ
ÚSTAV AUTOMOBILNÍHO A DOPRAVNÍHO INŽENÝRSTVÍ

FACULTY OF MECHANICAL ENGINEERING
INSTITUTE OF AUTOMOTIVE ENGINEERING

MODÁLNÍ VLASTNOSTI KLIKOVÉHO ÚSTROJÍ ŠESTIVÁLCOVÉHO TRAKTOROVÉHO MOTORU

MODAL PROPERTIES OF 6–CYLINDER TRACTOR ENGINE POWERTRAIN

PŘÍLOHA DIPLOMOVÉ PRÁCE
APPENDICES OF MASTER'S THESIS

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Table of Contents

Appendix 1 - Cranktrain Kinematics

1.1 Main Parameters of the Engine

$cyl = 6$		Number of Cylinders
$n_n = 2200 \cdot \frac{1}{min}$		Nominal Revolutions per Minute
$n = 1480 \cdot \frac{1}{min}$		Revolutions at Maximum Torque
$D = 105 \cdot mm$		Bore Diameter
$Z = 120 \cdot mm$		Stroke
$V_z = \frac{\pi \cdot D^2}{4} \cdot Z$	$V_z = 1039.082 \cdot cm^3$	Cylinder Displacement
$h = 136 \cdot mm$		Axial Cylinder Distance
$\varepsilon_c = 17.8$		Compression Ratio
$V_c = \frac{V_z}{\varepsilon_c - 1}$	$V_c = 61.85 \cdot cm^3$	Compression Volume
$r = \frac{Z}{2}$	$r = 60 \cdot mm$	Crank Radius
$l_c = 215 \cdot mm$		Effective Con Rod Length
$\lambda = \frac{r}{l_c}$	$\lambda = 0.279$	Crank/Rod Ratio
$m_{pa} = 2053.9 \cdot gm$		Weight of Piston Assembly
$m_{crec} = 907 \cdot gm$		Reciprocating Mass of Con Rod
$m_{crot} = 1645 \cdot gm$		Rotational Mass of Con Rod

1.2 Kinematics

$\omega = 2\cdot\pi\cdot n$ $\qquad$ $\omega = 154.985\frac{1}{s}$ $\qquad$ Angular Speed

Piston Position

$s_p(\alpha) = r\cdot\left[(1-\cos(\alpha)) + \frac{\lambda}{4}\cdot(1-\cos(2\cdot\alpha))\right]$ $\qquad$ Equation of Overall Piston Position

Piston Velocity

$v_p(\alpha) = r\cdot\omega\cdot\left(\sin(\alpha) + \frac{\lambda}{2}\cdot\sin(2\cdot\alpha)\right)$ $\qquad$ Equation of Overall Piston Velocity

Piston Acceleration

$a_p(\alpha) = r\cdot\omega^2\cdot(\cos(\alpha) + \lambda\cdot\cos(2\cdot\alpha))$ $\qquad$ Equation of Overall Piston Acceleration

Graphic Illustration of the Variables

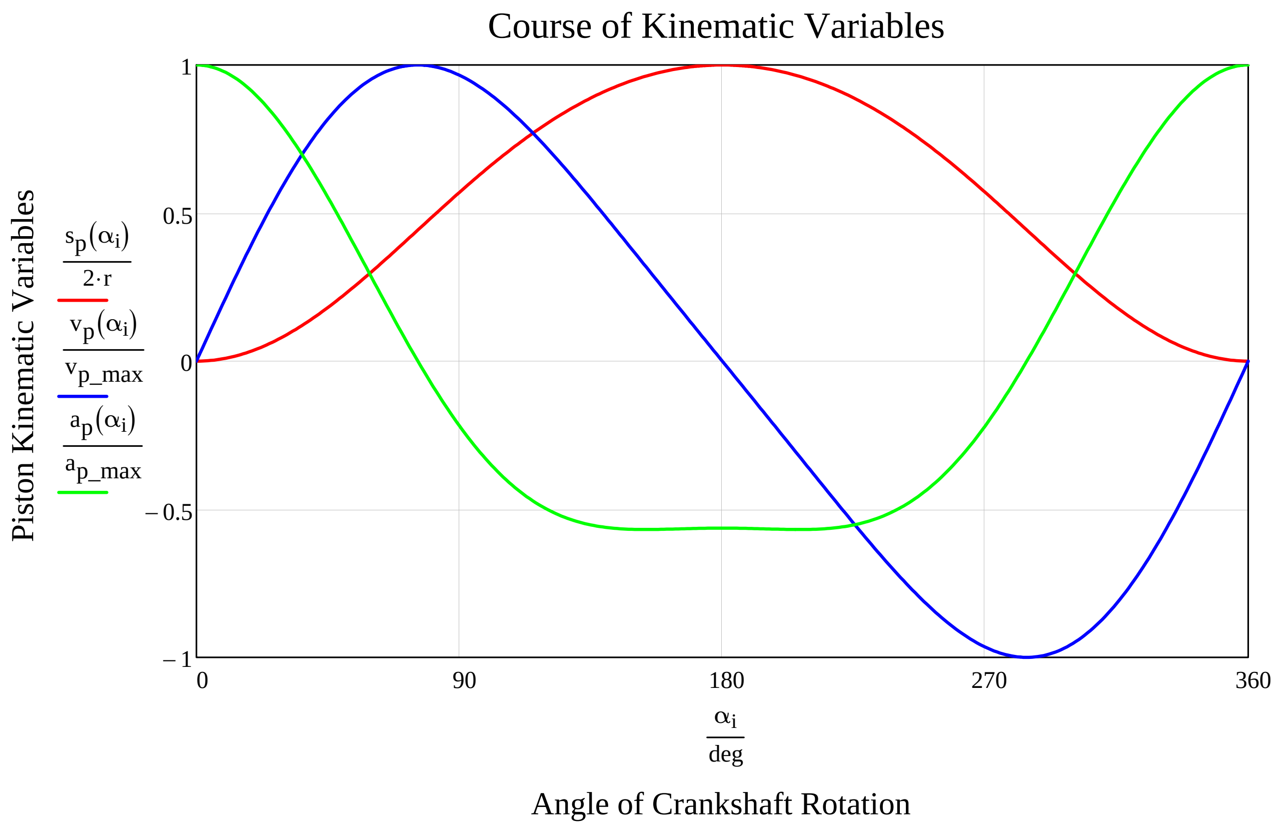
There will be used 1 degree of the crankshaft rotation as a step.

$i = 0..360$ $\qquad$ $\alpha_i = i\cdot deg$

Extremes:

$v_{p\_max} = \max(v_p(\alpha))$ $\qquad$ $v_{p\_max} = 9.632\frac{m}{s}$ $\qquad$ Maximum Speed

$a_{p\_max} = \max(a_p(\alpha))$ $\qquad$ $a_{p\_max} = 1843.428\frac{m}{s^2}$ $\qquad$ Maximum Acceleration



1.3 Indicator Diagram of the Cylinder Unit

$ip\_15 := READPRN("indiag\_1500rpm.dat")\cdot bar$ Reading of Indicated Pressures

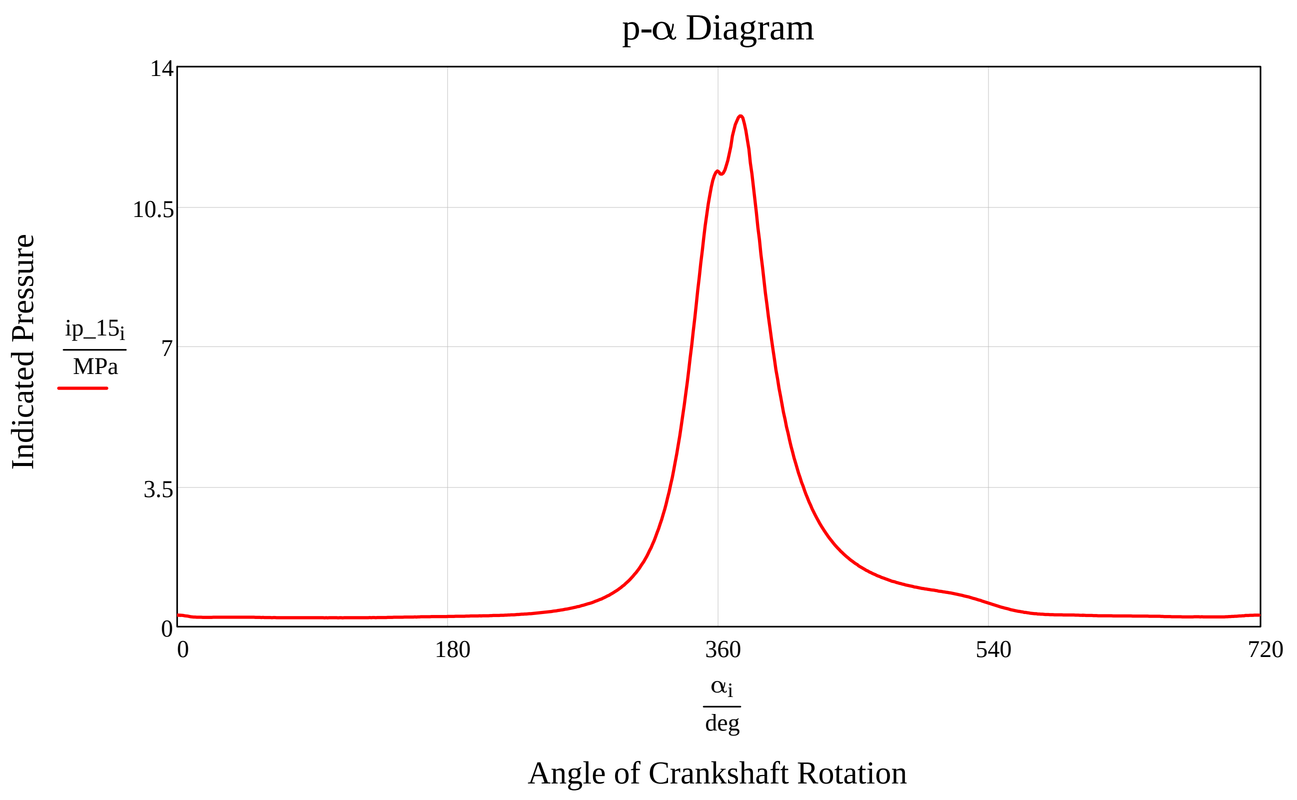
$i := 0..last(ip\_15)$ $\alpha_i := i\cdot deg$ Step of Crankshaft Rotation

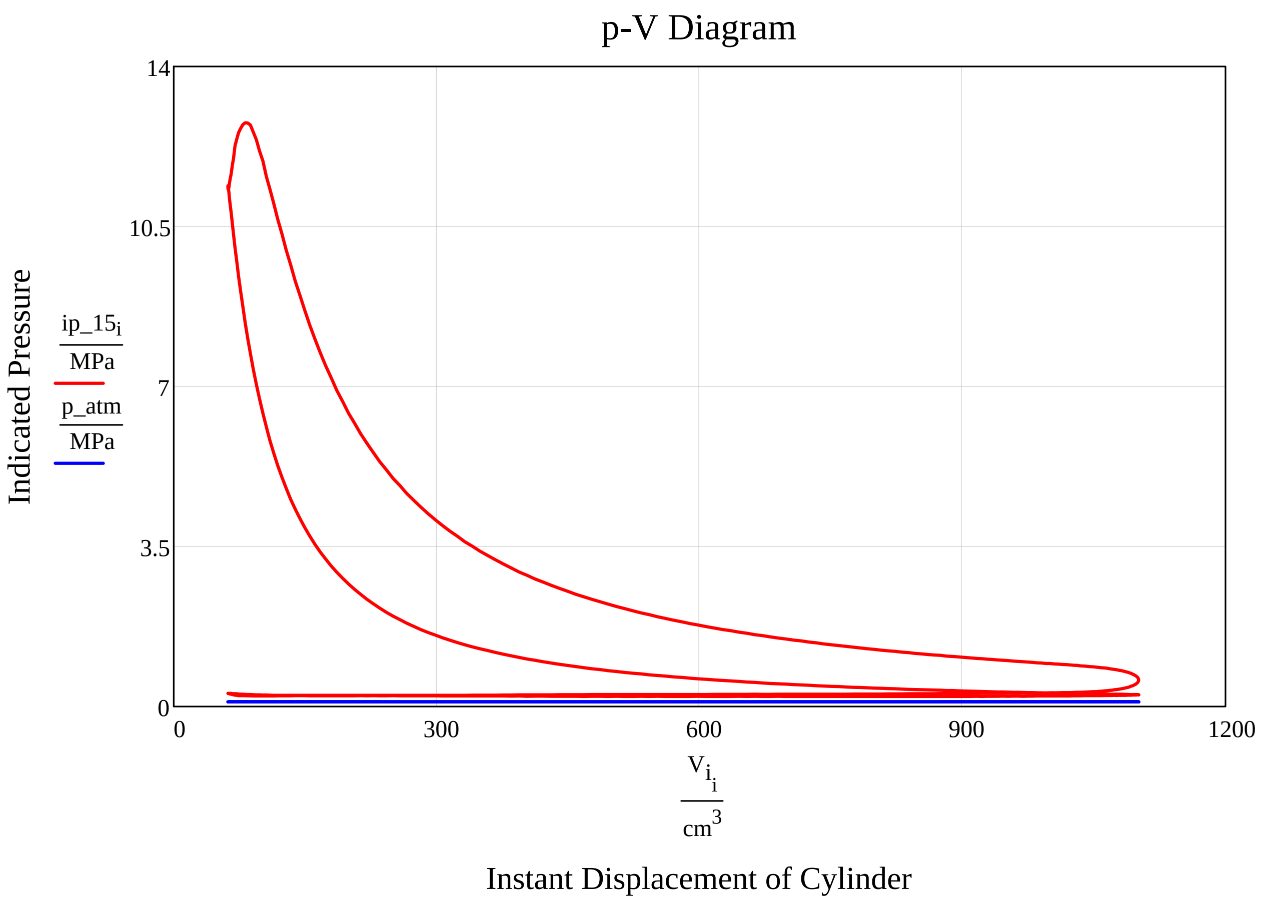
$p\_atm := 0.1\cdot MPa$ Atmosferic Pressure

$$V_i := V_c + \frac{\pi D^2}{4}\cdot s_p(\alpha)$$ Instant Displacement

$p_{i\_max} := max(ip\_15)$ $p_{i\_max} = 12.767\cdot MPa$ Maximum Indicated Pressure

$p_{i\_min} := min(ip\_15)$ $p_{i\_min} = 0.22\cdot MPa$ Minimum Indicated Pressure


p-α Diagram
Indicated Pressure
ip_15i
MPa
14
10.5
7
3.5
0
0
180
360
540
720
αi
deg
Angle of Crankshaft Rotation


p-V Diagram
Indicated Pressure
ip_15i
MPa
p_atm
MPa
14
10.5
7
3.5
0
0
300
600
900
1200
Vi
cm3
Instant Displacement of Cylinder

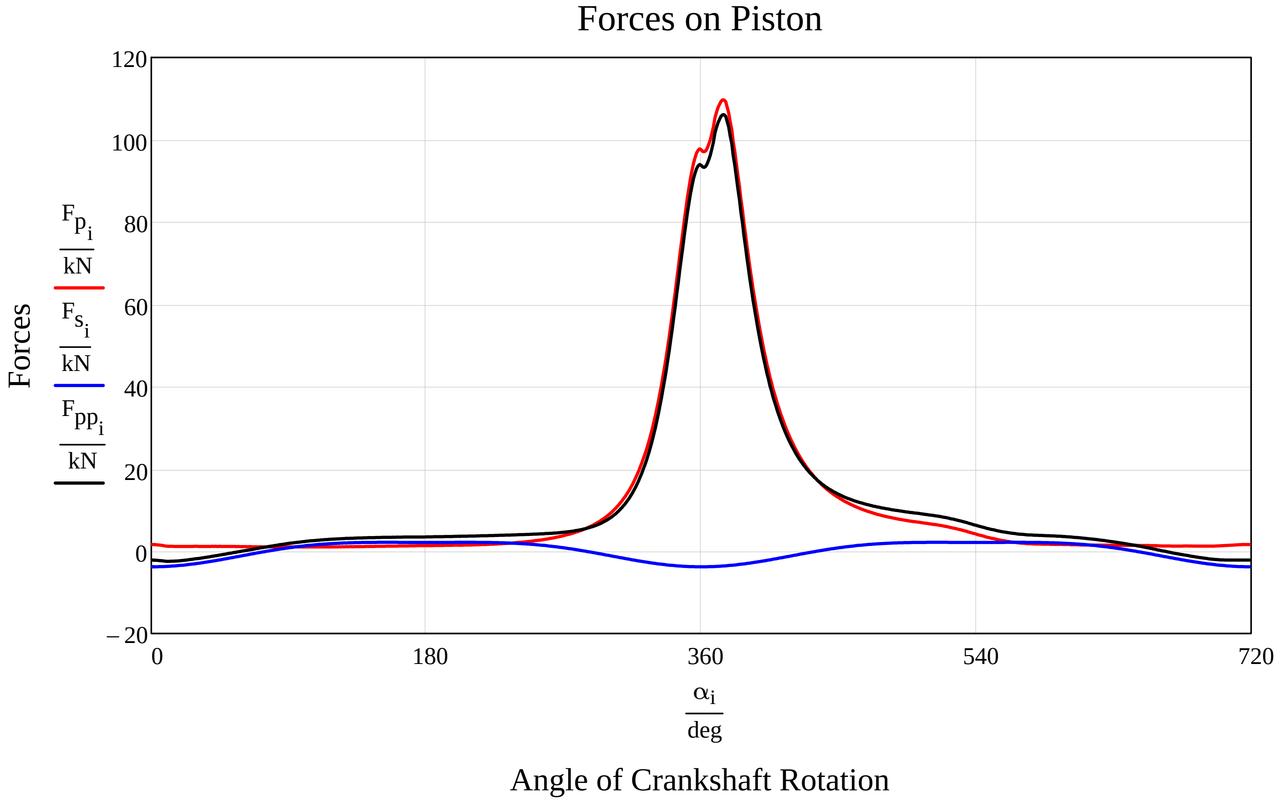
1.4 Force Transfer from Piston on Crankshaft

1.4.1 Forces on Piston Pin

$$F_{p_i} = \left(ip\_15_i - p\_atm\right)\cdot\frac{\pi\cdot D^2}{4}$$ Primary Force

$$F_{s_i} = -m_{pa}\cdot a_p\left(\alpha_i\right)$$ Secondary Force

$$F_{pp_i} = F_{p_i} + F_{s_i}$$ Overall Force on Piston Pin



Extremes:

$F_{p\_max} = \max\left(F_p\right)$ $\quad F_{p\_max} = 109.683\cdot kN$ Maximum Primary Force

$F_{s\_max} = \max\left(F_s\right)$ $\quad F_{s\_max} = 2.152\cdot kN$ Maximum Secondary Force

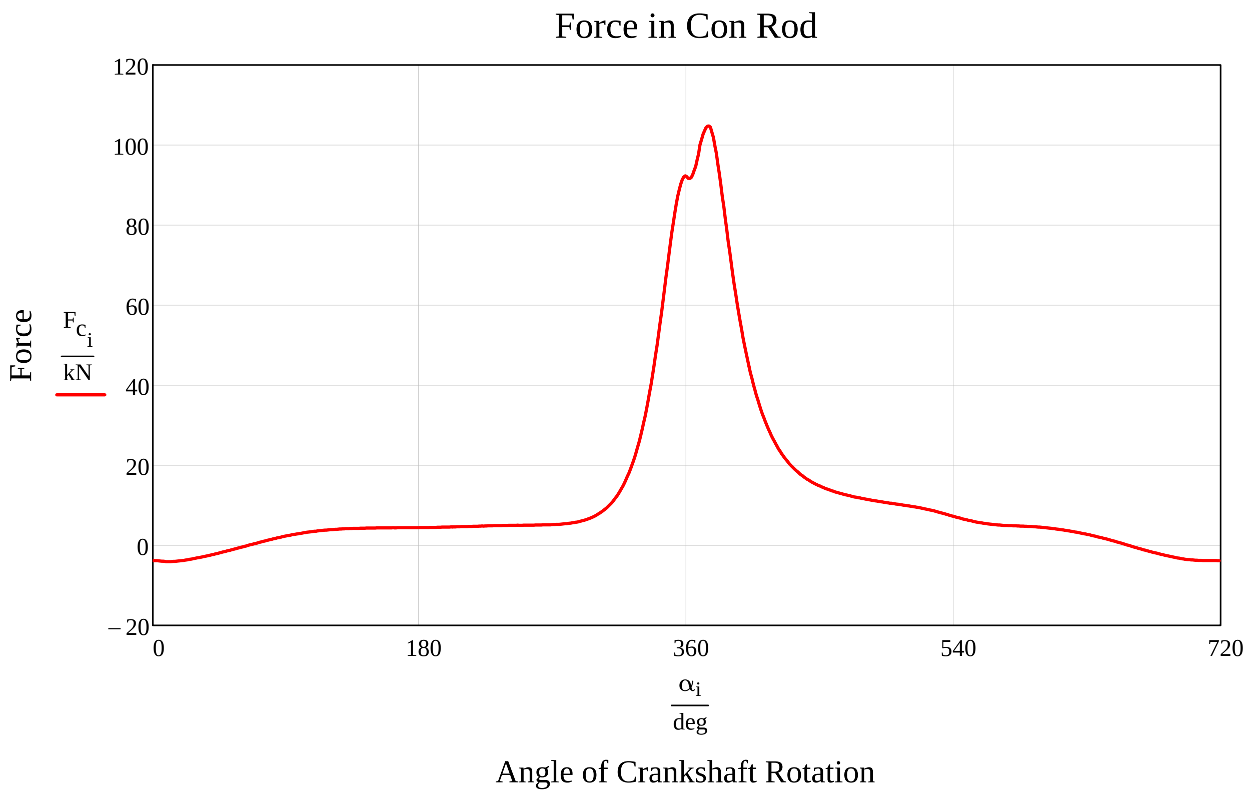
$F_{pp\_max} = \max\left(F_{pp}\right)$ $\quad F_{pp\_max} = 106.083\cdot kN$ Maximum Force on Piston Pin

1.4.2 Force in Con Rod

$\beta = \text{asin}(\lambda \cdot \sin(\alpha))$ Con Rod Declination Angle

$F_{cs_i} = -(m_{pa} + m_{crec}) \cdot a_p(\alpha_i)$ Inertia Force of all Reciprocating Masses

$F_{c_i} = \dfrac{F_{p_i} + F_{cs_i}}{\cos(\beta_i)}$ Force Transmitted trough Con Rod



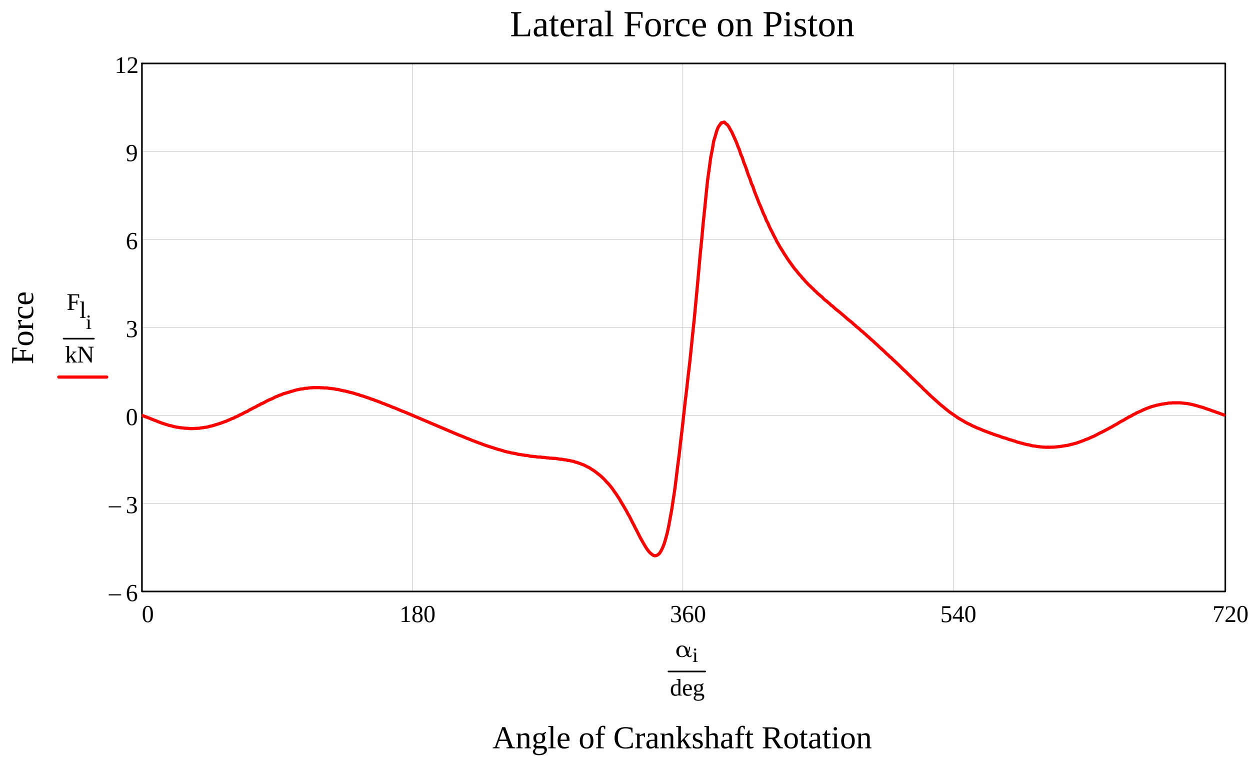
Extremes:

$F_{c\_max} = \max(F_c)$ $F_{c\_max} = 104.778 \cdot kN$ Maximum Force in Con Rod

$F_{c\_min} = \min(F_c)$ $F_{c\_min} = -4.093 \cdot kN$ Minimum Force in Con Rod

1.4.3 Lateral Force on Piston

$F_{l_i} = F_{c_i} \cdot \sin(\beta_i)$ Lateral Force on Piston



Extremes:

$F_{l\_max} = \max(F_l)$ $F_{l\_max} = 10.005 \cdot kN$ Maximum Lateral Force

$F_{l\_min} = \min(F_l)$ $F_{l\_min} = -4.783 \cdot kN$ Minimum Lateral Force

1.5 Torque of the One-Cylinder Unit

1.5.1 Forces on Crank Pin

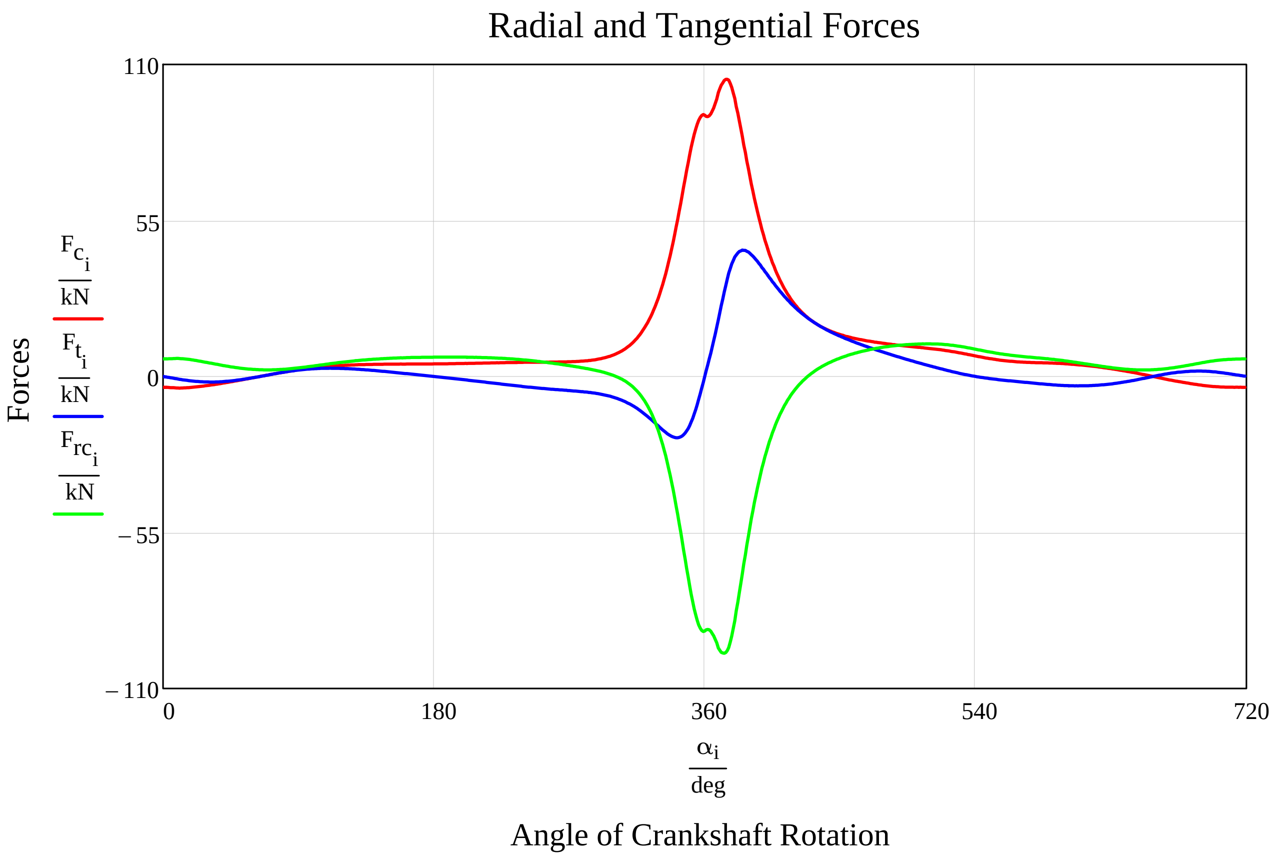
$F_{r_i} = -F_{c_i} \cdot \cos(\alpha_i + \beta_i)$ Radial Component of "Con Rod Force"

$F_{t_i} = F_{c_i} \cdot \sin(\alpha_i + \beta_i)$ Tangential Component of "Con Rod Force"

$F_{cent} = m_{crot} \cdot r \cdot \omega^2$ $F_{cent} = 2.371 \cdot kN$ Centrifugal Force of Rotating Mass Component of Con Rod

$F_{rc_i} = F_{r_i} + F_{cent}$ Overall Radial Force

$F_{cp_i} = \sqrt{\left(F_{t_i}\right)^2 + \left(F_{rc_i}\right)^2}$ Overall Force on Crank Pin



1.5.2 Torque on Crank Throw

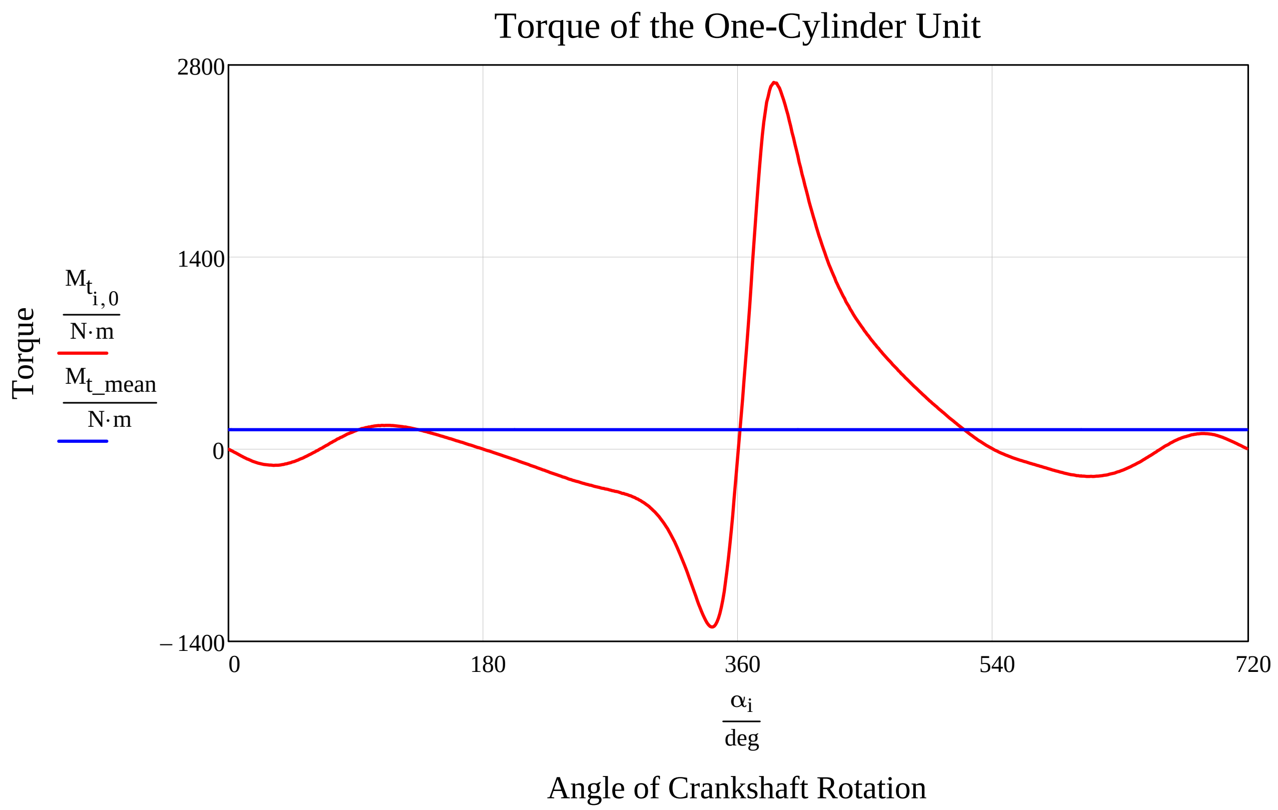
$M_{t_{i,0}} = F_{t_i} \cdot r$ Torque

Extremes:

$M_{t\_max} = \max\left(M_t^{\langle 0 \rangle}\right)$ $M_{t\_max} = 2673 \cdot N \cdot m$ Maximum Torque

$M_{t\_min} = \min\left(M_t^{\langle 0 \rangle}\right)$ $M_{t\_min} = -1295 \cdot N \cdot m$ Minimum Torque

$M_{t\_mean} = \text{mean}\left(M_t^{\langle 0 \rangle}\right)$ $M_{t\_mean} = 142 \cdot N \cdot m$ Average Torque



1.6 Torque Curves on Main Crank Pins and Crank Pins

$f\_order = (1\ \ 5\ \ 3\ \ 6\ \ 2\ \ 4)$ — Firing Order of the Engine

$M_{t_{i+720,0}} = M_{t_{i,0}}$ — Extension of Interval

1.6.1 Torque Curves of Individual Cylinders

$M_{t_{i,1}} = M_{t_{i,0}}$ — Torque of 1st Cylinder

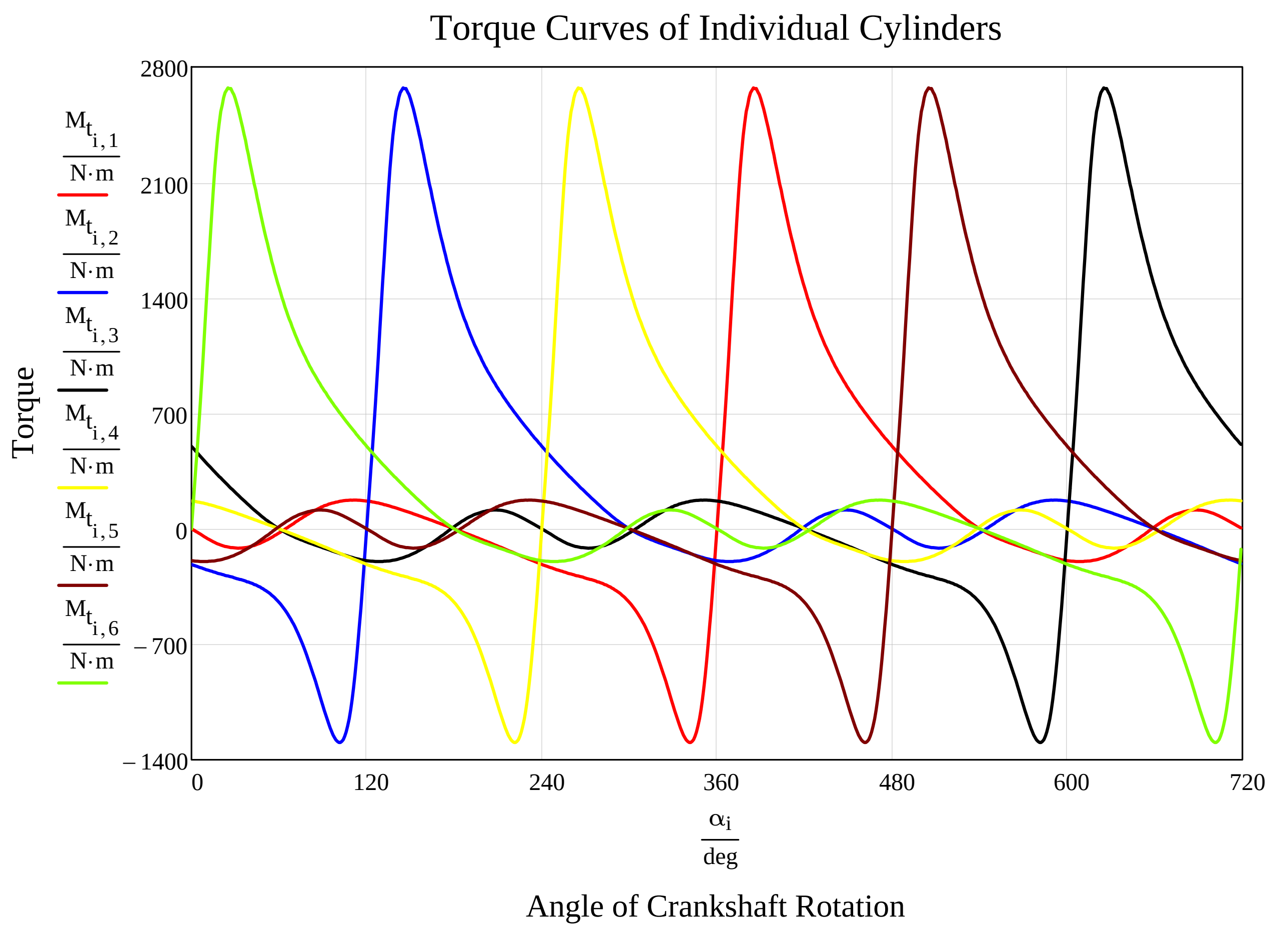
$M_{t_{i,2}} = M_{t_{i+240,0}}$ — Torque of 2nd Cylinder

$M_{t_{i,3}} = M_{t_{i+480,0}}$ — Torque of 3rd Cylinder

$M_{t_{i,4}} = M_{t_{i+120,0}}$ — Torque of 4th Cylinder

$M_{t_{i,5}} = M_{t_{i+600,0}}$ — Torque of 5th Cylinder

$M_{t_{i,6}} = M_{t_{i+360,0}}$ — Torque of 6th Cylinder



1.6.2 Torque on Main Crank Pins

With assumption there are no adjacent accessories and no friction in bearings.

$M_{tmj_{i,1}} = 0 \cdot N \cdot m$ Torque on 1st Main Crank Pin

$M_{tmj_{i,2}} = M_{tmj_{i,1}} + M_{t_{i,1}}$ Torque on 2nd Main Crank Pin

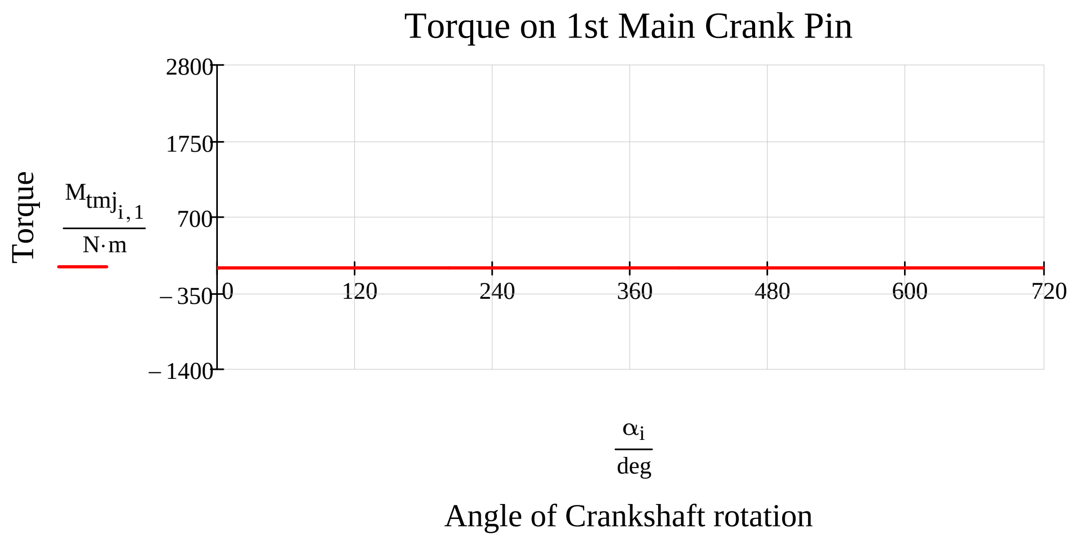
$M_{tmj_{i,3}} = M_{tmj_{i,2}} + M_{t_{i,2}}$ Torque on 3rd Main Crank Pin

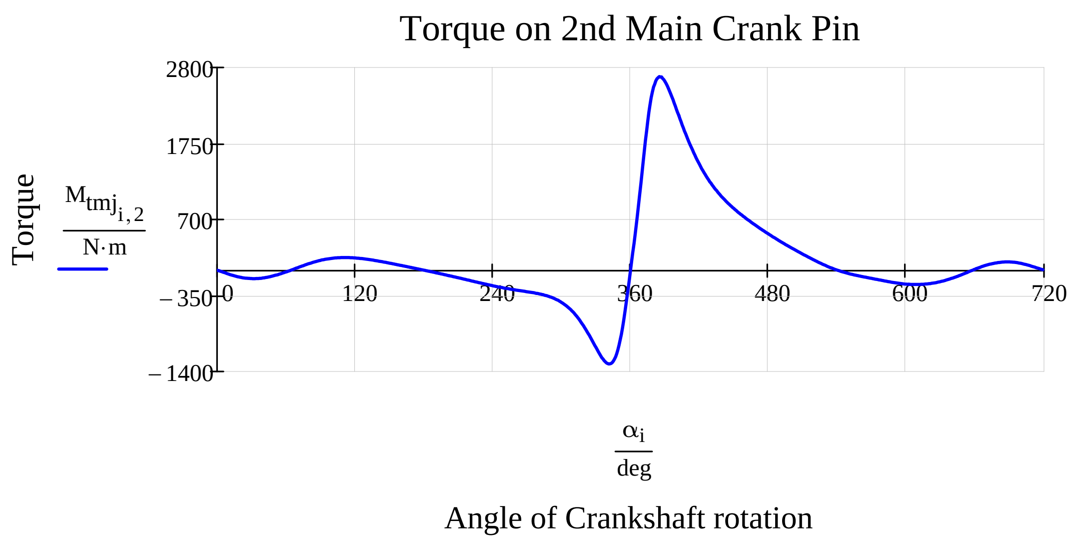
$M_{tmj_{i,4}} = M_{tmj_{i,3}} + M_{t_{i,3}}$ Torque on 4th Main Crank Pin

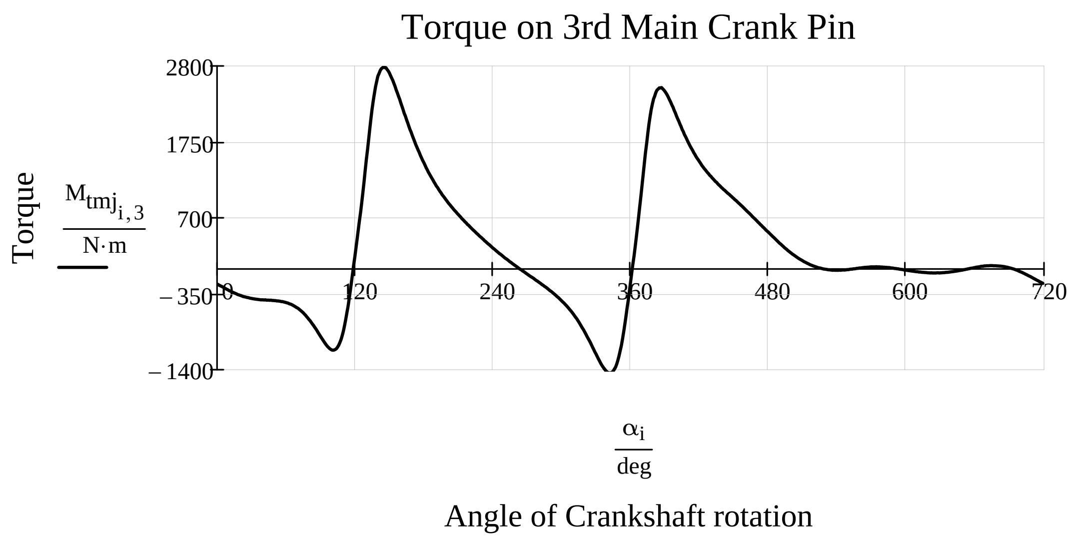
$M_{tmj_{i,5}} = M_{tmj_{i,4}} + M_{t_{i,4}}$ Torque on 5th Main Crank Pin

$M_{tmj_{i,6}} = M_{tmj_{i,5}} + M_{t_{i,5}}$ Torque on 6th Main Crank Pin

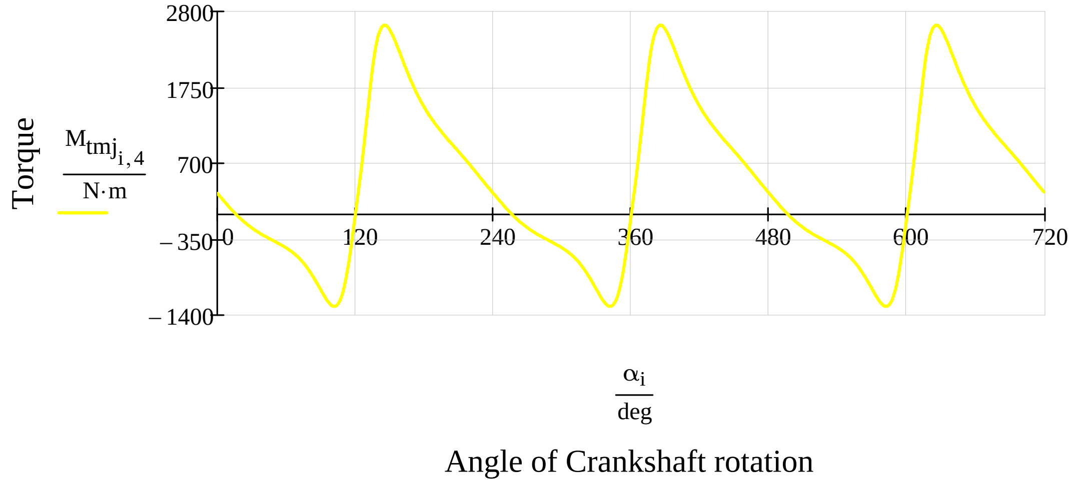
$M_{tmj_{i,7}} = M_{tmj_{i,6}} + M_{t_{i,6}}$ Torque on 7th Main Crank Pin


Torque on 1st Main Crank Pin
2800
1750
700
− 350
− 1400
0
120
240
360
480
600
720
Torque
Mtmji,1
N·m
αi
deg
Angle of Crankshaft rotation

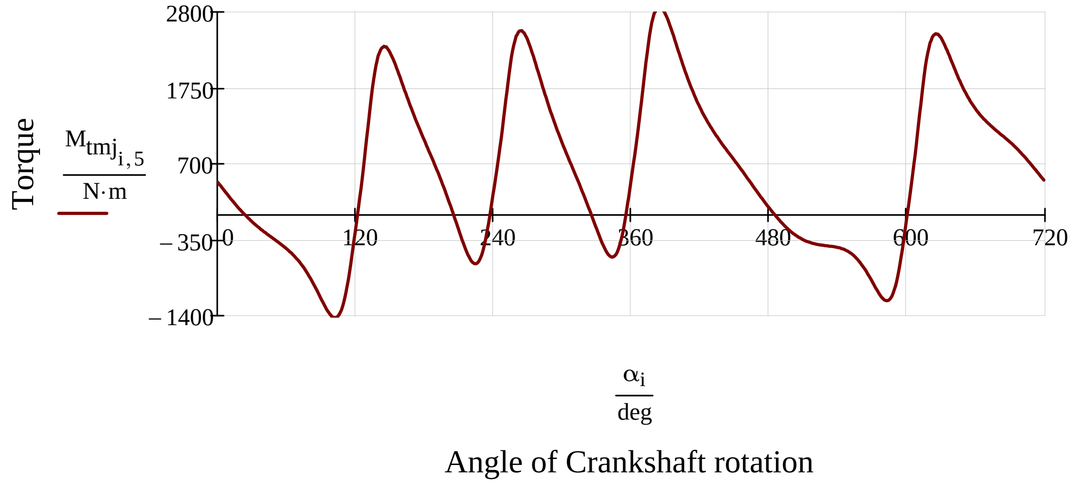

Torque on 2nd Main Crank Pin
2800
1750
700
− 350
− 1400
0
120
240
360
480
600
720
Torque
Mtmji,2
N·m
αi
deg
Angle of Crankshaft rotation


Torque on 3rd Main Crank Pin
2800
1750
700
− 350
− 1400
0
120
240
360
480
600
720
Torque
Mtmji,3
N·m
αi
deg
Angle of Crankshaft rotation

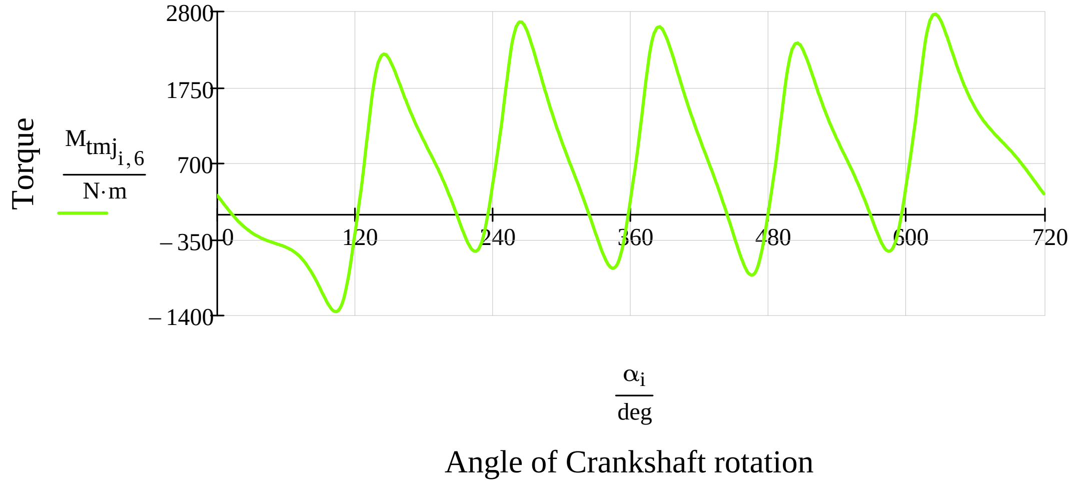
Torque on 4th Main Crank Pin



Torque on 5th Main Crank Pin



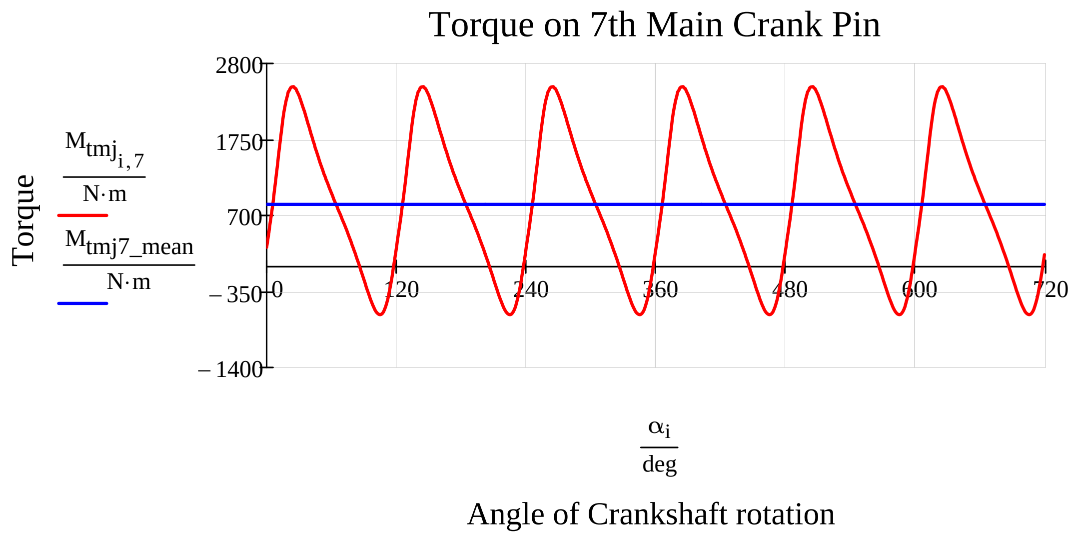
Torque on 6th Main Crank Pin



$$M_{tmj7\_mean} := \mathrm{mean}\left(M_{tmj}^{\langle 7 \rangle}\right)$$

$$M_{tmj7\_mean} = 853.447 \cdot N \cdot m$$

Mean Torque on Last Main Crank Pin



1.6.3 Torque on Crank Pins

With assumption there are no adjacent accessories and no friction in bearings.

$$M_{tcj_{i,1}} := \frac{1}{2} \cdot M_{t_{i,1}}$$
Torque on 1st Crank Pin

$$M_{tcj_{i,2}} := M_{tmj_{i,2}} + \frac{1}{2} \cdot M_{t_{i,2}}$$
Torque on 2nd Crank Pin

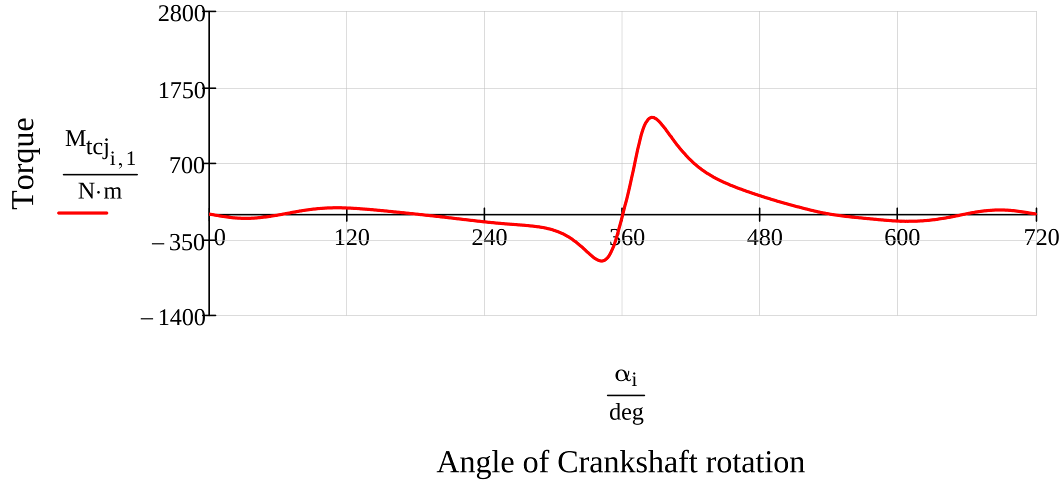
$$M_{tcj_{i,3}} := M_{tmj_{i,3}} + \frac{1}{2} \cdot M_{t_{i,3}}$$
Torque on 3rd Crank Pin

$$M_{tcj_{i,4}} := M_{tmj_{i,4}} + \frac{1}{2} \cdot M_{t_{i,4}}$$
Torque on 4th Crank Pin

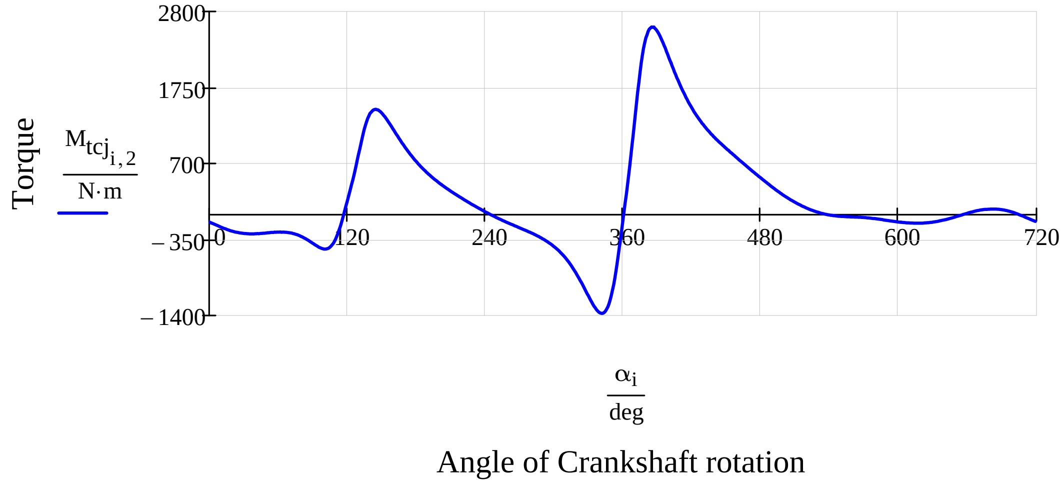
$$M_{tcj_{i,5}} := M_{tmj_{i,5}} + \frac{1}{2} \cdot M_{t_{i,5}}$$
Torque on 5th Crank Pin

$$M_{tcj_{i,6}} := M_{tmj_{i,6}} + \frac{1}{2} \cdot M_{t_{i,6}}$$
Torque on 6th Crank Pin

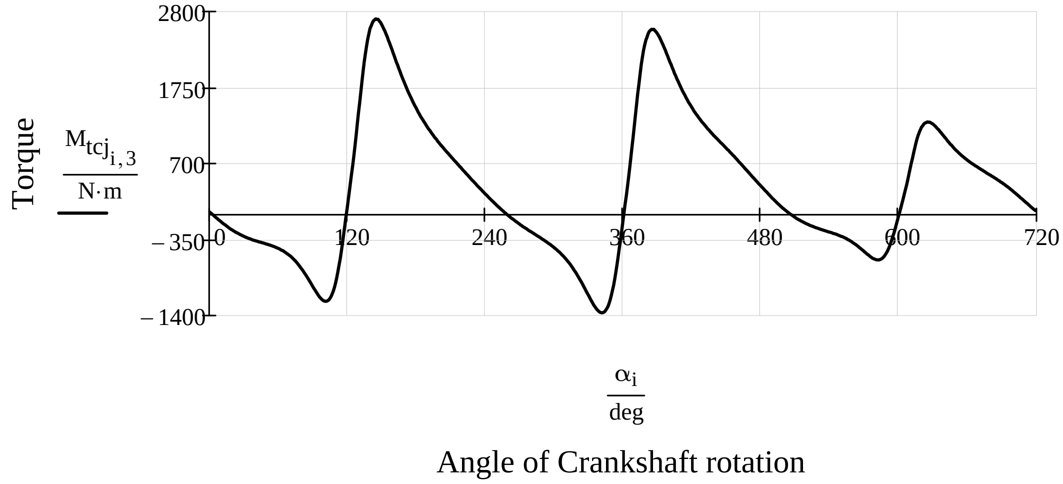
Torque on 1st Crank Pin

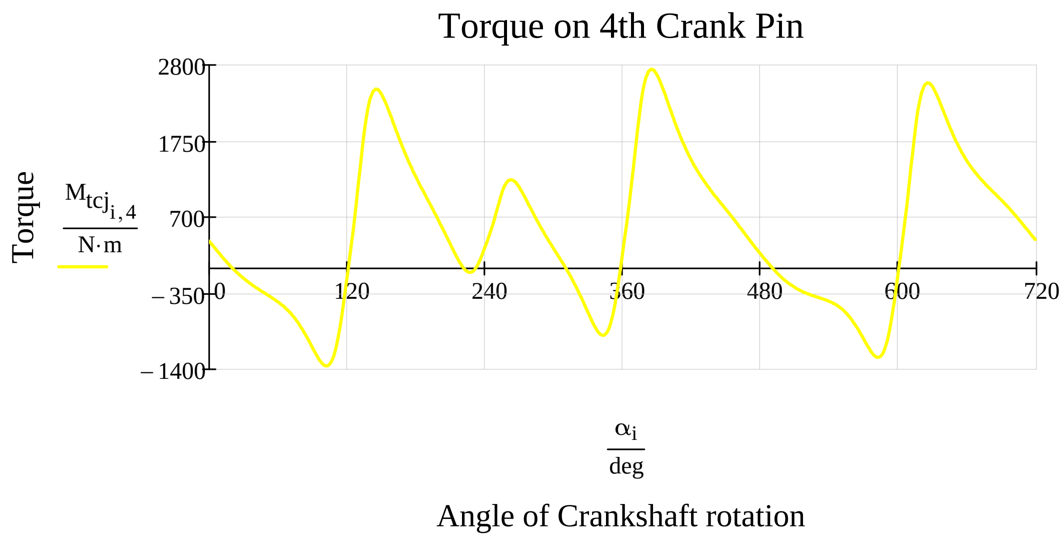


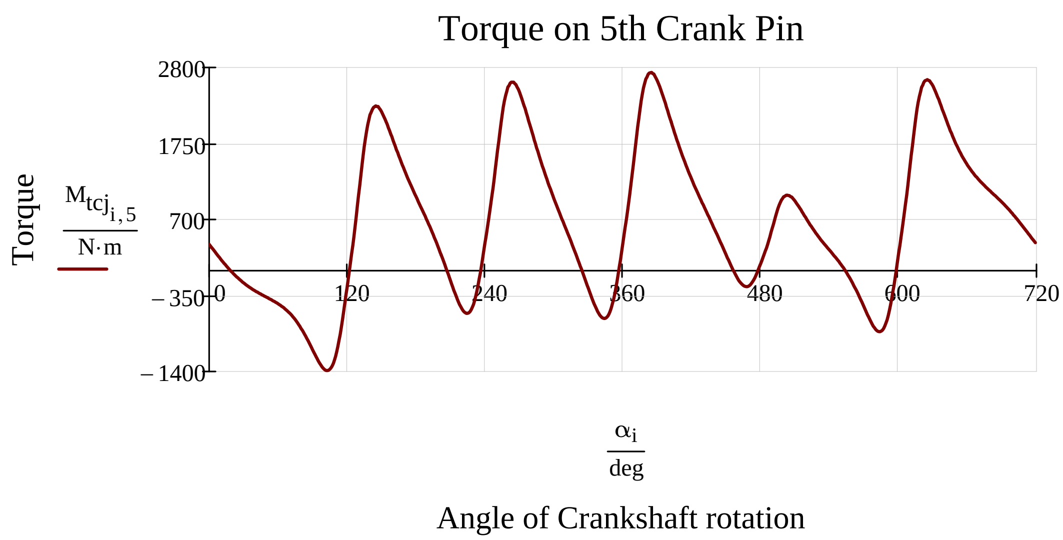
Torque on 2nd Crank Pin

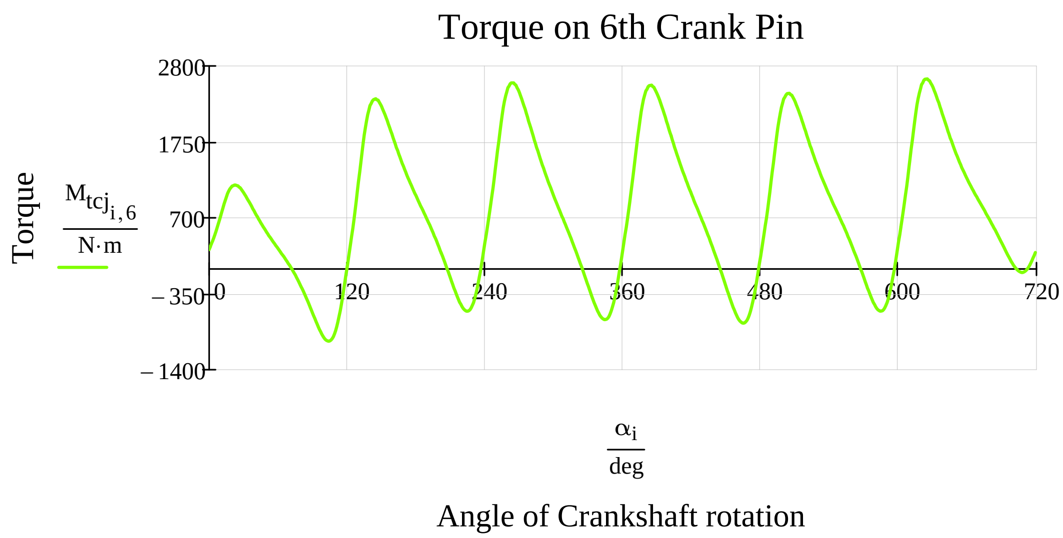


Torque on 3rd Crank Pin




Torque on 4th Crank Pin
2800
1750
700
– 350
– 1400
Torque
$\frac{M_{tcj_{i,4}}}{N \cdot m}$
0
120
240
360
480
600
720
$\frac{\alpha_i}{deg}$
Angle of Crankshaft rotation


Torque on 5th Crank Pin
2800
1750
700
– 350
– 1400
Torque
$\frac{M_{tcj_{i,5}}}{N \cdot m}$
0
120
240
360
480
600
720
$\frac{\alpha_i}{deg}$
Angle of Crankshaft rotation


Torque on 6th Crank Pin
2800
1750
700
– 350
– 1400
Torque
$\frac{M_{tcj_{i,6}}}{N \cdot m}$
0
120
240
360
480
600
720
$\frac{\alpha_i}{deg}$
Angle of Crankshaft rotation

1.6.4 Most Loaded Main Journal

$imj := 1..7$

$minM_{tmj_{imj}} := \min\left(M_{tmj}^{\langle imj \rangle}\right)$ Minimum Values of Torque on Main Journals

$maxM_{tmj_{imj}} := \max\left(M_{tmj}^{\langle imj \rangle}\right)$ Maximum Values of Torque on Main Journals

$diffM_{tmj} := maxM_{tmj} - minM_{tmj}$ Difference between Maximum and Minimum Torque on Main Journals

$\frac{maxM_{tmj_{imj}}}{N \cdot m} =$

0
2672.97
2785.414
2610.37
2874.883
2764.507
2478.584

$\frac{minM_{tmj_{imj}}}{N \cdot m} =$

0
-1295.473
-1446.353
-1279.07
-1429.185
-1345.36
-670.128

$\frac{diffM_{tmj_{imj}}}{N \cdot m} =$

0
3968.443
4231.767
3889.44
4304.068
4109.867
3148.712

1.6.5 Most Loaded Crank Journal

$icj := 1..6$

$minM_{tcj_{icj}} := \min\left(M_{tcj}^{\langle icj \rangle}\right)$ Minimum Values of Torque on Crank Journals

$maxM_{tcj_{icj}} := \max\left(M_{tcj}^{\langle icj \rangle}\right)$ Maximum Values of Torque on Crank Journals

$diffM_{tcj} := maxM_{tcj} - minM_{tcj}$ Difference between Maximum and Minimum Torque on Crank Journals

$\frac{maxM_{tcj_{icj}}}{N \cdot m} =$

1336.485
2585.448
2697.892
2742.626
2731.921
2621.546

$\frac{minM_{tcj_{icj}}}{N \cdot m} =$

-647.737
-1370.624
-1362.712
-1354.128
-1386.417
-1006.481

$\frac{diffM_{tcj_{icj}}}{N \cdot m} =$

1984.222
3956.072
4060.604
4096.754
4118.337
3628.027

Appendix 2 - Engine Balance

2.1 Rotating Inertia Force Balance

$m_{throw} = 3352 \cdot gm$ — Weight of Crank Throw

$e_{throw} = 41.829 \cdot mm$ — Excentricity of COG of Crank Throw

$F_{ro_0} = F_{cent} + m_{throw} \cdot e_{throw} \cdot \omega^2$ — Rotating Inertia Force of One Crank Throw

$F_{ro_1} = 2 \cdot \begin{pmatrix} F_{ro_0} \cdot \cos(90 \cdot deg) \\ F_{ro_0} \cdot \sin(90 \cdot deg) \end{pmatrix}$ — Rotating Inertia Force of 1st and 6th Crank Throw

$F_{ro_2} = 2 \cdot \begin{pmatrix} F_{ro_0} \cdot \cos(210 \cdot deg) \\ F_{ro_0} \cdot \sin(210 \cdot deg) \end{pmatrix}$ — Rotating Inertia Force of 2nd and 5th Crank Throw

$F_{ro_3} = 2 \cdot \begin{pmatrix} F_{ro_0} \cdot \cos(330 \cdot deg) \\ F_{ro_0} \cdot \sin(330 \cdot deg) \end{pmatrix}$ — Rotating Inertia Force of 3rd and 4th Crank Throw

$\sum_{j=1}^{3} F_{ro_j} = \begin{pmatrix} -0 \\ -0 \end{pmatrix} N$ — Sum of Rotating Inertia Forces

2.2 Primary Reciprocating Inertia Force Balance

$F_{I1}(\alpha) = -2 \cdot (m_{pa} + m_{crec}) \cdot r \cdot \omega^2 \cdot \cos(\alpha)$ — Primary Reciprocating Inertia Force of 1st and 6th Crank Throw

$F_{I2}(\alpha) = -2 \cdot (m_{pa} + m_{crec}) \cdot r \cdot \omega^2 \cdot \cos(\alpha + 120 \cdot deg)$ — Primary Reciprocating Inertia Force of 2nd and 5th Crank Throw

$F_{I3}(\alpha) = -2 \cdot (m_{pa} + m_{crec}) \cdot r \cdot \omega^2 \cdot \cos(\alpha + 240 \cdot deg)$ — Primary Reciprocating Inertia Force of 3rd and 4th Crank Throw

$F_I(\alpha) = F_{I1}(\alpha) + F_{I2}(\alpha) + F_{I3}(\alpha)$

$\max(F_I(\alpha)) = 0 \cdot kN$ — Sum of Primary Reciprocating Inertia Forces

2.3 Secondary Reciprocating Inertia Force Balance

$F_{II1}(\alpha) = -\lambda\cdot 2\cdot\left(m_{pa} + m_{crec}\right)\cdot r\cdot\omega^{2}\cdot\cos(2\cdot\alpha)$ — Secondary Reciprocating Inertia Force of 1st and 6th Crank Throw

$F_{II2}(\alpha) = -\lambda\cdot 2\cdot\left(m_{pa} + m_{crec}\right)\cdot r\cdot\omega^{2}\cdot\cos[2\cdot(\alpha + 120\cdot deg)]$ — Secondary Reciprocating Inertia Force of 2nd and 5th Crank Throw

$F_{II3}(\alpha) = -\lambda\cdot 2\cdot\left(m_{pa} + m_{crec}\right)\cdot r\cdot\omega^{2}\cdot\cos[2\cdot(\alpha + 240\cdot deg)]$ — Secondary Reciprocating Inertia Force of 3rd and 4th Crank Throw

$F_{II}(\alpha) = F_{II1}(\alpha) + F_{II2}(\alpha) + F_{II3}(\alpha)$

$\max\left(F_{II}(\alpha)\right) = 0\cdot kN$ — Sum of Secondary Reciprocating Inertia Forces

2.4 Moment from Rotating Inertia Force

$M_{ro_1} = \frac{2.5}{2}\cdot F_{ro_1}\cdot h$ — Moment from Rotating Inertia Force of 1st Crank Throw

$M_{ro_2} = \frac{1.5}{2}\cdot F_{ro_2}\cdot h$ — Moment from Rotating Inertia Force of 2nd Crank Throw

$M_{ro_3} = \frac{0.5}{2}\cdot F_{ro_3}\cdot h$ — Moment from Rotating Inertia Force of 3rd Crank Throw

$M_{ro_4} = -\frac{0.5}{2}\cdot F_{ro_3}\cdot h$ — Moment from Rotating Inertia Force of 4th Crank Throw

$M_{ro_5} = -\frac{1.5}{2}\cdot F_{ro_2}\cdot h$ — Moment from Rotating Inertia Force of 5th Crank Throw

$M_{ro_6} = -\frac{2.5}{2}\cdot F_{ro_1}\cdot h$ — Moment from Rotating Inertia Force of 6th Crank Throw

$\sum_{k=1}^{6} M_{ro_k} = \begin{pmatrix} -0 \\ -0 \end{pmatrix}\cdot N\cdot m$ — Sum of Moments from Rotating Inertia Forces

2.5 Moment from Primary Reciprocating Inertia Force

$M_{I1}(\alpha) = \frac{2.5}{2} \cdot F_{I1}(\alpha) \cdot h$ — Moment from Primary Reciprocating Inertia Force of 1st Crank Throw

$M_{I2}(\alpha) = \frac{1.5}{2} \cdot F_{I2}(\alpha) \cdot h$ — Moment from Primary Reciprocating Inertia Force of 2nd Crank Throw

$M_{I3}(\alpha) = \frac{0.5}{2} \cdot F_{I3}(\alpha) \cdot h$ — Moment from Primary Reciprocating Inertia Force of 3rd Crank Throw

$M_{I4}(\alpha) = -\frac{0.5}{2} \cdot F_{I3}(\alpha) \cdot h$ — Moment from Primary Reciprocating Inertia Force of 4th Crank Throw

$M_{I5}(\alpha) = -\frac{1.5}{2} \cdot F_{I2}(\alpha) \cdot h$ — Moment from Primary Reciprocating Inertia Force of 5th Crank Throw

$M_{I6}(\alpha) = -\frac{2.5}{2} \cdot F_{I1}(\alpha) \cdot h$ — Moment from Primary Reciprocating Inertia Force of 6th Crank Throw

$$M_I(\alpha) = M_{I1}(\alpha) + M_{I2}(\alpha) + M_{I3}(\alpha) + M_{I4}(\alpha) + M_{I5}(\alpha) + M_{I6}(\alpha)$$

$\max\left(M_I(\alpha)\right) = 0 \cdot N \cdot m$ — Sum of Moments from Primary Reciprocating Inertia Forces

2.6 Moment from Secondary Reciprocating Inertia Force

$M_{II1}(\alpha) = \frac{2.5}{2} \cdot F_{II1}(\alpha) \cdot h$ — Moment from Secondary Reciprocating Inertia Force of 1st Crank Throw

$M_{II2}(\alpha) = \frac{1.5}{2} \cdot F_{II2}(\alpha) \cdot h$ — Moment from Secondary Reciprocating Inertia Force of 2nd Crank Throw

$M_{II3}(\alpha) = \frac{0.5}{2} \cdot F_{II3}(\alpha) \cdot h$ — Moment from Secondary Reciprocating Inertia Force of 3rd Crank Throw

$M_{II4}(\alpha) = -\frac{0.5}{2} \cdot F_{II3}(\alpha) \cdot h$ — Moment from Secondary Reciprocating Inertia Force of 4th Crank Throw

$M_{II5}(\alpha) = -\frac{1.5}{2} \cdot F_{II2}(\alpha) \cdot h$ — Moment from Secondary Reciprocating Inertia Force of 5th Crank Throw

$M_{II6}(\alpha) = -\frac{2.5}{2} \cdot F_{II1}(\alpha) \cdot h$ — Moment from Secondary Reciprocating Inertia Force of 6th Crank Throw

$$M_{II}(\alpha) = M_{II1}(\alpha) + M_{II2}(\alpha) + M_{II3}(\alpha) + M_{II4}(\alpha) + M_{II5}(\alpha) + M_{II6}(\alpha)$$

$\max\left(M_{II}(\alpha)\right) = 0 \cdot N \cdot m$ Sum of Moments from Secondary Reciprocating Inertia Forces

2.7 Reaction Force in 4th Main Journal

$R_4 = 2 \cdot F_{ro_0} \cdot 0.5$ $R_4 = 5.739 \cdot kN$ Reaction in 4th Main Journal without Counterweights

$m_{cw} = 3410 \cdot gm$ Weight of Counterweight Couple

$e_{cw} = 48.33 \cdot mm$ Excentricity of COG of Counterweight Couple

$R_{4cw} = 2 \cdot \left(F_{ro_0} \cdot 0.5 - m_{cw} \cdot e_{cw} \cdot \omega^2 \cdot 0.5\right)$ $R_{4cw} = 1.78 \cdot kN$ Reaction in 4th Main Journal with Counterweights

$\eta_{red} = 1 - \dfrac{R_{4cw}}{R_4}$ $\eta_{red} = 68.982 \cdot \%$ Reduction of Reaction Force

Appendix 3 - Torsional Vibration without Torsional Damper

3.1 Equivalent Crankshaft System

3.1.1 Moments of Inertia

$I_{end\_fr} = 1.288 \cdot 10^{-3} \cdot kg \cdot m^2$ Moment of Inertia of Front End

$I_{pull} = 25.65 \cdot 10^{-3} \cdot kg \cdot m^2$ Moment of Inertia of Pulley

$I_{throw} = 26.095 \cdot 10^{-3} \cdot kg \cdot m^2$ Moment of Inertia of Crank Throw

$I_{end\_rr} = 4.327 \cdot 10^{-3} \cdot kg \cdot m^2$ Moment of Inertia of Rear End

$I_{flyw} = 872.949 \cdot 10^{-3} \cdot kg \cdot m^2$ Moment of Inertia of Flywheel

$I_{rot} = m_{crot} \cdot r^2$ $I_{rot} = 5.922 \cdot 10^{-3} \cdot kg \cdot m^2$ Moment of Inertia of Rotational Mass of Con Rod

$$I_{rec} = \left(m_{pa} + m_{crec}\right) \cdot \left(\frac{1}{2} + \frac{\lambda^2}{8}\right) \cdot r^2$$

$I_{rec} = 5.433 \cdot 10^{-3} \cdot kg \cdot m^2$ Moment of Inertia of Reciprocating Masses

3.1.2 Masses Reduction

$I_0 = I_{end\_fr} + I_{pull}$ $I_0 = 26.938 \cdot 10^{-3} \cdot kg \cdot m^2$ Reduced Moment of Inertia of Front End

$I_1 = I_{throw} + I_{rot} + I_{rec}$ $I_1 = 37.45 \cdot 10^{-3} \cdot kg \cdot m^2$ Reduced Moment of Inertia of 1st Crank Throw

$I_2 = I_{throw} + I_{rot} + I_{rec}$ $I_2 = 37.45 \cdot 10^{-3} \cdot kg \cdot m^2$ Reduced Moment of Inertia of 2nd Crank Throw

$I_3 = I_{throw} + I_{rot} + I_{rec}$ $I_3 = 37.45 \cdot 10^{-3} \cdot kg \cdot m^2$ Reduced Moment of Inertia of 3rd Crank Throw

$I_4 = I_{throw} + I_{rot} + I_{rec}$ $I_4 = 37.45 \cdot 10^{-3} \cdot kg \cdot m^2$ Reduced Moment of Inertia of 4th Crank Throw

$I_5 = I_{throw} + I_{rot} + I_{rec}$ $I_5 = 37.45 \cdot 10^{-3} \cdot kg \cdot m^2$ Reduced Moment of Inertia of 5th Crank Throw

$I_6 = I_{throw} + I_{rot} + I_{rec}$ $I_6 = 37.45 \cdot 10^{-3} \cdot kg \cdot m^2$ Reduced Moment of Inertia of 6th Crank Throw

$I_7 = I_{end\_rr} + I_{flyw}$ $I_7 = 877.276 \cdot 10^{-3} \cdot kg \cdot m^2$ Reduced Moment of Inertia of Rear End

$I_8 = \sum_{n=0}^{7} I_n$ $I_8 = 1.129 \cdot kg \cdot m^2$ Moment of Inertia of Whole Torsional System

3.1.3 Length Reduction

$d_{mj} = 88 \cdot mm$ Main Crank Pin Diameter

$l_{mj} = 44 \cdot mm$ Main Crank Pin Length

$l_{pull} = 33 \cdot mm$ Pulley Pin Length

$d_{pull} = 40 \cdot mm$ Pulley Pin Diameter

$d_{rel} = 30 \cdot mm$ Relieving Holes Diameter

$d_{cj} = 66 \cdot mm$ Crank Pin Diameter

$l_{cj} = 40 \cdot mm$ Crank Pin Length

$l_{web} = 18 \cdot mm$ Crank Web Length

$b_{web} = 106 \cdot mm$ Crank Web Width

$l_{web\_dist} = 56 \cdot mm$ Distance between Webs of One Crank Throw

$d_{flyw} = 80 \cdot mm$ Flywheel Flange Diameter

$l_{flyw} = 35 \cdot mm$ Flywheel Flange Length

$E_m = 210 \cdot GPa$ Young's modulus

$G_m = 81 \cdot GPa$ Shear modulus

$$l_{throw\_red} = l_{mj} + l_{cj}\cdot\frac{d_{mj}^{4}}{d_{cj}^{4} - d_{rel}^{4}} + 0.7\cdot l_{web}\cdot\frac{d_{mj}^{4}\cdot\left(b_{web}^{2} + r^{2}\right)}{b_{web}^{3}\cdot r^{3}} + 2.36\cdot\frac{G_m}{E_m}\cdot r\cdot\frac{d_{mj}^{4}}{l_{web}\cdot b_{web}^{3}}$$

$l_{throw\_red} = 0.372\,m$ Reduced Crank Throw Length

$$l_{end\_fr\_red} = l_{pull}\cdot\left(\frac{d_{mj}}{d_{pull}}\right)^{4} + \frac{l_{mj}}{2} + \frac{l_{throw\_red}}{2}$$

$l_{end\_fr\_red} = 0.981\cdot m$ Reduced Front End Length

$$l_{end\_rr\_red} = l_{flyw}\cdot\left(\frac{d_{mj}}{d_{flyw}}\right)^{4} + \frac{l_{mj}}{2} + \frac{l_{throw\_red}}{2}$$

$l_{end\_rr\_red} = 0.259\,m$ Reduced Rear End Length

3.1.4 Torsional Stiffness

$I_p = \frac{\pi\cdot d_{mj}^{4}}{32}$ $I_p = 5.887\times10^{-6}\,m^{4}$ Polar Moment of Inertia of Reduced Shaft

$c_0 = \frac{G_m\cdot I_p}{l_{end\_fr\_red}}$ $c_0 = 4.86\times10^{5}\cdot N\cdot m\cdot rad^{-1}$ Front End Torsional Stiffness

$c_1 = \frac{G_m\cdot I_p}{l_{throw\_red}}$ $c_1 = 1.281\times10^{6}\cdot N\cdot m\cdot rad^{-1}$ 1st Crank Throw Torsional Stiffness

$c_2 = \frac{G_m\cdot I_p}{l_{throw\_red}}$ $c_2 = 1.281\times10^{6}\cdot N\cdot m\cdot rad^{-1}$ 2nd Crank Throw Torsional Stiffness

$c_3 = \frac{G_m\cdot I_p}{l_{throw\_red}}$ $c_3 = 1.281\times10^{6}\cdot N\cdot m\cdot rad^{-1}$ 3rd Crank Throw Torsional Stiffness

$c_4 = \frac{G_m\cdot I_p}{l_{throw\_red}}$ $c_4 = 1.281\times10^{6}\cdot N\cdot m\cdot rad^{-1}$ 4th Crank Throw Torsional Stiffness

$$c_5 = \frac{G_m \cdot I_p}{l_{throw\_red}} \qquad c_5 = 1.281 \times 10^6 \cdot N \cdot m \cdot rad^{-1}$$ 5th Crank Throw Torsional Stiffness

$$c_6 = \frac{G_m \cdot I_p}{l_{throw\_red}} \qquad c_6 = 1.281 \times 10^6 \cdot N \cdot m \cdot rad^{-1}$$ 6th Crank Throw Torsional Stiffness

$$c_7 = \frac{G_m \cdot I_p}{l_{end\_rr\_red}} \qquad c_7 = 1.838 \times 10^6 \cdot N \cdot m \cdot rad^{-1}$$ Rear End Torsional Stiffness

3.2 Free Torsional Vibration

$$M = \begin{pmatrix} I_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_8 \end{pmatrix}$$

Diagonal Mass Matrix

$$C = \begin{pmatrix} c_0 & -c_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -c_0 & c_0 + c_1 & -c_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -c_1 & c_1 + c_2 & -c_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -c_2 & c_2 + c_3 & -c_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c_3 & c_3 + c_4 & -c_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -c_4 & c_4 + c_5 & -c_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -c_5 & c_5 + c_6 & -c_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -c_6 & c_6 + c_7 & -c_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_7 & c_7 \end{pmatrix}$$

Diagonal Stiffness Matrix

$A = M^{-1} \cdot C$ Conversion into Eigenvalues Problem

$\chi = \text{eigenvals}(A)$

$\Omega = \overrightarrow{\sqrt{\chi}}$ Vector of Inherent Frequencies

$$\chi = \begin{pmatrix} 129292034.167 \\ 108608206.718 \\ 79624781.131 \\ 49564242.036 \\ 26948675.394 \\ 12700790.473 \\ 3983318.747 \\ 1599040.583 \\ -0 \end{pmatrix} \cdot s^{-2} \qquad \Omega = \begin{pmatrix} 11370.666 \\ 10421.526 \\ 8923.272 \\ 7040.188 \\ 5191.211 \\ 3563.817 \\ 1995.825 \\ 1264.532 \\ 0i \end{pmatrix} \cdot s^{-1}$$

$w = \text{eigenvecs}(A)$ Modal Matrix

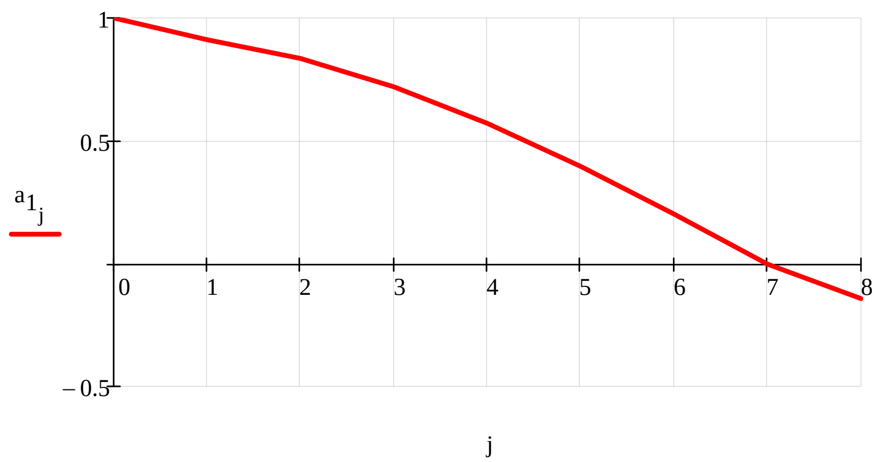
$$w^T = \begin{pmatrix} -0.027 & 0.164 & -0.384 & 0.52 & -0.542 & 0.445 & -0.252 & 0.003 & -0 \\ -0.063 & 0.317 & -0.546 & 0.325 & 0.163 & -0.517 & 0.445 & -0.006 & 0 \\ 0.13 & -0.445 & 0.373 & 0.323 & -0.479 & -0.165 & 0.533 & -0.01 & 0 \\ -0.291 & 0.509 & 0.075 & -0.468 & -0.332 & 0.285 & 0.489 & -0.016 & 0.001 \\ 0.621 & -0.307 & -0.417 & -0.199 & 0.176 & 0.412 & 0.324 & -0.02 & 0.001 \\ -0.691 & -0.205 & 0.056 & 0.296 & 0.426 & 0.398 & 0.222 & -0.037 & 0.005 \\ -0.565 & -0.44 & -0.342 & -0.203 & -0.041 & 0.126 & 0.278 & 0.398 & -0.275 \\ -0.528 & -0.481 & -0.441 & -0.38 & -0.301 & -0.209 & -0.106 & 0.001 & 0.075 \\ 0.333 & 0.333 & 0.333 & 0.333 & 0.333 & 0.333 & 0.333 & 0.333 & 0.333 \end{pmatrix}$$

$j = 0..8$

$a_{1_j} = \frac{w_{j,7}}{w_{0,7}}$ First Shape of Inherent Torsional Deviations

$a_1^T = (1 \;\; 0.911 \;\; 0.835 \;\; 0.72 \;\; 0.571 \;\; 0.395 \;\; 0.201 \;\; -0.003 \;\; -0.142)$

First Shape of Inherent Oscillation

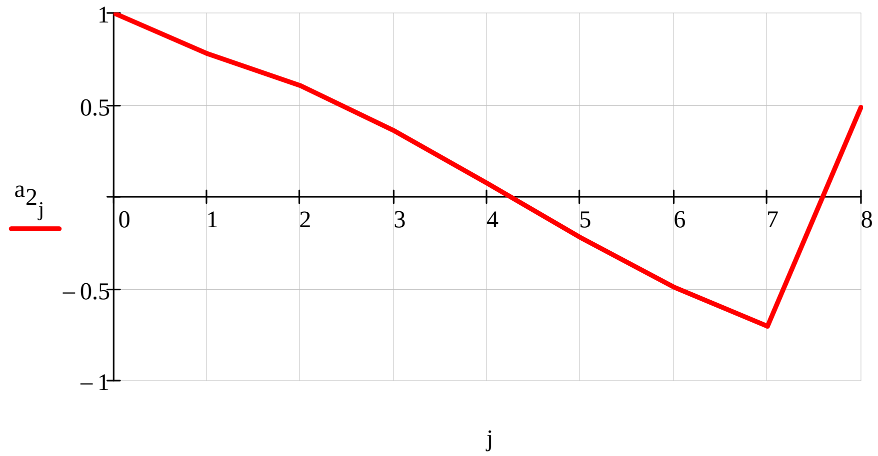


$$a_{2_j} = \frac{w_{j,6}}{w_{0,6}}$$

Second Shape of Inherent Torsional Deviations

$$a_2^{T} = (1 \quad 0.779 \quad 0.605 \quad 0.36 \quad 0.073 \quad -0.223 \quad -0.492 \quad -0.704 \quad 0.487)$$

Second Shape of Inherent Oscillation



$\Omega_7 = 1264.532 \cdot s^{-1}$ $\Omega_{wd1} = \Omega_7$

$N_1 = \frac{\Omega_7}{2 \cdot \pi}$ $N_1 = 201.256 \cdot Hz$ First Inherent Frequency of Equivalent Crankshaft System

$\Omega_6 = 1995.825 \cdot s^{-1}$ $\Omega_{wd2} = \Omega_6$

$N_2 = \frac{\Omega_6}{2 \cdot \pi}$ $N_2 = 317.645 \cdot Hz$ Second Inherent Frequency of Equivalent Crankshaft System

3.3 Forced Torsional Vibrations

3.3.1 Fourier Transform

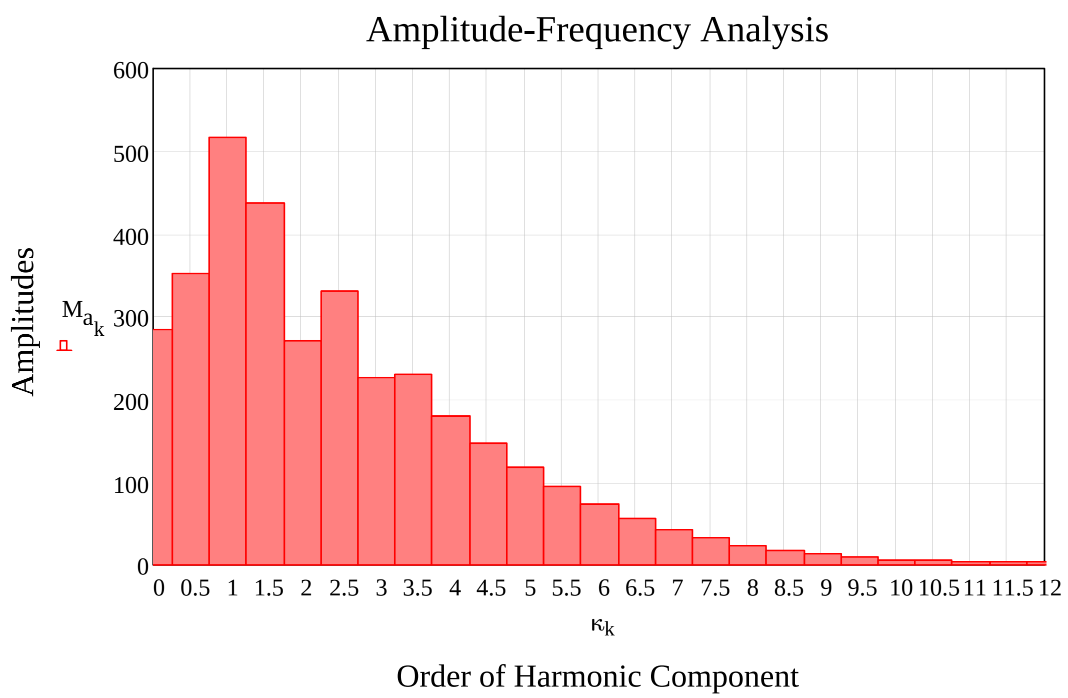
$n_s = 720$ — Number of Samples

$k = 0..24$ — Number of Harmonic Component

$\kappa_k = \frac{k}{2}$ — Order of Harmonic Component by 4-Stroke Engine

$$M_{a_k} = \left| \frac{2}{n_s} \cdot \sum_{i=0}^{n_s-1} \left[M_{t_{i,0}} \cdot e^{i \cdot \left(k \cdot 2 \cdot \pi \cdot \frac{i}{n_s} \right)} \right] \right|$$

Torque Amplitude of Harmonic Component



3.3.2 Critical Speeds

$\kappa = 0.5, 1 .. 12$ — Orders of Harmonic Components

$n_{1res}(\kappa) = \frac{N_1}{\kappa}$ — Critical Speeds of First Inherent Frequency

$n_{2res}(\kappa) = \frac{N_2}{\kappa}$ — Critical Speeds of Second Inherent Frequency

$$\kappa = \begin{pmatrix} 0.5 \\ 1 \\ 1.5 \\ 2 \\ 2.5 \\ 3 \\ 3.5 \\ 4 \\ 4.5 \\ 5 \\ 5.5 \\ 6 \\ 6.5 \\ 7 \\ 7.5 \\ 8 \\ 8.5 \\ 9 \\ 9.5 \\ 10 \\ 10.5 \\ 11 \\ 11.5 \\ 12 \end{pmatrix} \qquad \frac{n_{1res}(\kappa)}{min^{-1}} = \begin{pmatrix} 24150.778 \\ 12075.389 \\ 8050.259 \\ 6037.694 \\ 4830.156 \\ 4025.13 \\ 3450.111 \\ 3018.847 \\ 2683.42 \\ 2415.078 \\ 2195.525 \\ 2012.565 \\ 1857.752 \\ 1725.056 \\ 1610.052 \\ 1509.424 \\ 1420.634 \\ 1341.71 \\ 1271.094 \\ 1207.539 \\ 1150.037 \\ 1097.763 \\ 1050.034 \\ 1006.282 \end{pmatrix} \qquad \frac{n_{2res}(\kappa)}{min^{-1}} = \begin{pmatrix} 38117.456 \\ 19058.728 \\ 12705.819 \\ 9529.364 \\ 7623.491 \\ 6352.909 \\ 5445.351 \\ 4764.682 \\ 4235.273 \\ 3811.746 \\ 3465.223 \\ 3176.455 \\ 2932.112 \\ 2722.675 \\ 2541.164 \\ 2382.341 \\ 2242.203 \\ 2117.636 \\ 2006.182 \\ 1905.873 \\ 1815.117 \\ 1732.612 \\ 1657.281 \\ 1588.227 \end{pmatrix}$$

3.3.3 Resonance Yield of First Inherent Frequency

$\delta = (0 \quad 0 \quad 480 \quad 240 \quad 600 \quad 120 \quad 360)\cdot deg$ Angles of Firings

$\kappa_k = \frac{k}{2}$ Orders of Harmonic Components

$\psi_1 = \kappa_1 \cdot \delta$ Phase Shift

$$\varepsilon_1 = \left| \sqrt{\left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \sin\left(\psi_1^{\langle i \rangle}\right)\right)\right]^2 + \left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \cos\left(\psi_1^{\langle i \rangle}\right)\right)\right]^2} \right|$$

$\varepsilon_1 = 0.486$ Resonance Yield for Orders κ = 0,5; 3,5; 6,5; 9,5

$\psi_2 = \kappa_2 \cdot \delta$ Phase Shift

$$\varepsilon_2 = \left| \sqrt{\left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \sin\left(\psi_2^{\langle i \rangle} \right) \right) \right]^2 + \left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \cos\left(\psi_2^{\langle i \rangle} \right) \right) \right]^2} \right|$$

$\varepsilon_2 = 0.157$ Resonance Yield for Orders κ = 1; 4; 7; 10

$\psi_3 = \kappa_3 \cdot \delta$ Phase Shift

$$\varepsilon_3 = \left| \sqrt{\left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \sin\left(\psi_3^{\langle i \rangle} \right) \right) \right]^2 + \left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \cos\left(\psi_3^{\langle i \rangle} \right) \right) \right]^2} \right|$$

$\varepsilon_3 = 1.299$ Resonance Yield for Orders κ = 1,5; 4,5; 7,5; 10,5

$\psi_4 = \kappa_4 \cdot \delta$ Phase Shift

$$\varepsilon_4 = \left| \sqrt{\left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \sin\left(\psi_4^{\langle i \rangle} \right) \right) \right]^2 + \left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \cos\left(\psi_4^{\langle i \rangle} \right) \right) \right]^2} \right|$$

$\varepsilon_4 = 0.157$ Resonance Yield for Orders κ = 2; 5; 8; 11

$\psi_5 = \kappa_5 \cdot \delta$ Phase Shift

$$\varepsilon_5 = \left| \sqrt{\left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \sin\left(\psi_5^{\langle i \rangle} \right) \right) \right]^2 + \left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \cos\left(\psi_5^{\langle i \rangle} \right) \right) \right]^2} \right|$$

$\varepsilon_5 = 0.486$ Resonance Yield for Orders κ = 2,5; 5,5; 8,5; 11,5

$\psi_6 = \kappa_6 \cdot \delta$ Phase Shift

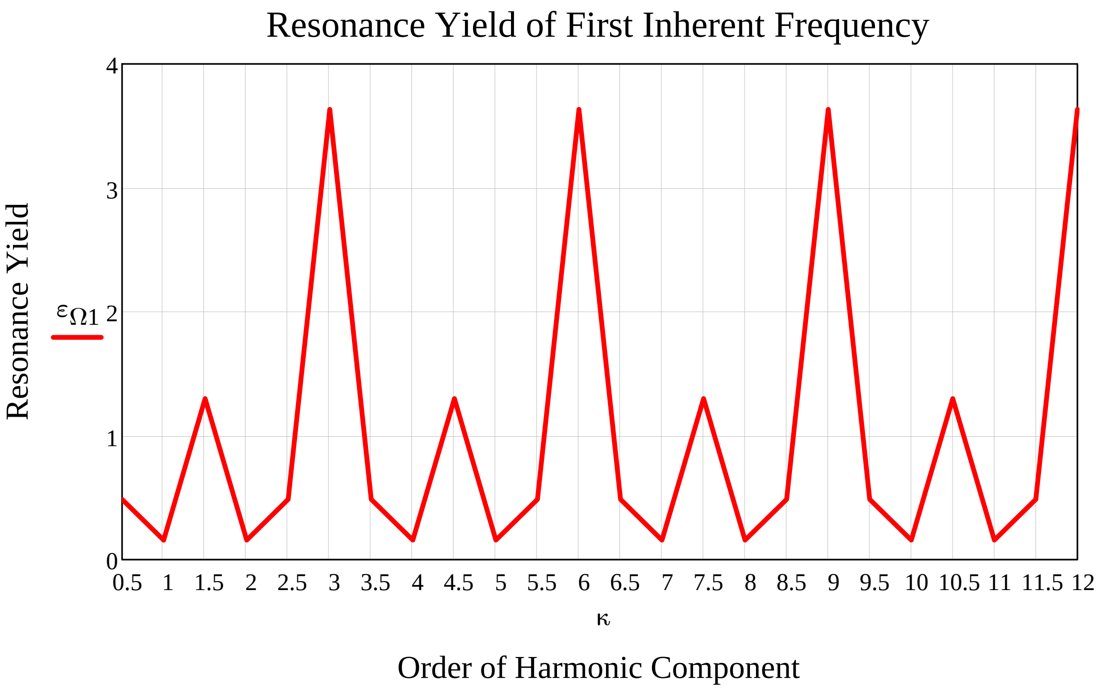
$$\varepsilon_6 = \left| \sqrt{\left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \sin\left(\psi_6^{\langle i \rangle} \right) \right) \right]^2 + \left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \cos\left(\psi_6^{\langle i \rangle} \right) \right) \right]^2} \right|$$

$\varepsilon_6 = 3.633$ Resonance Yield for Orders κ = 3; 6; 9; 12

$$\varepsilon_{\Omega 1} = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \\ \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \\ \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \\ \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{pmatrix}$$

$j = 0..23$

$\kappa_j = j \cdot 0.5 + 0.5$



3.3.4 Resonance Yield of Second Inherent Frequency

$$\varepsilon'_1 = \left|\sqrt{\left[\sum_{i=1}^{6}\left(a2_i\cdot\sin\left(\psi_1^{\langle i\rangle}\right)\right)\right]^2+\left[\sum_{i=1}^{6}\left(a2_i\cdot\cos\left(\psi_1^{\langle i\rangle}\right)\right)\right]^2}\right|$$

$\varepsilon'_1 = 0.854$ Resonance Yield for Orders κ = 0,5; 3,5; 6,5; 9,5

$$\varepsilon'_2 = \left|\sqrt{\left[\sum_{i=1}^{6}\left(a2_i\cdot\sin\left(\psi_2^{\langle i\rangle}\right)\right)\right]^2+\left[\sum_{i=1}^{6}\left(a2_i\cdot\cos\left(\psi_2^{\langle i\rangle}\right)\right)\right]^2}\right|$$

$\varepsilon'_2 = 0.128$ Resonance Yield for Orders κ = 1; 4; 7; 10

$$\varepsilon'_3 = \left|\sqrt{\left[\sum_{i=1}^{6}\left(a2_i\cdot\sin\left(\psi_3^{\langle i\rangle}\right)\right)\right]^2+\left[\sum_{i=1}^{6}\left(a2_i\cdot\cos\left(\psi_3^{\langle i\rangle}\right)\right)\right]^2}\right|$$

$\varepsilon'_3 = 2.385$ Resonance Yield for Orders κ = 1,5; 4,5; 7,5; 10,5

$$\varepsilon'_4 = \left|\sqrt{\left[\sum_{i=1}^{6}\left(a2_i\cdot\sin\left(\psi_4^{\langle i\rangle}\right)\right)\right]^2+\left[\sum_{i=1}^{6}\left(a2_i\cdot\cos\left(\psi_4^{\langle i\rangle}\right)\right)\right]^2}\right|$$

$\varepsilon'_4 = 0.128$ Resonance Yield for Orders κ = 2; 5; 8; 11

$$\varepsilon'_5 = \left|\sqrt{\left[\sum_{i=1}^{6}\left(a2_i\cdot\sin\left(\psi_5^{\langle i\rangle}\right)\right)\right]^2+\left[\sum_{i=1}^{6}\left(a2_i\cdot\cos\left(\psi_5^{\langle i\rangle}\right)\right)\right]^2}\right|$$

$\varepsilon'_5 = 0.854$ Resonance Yield for Orders κ = 2,5; 5,5; 8,5; 11,5

$$\varepsilon'_6 = \left| \sqrt{\left[\sum_{i=1}^{6} \left(a_{2_i}\cdot\sin\left(\psi_6^{\langle i\rangle}\right)\right)\right]^2 + \left[\sum_{i=1}^{6} \left(a_{2_i}\cdot\cos\left(\psi_6^{\langle i\rangle}\right)\right)\right]^2} \right|$$

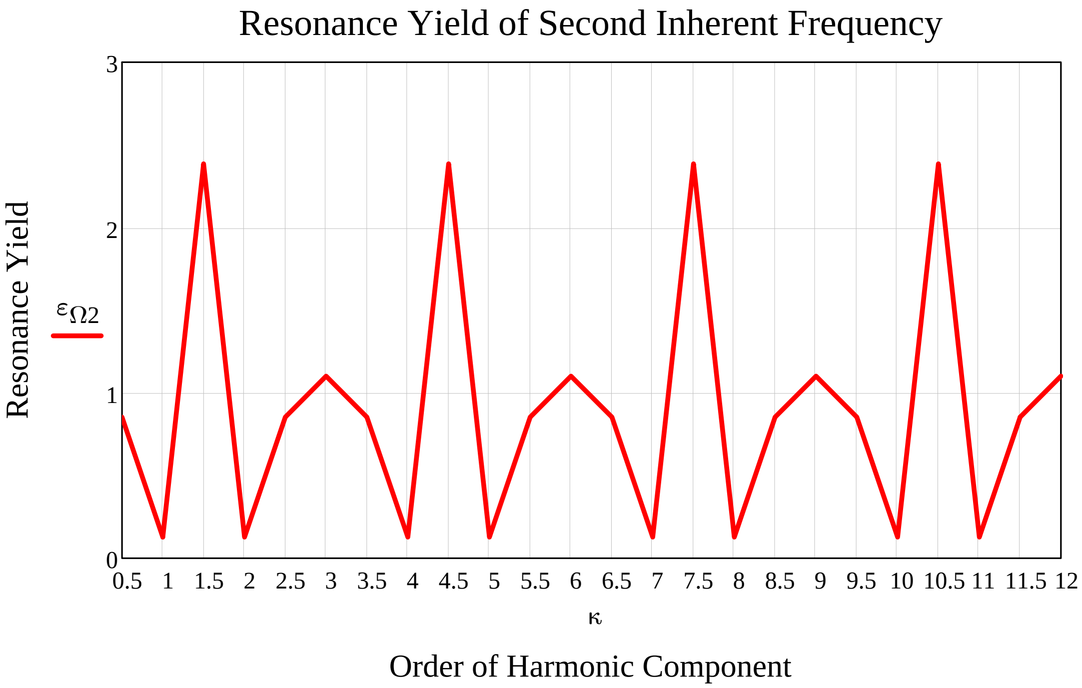
$\varepsilon'_6 = 1.101$

Resonance Yield for Orders κ = 3; 6; 9; 12

$$\varepsilon_{\Omega 2} = \begin{pmatrix} \varepsilon'_1 \\ \varepsilon'_2 \\ \varepsilon'_3 \\ \varepsilon'_4 \\ \varepsilon'_5 \\ \varepsilon'_6 \\ \varepsilon'_1 \\ \varepsilon'_2 \\ \varepsilon'_3 \\ \varepsilon'_4 \\ \varepsilon'_5 \\ \varepsilon'_6 \\ \varepsilon'_1 \\ \varepsilon'_2 \\ \varepsilon'_3 \\ \varepsilon'_4 \\ \varepsilon'_5 \\ \varepsilon'_6 \\ \varepsilon'_1 \\ \varepsilon'_2 \\ \varepsilon'_3 \\ \varepsilon'_4 \\ \varepsilon'_5 \\ \varepsilon'_6 \end{pmatrix}$$

$j = 0..23$

$\kappa_j = j\cdot 0.5 + 0.5$



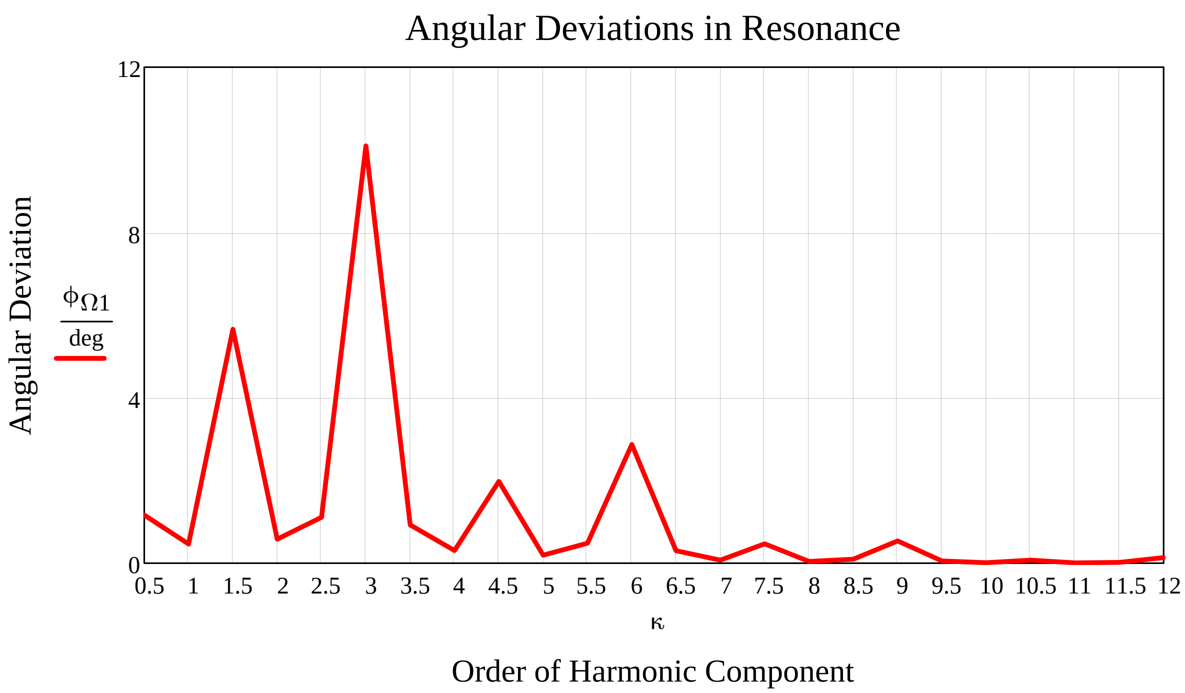
3.3.5 Angular Deviations in Resonance

$o := 0..8$

$\xi := 1.5 \cdot N \cdot m \cdot s \cdot rad^{-1}$

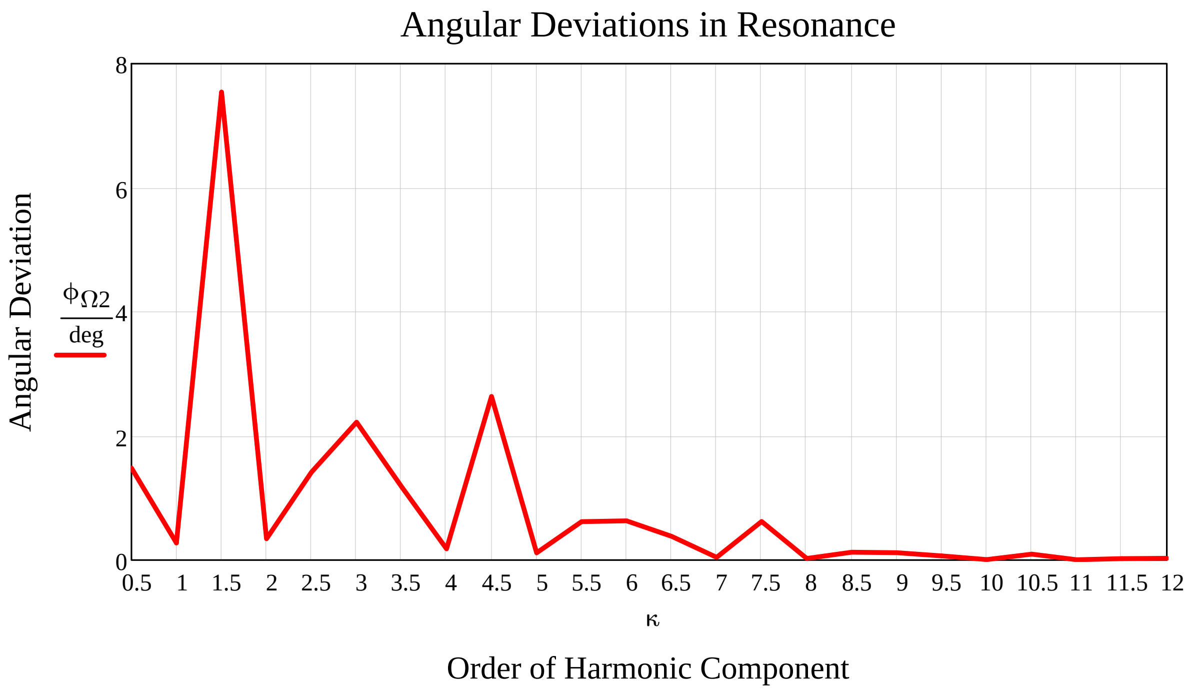
$$\phi\Omega1_j := \frac{M_{a_j} \cdot \varepsilon\Omega1_j}{\Omega_7 \cdot \xi \cdot \left[\sum_o \left(a1_o\right)^2\right]}$$

Angular Deviations of First Inherent Frequency



$$\phi\Omega2_j := \frac{M_{a_j} \cdot \varepsilon\Omega2_j}{\Omega_6 \cdot \xi \cdot \left[\sum_o \left(a2_o\right)^2\right]}$$

Angular Deviations of Second Inherent Frequency



3.3.6 Additional Stress in Resonance

$W_{\tau cj} = \frac{\pi}{16} \cdot d_{cj}^{\ 3}$ $\qquad W_{\tau cj} = 5.645 \times 10^{-5} \cdot m^3$ $\qquad$ Shearing Section Modulus of Crank Pin

$\Delta a_1 = a1_6 - a1_7$ $\qquad \Delta a_1 = 0.204$ $\qquad$ Relative Twist Angle of First Shape

$M_{a1} = \phi\Omega 1_{11} \cdot \Delta a_1 \cdot c_6$ $\qquad M_{a1} = 13069.038 \cdot N \cdot m$ $\qquad$ Additional Torque in Resonance

$\tau_{acj1} = \frac{M_{a1}}{W_{\tau cj}}$ $\qquad \tau_{acj1} = 231.516 \cdot MPa$ $\qquad$ Additional Stress in Resonance

$\Delta a_2 = \left| a2_7 - a2_8 \right|$ $\qquad \Delta a_2 = 1.191$ $\qquad$ Relative Twist Angle of Second Shape

$M_{a2} = \phi\Omega 2_{17} \cdot \Delta a_2 \cdot c_1$ $\qquad M_{a2} = 3160.669 \cdot N \cdot m$ $\qquad$ Additional Torque in Resonance

$\tau_{acj2} = \frac{M_{a2}}{W_{\tau cj}}$ $\qquad \tau_{acj2} = 55.991 \cdot MPa$ $\qquad$ Additional Stress in Resonance

Appendix 4 - Torsional Vibration with Damper

4.1 Equivalent Crankshaft System

4.1.1 Damper Properties

$I_{damp} = 85.384 \cdot 10^{-3} \cdot kg \cdot m^2$ Moment of Inertia of Torsional Damper

$\Omega_{damp1} = 146 \cdot Hz$ First Inherent Frequency of Damper

$\Omega_{damp2} = 158.3 \cdot Hz$ Second Inherent Frequency of Damper

4.1.2 Masses Reduction

$I_0 = I_{damp}$ $I_0 = 85.384 \cdot 10^{-3} \cdot kg \cdot m^2$ Reduced Moment of Inertia of Damper

$I_1 = I_{end\_fr} + I_{pull}$ $I_1 = 26.938 \cdot 10^{-3} \cdot kg \cdot m^2$ Reduced Moment of Inertia of Front End

$I_2 = I_{throw} + I_{rot} + I_{rec}$ $I_2 = 37.45 \cdot 10^{-3} \cdot kg \cdot m^2$ Reduced Moment of Inertia of 1st Crank Throw

$I_3 = I_{throw} + I_{rot} + I_{rec}$ $I_3 = 37.45 \cdot 10^{-3} \cdot kg \cdot m^2$ Reduced Moment of Inertia of 2nd Crank Throw

$I_4 = I_{throw} + I_{rot} + I_{rec}$ $I_4 = 37.45 \cdot 10^{-3} \cdot kg \cdot m^2$ Reduced Moment of Inertia of 3rd Crank Throw

$I_5 = I_{throw} + I_{rot} + I_{rec}$ $I_5 = 37.45 \cdot 10^{-3} \cdot kg \cdot m^2$ Reduced Moment of Inertia of 4th Crank Throw

$I_6 = I_{throw} + I_{rot} + I_{rec}$ $I_6 = 37.45 \cdot 10^{-3} \cdot kg \cdot m^2$ Reduced Moment of Inertia of 5th Crank Throw

$I_7 = I_{throw} + I_{rot} + I_{rec}$ $I_7 = 37.45 \cdot 10^{-3} \cdot kg \cdot m^2$ Reduced Moment of Inertia of 6th Crank Throw

$I_8 = I_{end\_rr} + I_{flyw}$ $I_8 = 877.276 \cdot 10^{-3} \cdot kg \cdot m^2$ Reduced Moment of Inertia of Rear End

$I_9 = \sum_{n=0}^{8} I_n$ $I_9 = 1.214 \cdot kg \cdot m^2$ Moment of Inertia of Whole Torsional System

4.1.3 Torsional Stiffness

$c_0 = I_{damp} \cdot \Omega_{damp1}^2$ $\quad$ $c_0 = 1.82 \times 10^3 \cdot N \cdot m \cdot rad^{-1}$ Damper Stiffness

$c_1 = \dfrac{G_m \cdot I_p}{l_{end\_fr\_red}}$ $\quad$ $c_1 = 4.86 \times 10^5 \cdot N \cdot m \cdot rad^{-1}$ Front End Torsional Stiffness

$c_2 = \dfrac{G_m \cdot I_p}{l_{throw\_red}}$ $\quad$ $c_2 = 1.281 \times 10^6 \cdot N \cdot m \cdot rad^{-1}$ 1st Crank Throw Torsional Stiffness

$c_3 = \dfrac{G_m \cdot I_p}{l_{throw\_red}}$ $\quad$ $c_3 = 1.281 \times 10^6 \cdot N \cdot m \cdot rad^{-1}$ 2nd Crank Throw Torsional Stiffness

$c_4 = \dfrac{G_m \cdot I_p}{l_{throw\_red}}$ $\quad$ $c_4 = 1.281 \times 10^6 \cdot N \cdot m \cdot rad^{-1}$ 3rd Crank Throw Torsional Stiffness

$c_5 = \dfrac{G_m \cdot I_p}{l_{throw\_red}}$ $\quad$ $c_5 = 1.281 \times 10^6 \cdot N \cdot m \cdot rad^{-1}$ 4th Crank Throw Torsional Stiffness

$c_6 = \dfrac{G_m \cdot I_p}{l_{throw\_red}}$ $\quad$ $c_6 = 1.281 \times 10^6 \cdot N \cdot m \cdot rad^{-1}$ 5th Crank Throw Torsional Stiffness

$c_7 = \dfrac{G_m \cdot I_p}{l_{throw\_red}}$ $\quad$ $c_7 = 1.281 \times 10^6 \cdot N \cdot m \cdot rad^{-1}$ 6th Crank Throw Torsional Stiffness

$c_8 = \dfrac{G_m \cdot I_p}{l_{end\_rr\_red}}$ $\quad$ $c_8 = 1.838 \times 10^6 \cdot N \cdot m \cdot rad^{-1}$ Rear End Torsional Stiffness

4.2 Free Torsional Vibration

$$M = \begin{pmatrix} I_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_9 \end{pmatrix}$$

Diagonal Mass Matrix

$$C = \begin{pmatrix} c_0 & -c_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -c_0 & c_0 + c_1 & -c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -c_1 & c_1 + c_2 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -c_2 & c_2 + c_3 & -c_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c_3 & c_3 + c_4 & -c_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -c_4 & c_4 + c_5 & -c_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -c_5 & c_5 + c_6 & -c_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -c_6 & c_6 + c_7 & -c_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_7 & c_7 + c_8 & -c_8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_8 & c_8 \end{pmatrix}$$

Diagonal Stiffness Matrix

$$A = M^{-1} \cdot C \qquad \chi = \text{eigenvals}(A) \qquad \Omega = \overrightarrow{\sqrt{\chi}}$$

$$\chi = \begin{pmatrix} 129292068.177 \\ 108608398.739 \\ 79625603.41 \\ 49568426.021 \\ 26969454.824 \\ 12725858.843 \\ 3890704.569 \\ 1593127.313 \\ 21842.302 \\ 0 \end{pmatrix} \cdot s^{-2} \qquad \Omega = \begin{pmatrix} 11370.667 \\ 10421.535 \\ 8923.318 \\ 7040.485 \\ 5193.212 \\ 3567.332 \\ 1972.487 \\ 1262.191 \\ 147.791 \\ 0 \end{pmatrix} \cdot s^{-1}$$

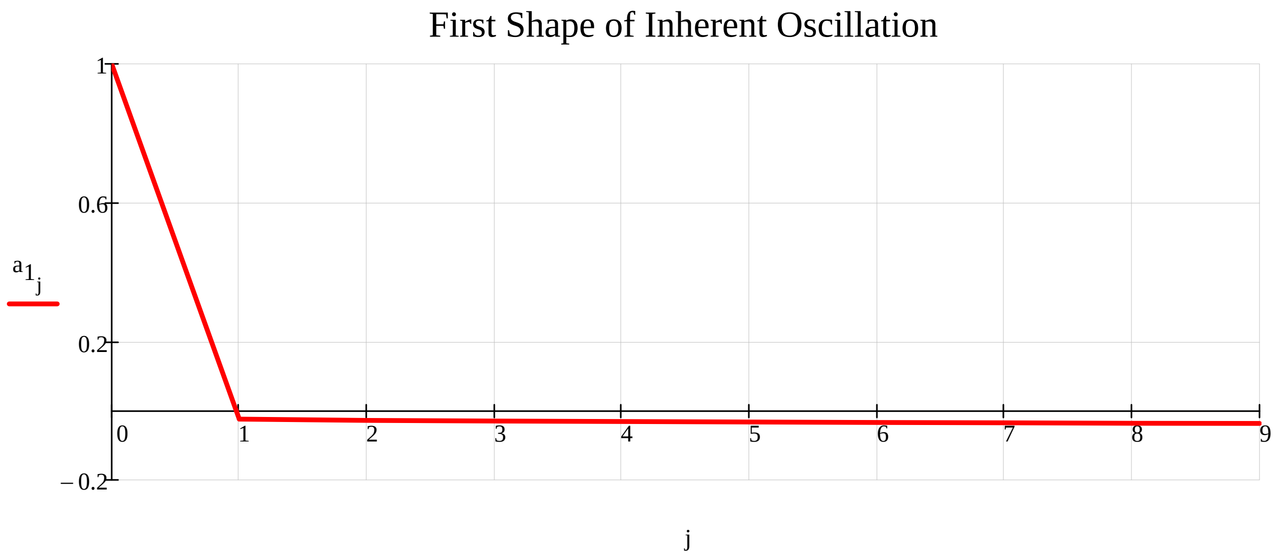
$w := \mathrm{eigenvecs}(A)$ Modal Matrix

$$w^T = \begin{pmatrix} 0 & -0.027 & 0.164 & -0.384 & 0.52 & -0.542 & 0.445 & -0.252 & 0.003 & -0 \\ -0 & 0.063 & -0.317 & 0.546 & -0.325 & -0.163 & 0.517 & -0.445 & 0.006 & -0 \\ 0 & -0.131 & 0.445 & -0.373 & -0.323 & 0.479 & 0.165 & -0.533 & 0.01 & -0 \\ 0 & -0.292 & 0.509 & 0.075 & -0.468 & -0.332 & 0.285 & 0.489 & -0.016 & 0 \\ -0 & 0.623 & -0.306 & -0.417 & -0.199 & 0.176 & 0.412 & 0.323 & -0.02 & 0.001 \\ 0.001 & -0.691 & -0.206 & 0.055 & 0.295 & 0.426 & 0.398 & 0.222 & -0.036 & 0.005 \\ 0.003 & -0.569 & -0.449 & -0.352 & -0.215 & -0.054 & 0.114 & 0.268 & 0.392 & -0.25 \\ 0.007 & -0.524 & -0.48 & -0.441 & -0.381 & -0.304 & -0.212 & -0.111 & -0.004 & 0.073 \\ 0.995 & -0.025 & -0.028 & -0.03 & -0.031 & -0.033 & -0.034 & -0.035 & -0.037 & -0.037 \\ 0.316 & 0.316 & 0.316 & 0.316 & 0.316 & 0.316 & 0.316 & 0.316 & 0.316 & 0.316 \end{pmatrix}$$

$j := 0..9$

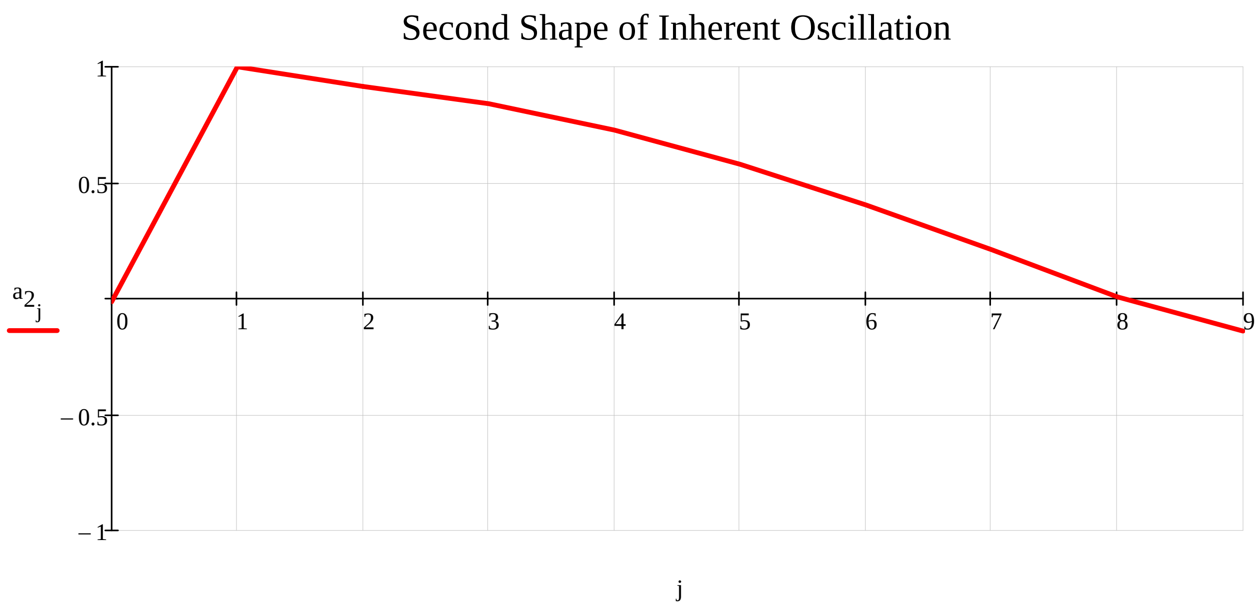
$$a1_j := \frac{w_{j,8}}{w_{0,8}}$$ First Shape of Inherent Torsional Deviations

$$a1^T = (1 \;\; -0.025 \;\; -0.028 \;\; -0.03 \;\; -0.031 \;\; -0.033 \;\; -0.034 \;\; -0.035 \;\; -0.037 \;\; -0.037)$$



$$a_{2_j} = \frac{w_{j,7}}{w_{1,7}}$$ Second Shape of Inherent Torsional Deviations

$$a_2^T = (-0.014 \;\; 1 \;\; 0.915 \;\; 0.841 \;\; 0.727 \;\; 0.579 \;\; 0.404 \;\; 0.211 \;\; 0.007 \;\; -0.14)$$



$$N_1 = \frac{\Omega_7}{2\cdot\pi}$$ $N_1 = 200.884\cdot Hz$ First Inherent Frequency of Equivalent Crankshaft System

$$N_2 = \frac{\Omega_6}{2\cdot\pi}$$ $N_2 = 313.931\cdot Hz$ Second Inherent Frequency of Equivalent Crankshaft System

4.3 Forced Torsional Vibrations

4.3.1 Critical Speeds

$\kappa = 0.5, 1 .. 12$ Orders of Harmonic Components

$$n_{1res}(\kappa) = \frac{N_1}{\kappa} \qquad n_{2res}(\kappa) = \frac{N_2}{\kappa}$$ Critical Speeds

$$\kappa = \begin{pmatrix} 0.5 \\ 1 \\ 1.5 \\ 2 \\ 2.5 \\ 3 \\ 3.5 \\ 4 \\ 4.5 \\ 5 \\ 5.5 \\ 6 \\ 6.5 \\ 7 \\ 7.5 \\ 8 \\ 8.5 \\ 9 \\ 9.5 \\ 10 \\ 10.5 \\ 11 \\ 11.5 \\ 12 \end{pmatrix} \qquad \frac{n_{1res}(\kappa)}{min^{-1}} = \begin{pmatrix} 24106.081 \\ 12053.041 \\ 8035.36 \\ 6026.52 \\ 4821.216 \\ 4017.68 \\ 3443.726 \\ 3013.26 \\ 2678.453 \\ 2410.608 \\ 2191.462 \\ 2008.84 \\ 1854.314 \\ 1721.863 \\ 1607.072 \\ 1506.63 \\ 1418.005 \\ 1339.227 \\ 1268.741 \\ 1205.304 \\ 1147.909 \\ 1095.731 \\ 1048.09 \\ 1004.42 \end{pmatrix} \qquad \frac{n_{2res}(\kappa)}{min^{-1}} = \begin{pmatrix} 37671.725 \\ 18835.862 \\ 12557.242 \\ 9417.931 \\ 7534.345 \\ 6278.621 \\ 5381.675 \\ 4708.966 \\ 4185.747 \\ 3767.172 \\ 3424.702 \\ 3139.31 \\ 2897.825 \\ 2690.837 \\ 2511.448 \\ 2354.483 \\ 2215.984 \\ 2092.874 \\ 1982.722 \\ 1883.586 \\ 1793.892 \\ 1712.351 \\ 1637.901 \\ 1569.655 \end{pmatrix}$$

4.3.2 Resonance Yield of First Inherent Frequency

$\delta = (0 \quad 0 \quad 480 \quad 240 \quad 600 \quad 120 \quad 360)\cdot deg$ Angles of Firings

$\kappa_k = \dfrac{k}{2}$ Orders of Harmonic Components

$\psi_1 = \kappa_1 \cdot \delta$ Phase Shift

$$\varepsilon_1 = \left| \sqrt{\left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \sin\left(\psi_1^{\langle i \rangle}\right)\right)\right]^2 + \left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \cos\left(\psi_1^{\langle i \rangle}\right)\right)\right]^2} \right|$$

$\varepsilon_1 = 0.007$ Resonance Yield for Orders κ = 0,5; 3,5; 6,5; 9,5

$\psi_2 = \kappa_2 \cdot \delta$ Phase Shift

$$\varepsilon_2 = \left| \sqrt{\left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \sin\left(\psi_2^{\langle i \rangle}\right)\right)\right]^2 + \left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \cos\left(\psi_2^{\langle i \rangle}\right)\right)\right]^2} \right|$$

$\varepsilon_2 = 0.002$ Resonance Yield for Orders κ = 1; 4; 7; 10

$\psi_3 = \kappa_3 \cdot \delta$ Phase Shift

$$\varepsilon_3 = \left| \sqrt{\left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \sin\left(\psi_3^{\langle i \rangle}\right)\right)\right]^2 + \left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \cos\left(\psi_3^{\langle i \rangle}\right)\right)\right]^2} \right|$$

$\varepsilon_3 = 0.015$ Resonance Yield for Orders κ = 1,5; 4,5; 7,5; 10,5

$\psi_4 = \kappa_4 \cdot \delta$ Phase Shift

$$\varepsilon_4 = \left| \sqrt{\left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \sin\left(\psi_4^{\langle i \rangle}\right)\right)\right]^2 + \left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \cos\left(\psi_4^{\langle i \rangle}\right)\right)\right]^2} \right|$$

$\varepsilon_4 = 0.002$ Resonance Yield for Orders κ = 2; 5; 8; 11

$\psi_5 = \kappa_5 \cdot \delta$ Phase Shift

$$\varepsilon_5 = \left| \sqrt{\left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \sin\left(\psi_5^{\langle i \rangle}\right)\right)\right]^2 + \left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \cos\left(\psi_5^{\langle i \rangle}\right)\right)\right]^2} \right|$$

$\varepsilon_5 = 0.007$ Resonance Yield for Orders κ = 2,5; 5,5; 8,5; 11,5

$\psi_6 = \kappa_6 \cdot \delta$ Phase Shift

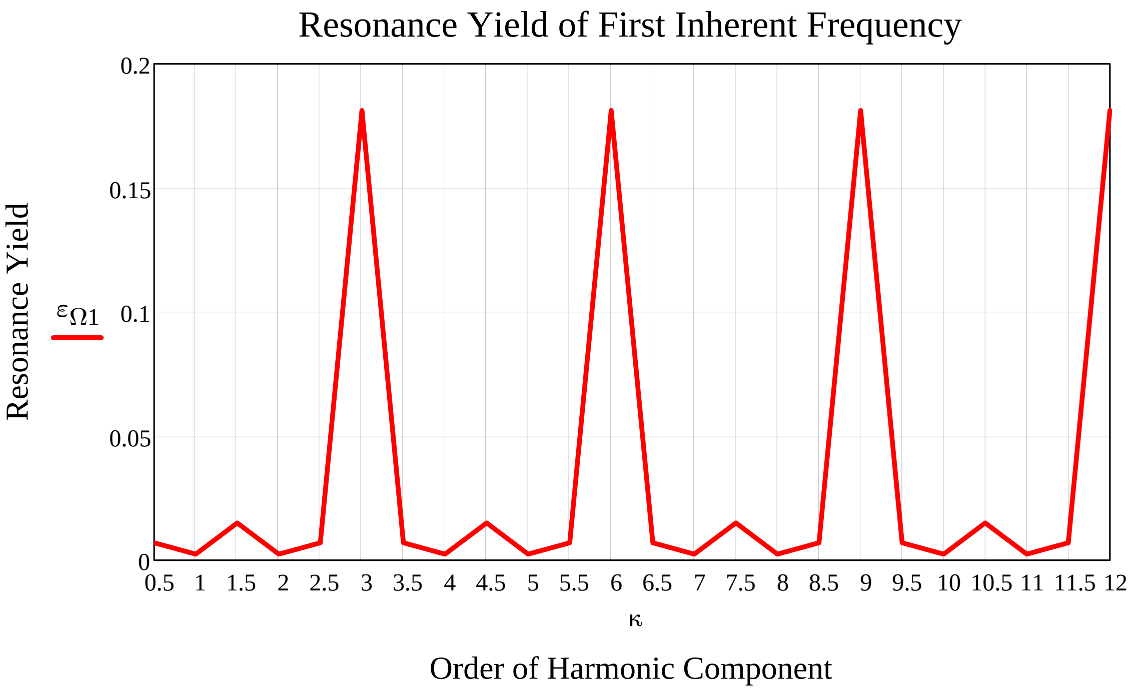
$$\varepsilon_6 = \left| \sqrt{\left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \sin\left(\psi_6^{\langle i \rangle}\right)\right)\right]^2 + \left[\sum_{i=1}^{6} \left(a_{1_i} \cdot \cos\left(\psi_6^{\langle i \rangle}\right)\right)\right]^2} \right|$$

$\varepsilon_6 = 0.181$ Resonance Yield for Orders κ = 3; 6; 9; 12

$$\varepsilon_{\Omega 1} = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \\ \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \\ \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \\ \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{pmatrix}$$

$$j := 0..23$$

$$\kappa_j := j \cdot 0.5 + 0.5$$



4.3.3 Resonance Yield of Second Inherent Frequency

$$\varepsilon'_1 = \left|\sqrt{\left[\sum_{i=1}^{6}\left(a_{2_i}\cdot\sin\left(\psi_1^{\langle i\rangle}\right)\right)\right]^2 + \left[\sum_{i=1}^{6}\left(a_{2_i}\cdot\cos\left(\psi_1^{\langle i\rangle}\right)\right)\right]^2}\right|$$

$\varepsilon'_1 = 0.418$ Resonance Yield for Orders κ = 0,5; 3,5; 6,5; 9,5

$$\varepsilon'_2 = \left|\sqrt{\left[\sum_{i=1}^{6}\left(a_{2_i}\cdot\sin\left(\psi_2^{\langle i\rangle}\right)\right)\right]^2 + \left[\sum_{i=1}^{6}\left(a_{2_i}\cdot\cos\left(\psi_2^{\langle i\rangle}\right)\right)\right]^2}\right|$$

$\varepsilon'_2 = 0.142$ Resonance Yield for Orders κ = 1; 4; 7; 10

$$\varepsilon'_3 = \left|\sqrt{\left[\sum_{i=1}^{6}\left(a_{2_i}\cdot\sin\left(\psi_3^{\langle i\rangle}\right)\right)\right]^2 + \left[\sum_{i=1}^{6}\left(a_{2_i}\cdot\cos\left(\psi_3^{\langle i\rangle}\right)\right)\right]^2}\right|$$

$\varepsilon'_3 = 1.046$ Resonance Yield for Orders κ = 1,5; 4,5; 7,5; 10,5

$$\varepsilon'_4 = \left|\sqrt{\left[\sum_{i=1}^{6}\left(a_{2_i}\cdot\sin\left(\psi_4^{\langle i\rangle}\right)\right)\right]^2 + \left[\sum_{i=1}^{6}\left(a_{2_i}\cdot\cos\left(\psi_4^{\langle i\rangle}\right)\right)\right]^2}\right|$$

$\varepsilon'_4 = 0.142$ Resonance Yield for Orders κ = 2; 5; 8; 11

$$\varepsilon'_5 = \left|\sqrt{\left[\sum_{i=1}^{6}\left(a_{2_i}\cdot\sin\left(\psi_5^{\langle i\rangle}\right)\right)\right]^2 + \left[\sum_{i=1}^{6}\left(a_{2_i}\cdot\cos\left(\psi_5^{\langle i\rangle}\right)\right)\right]^2}\right|$$

$\varepsilon'_5 = 0.418$ Resonance Yield for Orders κ = 2,5; 5,5; 8,5; 11,5

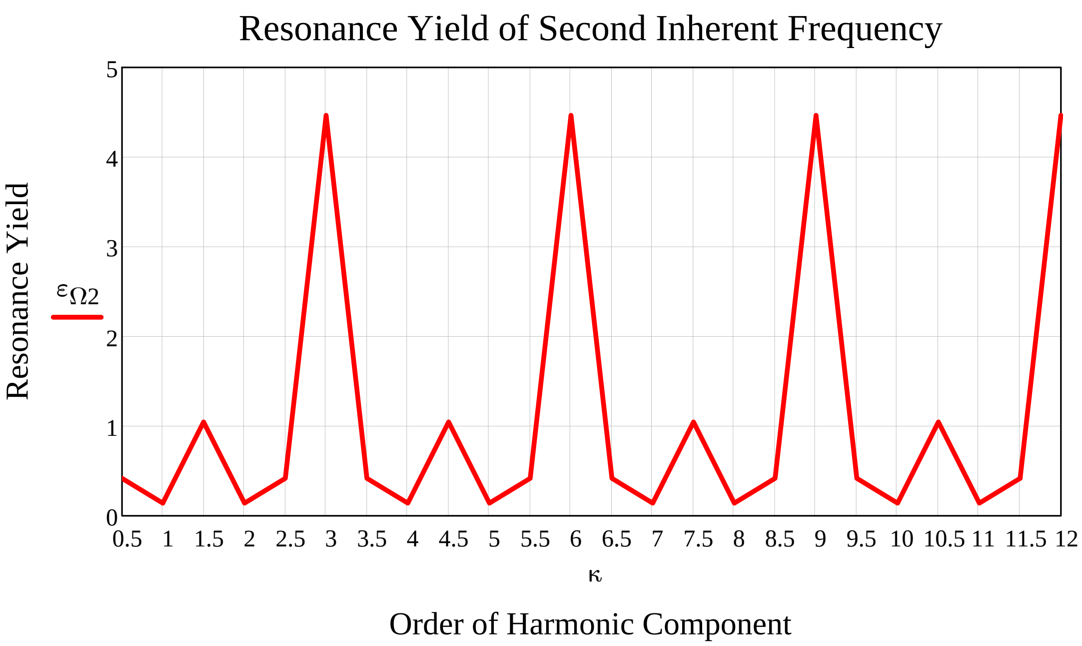
$$\varepsilon'_6 = \left|\sqrt{\left[\sum_{i=1}^{6}\left(a_{2_i}\cdot\sin\left(\psi_6^{\langle i\rangle}\right)\right)\right]^2 + \left[\sum_{i=1}^{6}\left(a_{2_i}\cdot\cos\left(\psi_6^{\langle i\rangle}\right)\right)\right]^2}\right|$$

$\varepsilon'_6 = 4.467$ Resonance Yield for Orders κ = 3; 6; 9; 12

$$\varepsilon_{\Omega 2} = \begin{pmatrix} \varepsilon'_1 \\ \varepsilon'_2 \\ \varepsilon'_3 \\ \varepsilon'_4 \\ \varepsilon'_5 \\ \varepsilon'_6 \\ \varepsilon'_1 \\ \varepsilon'_2 \\ \varepsilon'_3 \\ \varepsilon'_4 \\ \varepsilon'_5 \\ \varepsilon'_6 \\ \varepsilon'_1 \\ \varepsilon'_2 \\ \varepsilon'_3 \\ \varepsilon'_4 \\ \varepsilon'_5 \\ \varepsilon'_6 \\ \varepsilon'_1 \\ \varepsilon'_2 \\ \varepsilon'_3 \\ \varepsilon'_4 \\ \varepsilon'_5 \\ \varepsilon'_6 \end{pmatrix}$$

$j = 0..23$

$\kappa_j = j \cdot 0.5 + 0.5$



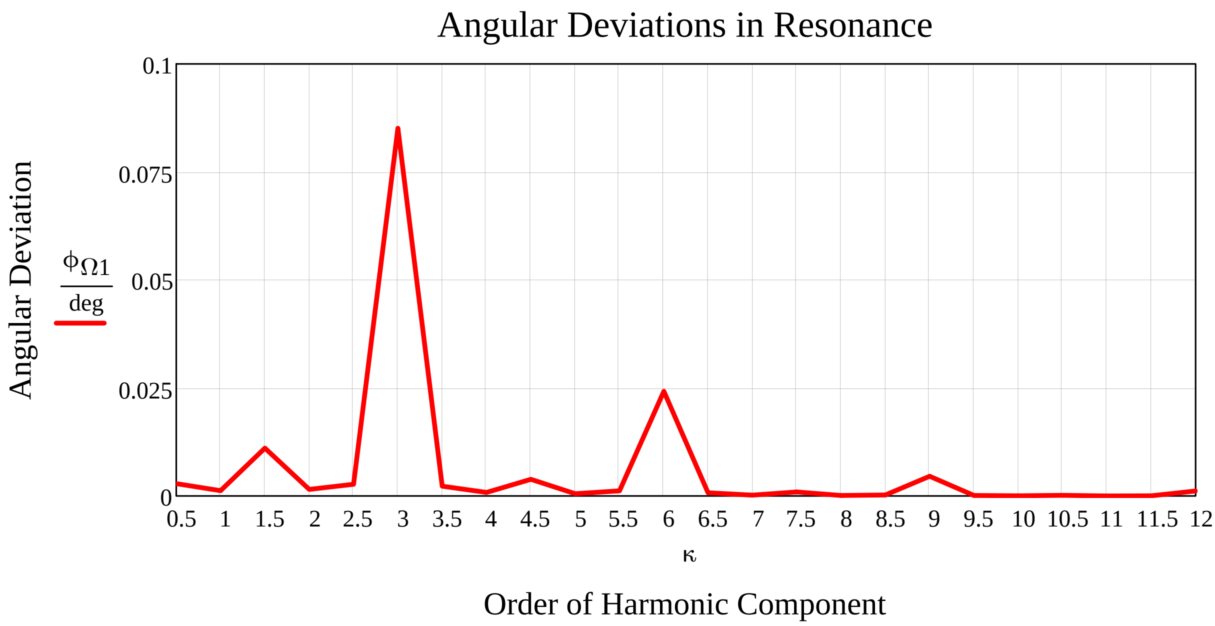
4.3.4 Angular Deviations in Resonance

$\gamma = 0.09$ Relative Loss

$\xi_{damp} = 2 \cdot \gamma \cdot I_{damp} \cdot \Omega_{wd1}$ Damping Resistance of Damper

$$\phi_{\Omega 1_j} = \frac{M_{a_j} \cdot \varepsilon_{\Omega 1_j}}{\Omega_6 \cdot \left[\xi \cdot \sum_{o=1}^{9} \left(a_{1_o}\right)^2 + \xi_{damp} \cdot \left(a_{1_0} - a_{1_1}\right)^2 \right]}$$

Angular Deviations of First Inherent Frequency

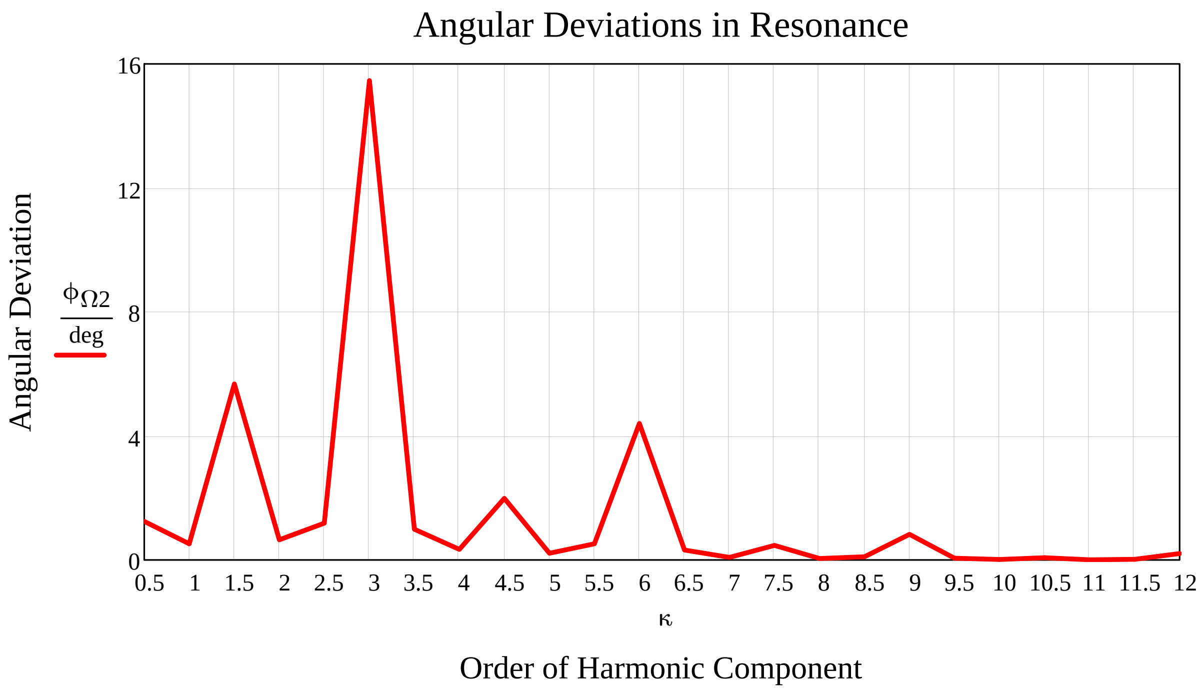


$$\xi_{damp} = 2 \cdot \gamma \cdot I_{damp} \cdot \Omega_{wd2}$$

Damping resistance of Damper

$$\phi_{\Omega 2_j} = \frac{M_{a_j} \cdot \varepsilon_{\Omega 2_j}}{\Omega_8 \cdot \left[\xi \cdot \sum_{o=1}^{9} \left(a_{2_o}\right)^2 + \xi_{damp} \cdot \left(a_{2_0} - a_{2_1}\right)^2 \right]}$$

Angular Deviations of Second Inherent Frequency



4.3.5 Additional Stress in Resonance

$\Delta a_1 = a_{1_1} - a_{1_2}$ $\qquad \Delta a_1 = 0.004$ $\qquad$ Relative Twist Angle

$M_{a1} = \phi_{\Omega 1_6} \cdot \Delta a_1 \cdot c_6$ $\qquad M_{a1} = 0.192 \cdot N \cdot m$ $\qquad$ Additional Torque in Resonance

$\tau_{acj1} = \dfrac{M_{a1}}{W_{\tau cj}}$ $\qquad \tau_{acj1} = 0.003 \cdot MPa$ $\qquad$ Additional Stress in Resonance

$\Delta a_2 = \left| a_{2_7} - a_{2_8} \right|$ $\qquad \Delta a_2 = 0.203$ $\qquad$ Relative Twist Angle

$M_{a2} = \phi_{\Omega 2_{17}} \cdot \Delta a_2 \cdot c_1$ $\qquad M_{a2} = 1425.788 \cdot N \cdot m$ $\qquad$ Additional Torque in Resonance

$\tau_{acj2} = \dfrac{M_{a2}}{W_{\tau cj}}$ $\qquad \tau_{acj2} = 25.258 \cdot MPa$ $\qquad$ Additional Stress in Resonance