

Derivation of losses from impedance spectrum for contour modes of ceramic resonator

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ABSTRACT

Method of loss coefficients calculation from mechanical quality factors of transversal vibration modes is presented. Theoretical formulae are derived for the contour mode of thin square plate and width extensional mode of a thin rectangular plate. Mechanical quality factors are expressed as a function of loss coefficients for ceramic with known Poisson's ratio. Measured impedance spectra are compared with calculations and used for the characterization of losses on lead-free BCZT and lead-based PZT piezoelectric ceramics. Full set of loss coefficients active for planar vibration modes is characterized on a single sample. Mechanical quality factor at resonance is the same for the contour extensional mode of a thin square plate and for the width extensional mode of a thin rectangular plate. Loss coefficients reach the biggest values for the piezoelectric losses, followed by the dielectric and elastic losses. Our results show that the BCZT ceramics have losses of the same magnitude as soft PZT ceramic.

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1. Introduction

Heat generation occurring during the operation of piezoelectric resonator at resonance is substantial for its high-power properties. Heat originates from elastic, dielectric and piezoelectric losses, which are described by general formalism by the imaginary parts of tensor material constants [1]. Usually, losses for all tensor coefficients active for the vibration mode are not known. One of the methods for loss characterization is nonlinear least-square fit of impedance/admittance frequency spectra in the vicinity of resonance/antiresonance using imaginary parts of material coefficients [2–6]. Such fit is however special for each resonator type and vibration mode while impedance/admittance analytical formula is available just for simple resonators. Fit uses a full frequency spectrum not just resonance/antiresonance frequency points. Simpler method for losses characterization is by dielectric loss tangent ($\tan \delta$) measured directly by LCR meter and mechanical quality factor Q_m analyzed from impedance spectrum by impedance analyzer. Losses are derived by 3dB method as a function of all active loss coefficients for every specific vibration mode. The method uses just resonance/antiresonance data from impedance/admittance spectrum. Results of losses calculation from impedance frequency spectra were published for standard ceramic resonators [7–10]. The method is based on mechanical quality factors at resonance Q_r and antiresonance Q_a measured at resonance/antiresonance impedance peaks [11]. A complete set of loss coefficients is obtained from the combination of results from several

samples of appropriate shape and vibration mode. Poisson's ratio σ for studied ceramic causes problems while it is not known and it could not be easily measured. For PZT ceramic, it is assumed to be known, or it must be obtained by other methods than impedance spectrum measurement. For newly developed lead-free types of piezoelectric ceramic materials, it is however not an applicable method while we typically have unstable properties from sample to sample or we have a lack of appropriate sample shapes. Losses measurement methodology using IEC483 Standard for the Poisson's ratio measurement can combine radially vibrating disc and square plate resonator data [12]. The present contribution is focused on the derivation of set of dielectric, elastic and piezoelectric losses obtained on a single ceramic sample by using contour mode of square plate and width extensional mode of rectangular plate resonators. Square plate (k_p -mode, CE) could be shaped by polishing down to the rectangular plate (k'_{31} -mode, WE) and further to slender bar shape (k_{31} -mode, LE). Losses are calculated from measured mechanical quality factors at these resonance modes from a single ceramic sample. Derivation of losses helps also in designing material properties of piezoelectric ceramic materials.

2. Theory

Losses active in contour vibrations of piezoelectric resonator (e.g. disc, plate or bar vibrating in transversal mode) are represented by the loss tangents

$$\epsilon_{33}^{*T} = \epsilon_{33}^T \left(1 - j \tan \delta'_{33}\right) \quad (1a)$$

$$s_{11}^{*E} = s_{11}^E \left(1 - j \tan \phi'_{11}\right) \quad (1b)$$

$$s_{12}^{*E} = s_{12}^E \left(1 - j \tan \phi'_{12}\right) \quad (1c)$$

$$d_{31}^* = d_{31} \left(1 - j \tan \theta'_{31}\right) \quad (1d)$$

with its own loss factor for each tensor component of dielectric, elastic and piezoelectric coefficients. We adopt the same formalism as in Ref [8].

2.1. Contour mode of square plate resonator

Let us assume resonator in shape of square plate with electrodes deposited on major faces and poled in thickness direction. Dimensions and coordinate system are defined in Figure 1. Such a resonator vibrates in the transversal mode under AC voltage driving. Its vibration mode is contour (breathing) mode with equal transversal displacements due to the equal size of planar dimensions and isotropy. It reduces the problem to one-dimensional calculation. For the calculation of losses, we can start with the admittance of square plate resonator [13]:

$$Y = j\omega \left(\epsilon_{33}^T \frac{a^2}{b} \right) \left(1 - k_p^2 \right) \left(1 + \frac{k_p^2 \tan \eta}{1 - k_p^2} \right) \quad (2)$$

where the meaning of the individual variables is as follows:

wave number $\eta = 2\pi f \frac{a}{2} \sqrt{\rho s_{11}^E (1 - \sigma^2)}$, Poisson's ratio $\sigma = -\frac{s_{12}^E}{s_{11}^E}$, planar electromechanical coupling factor $k_p^2 = \frac{2d_{31}^2}{\epsilon_{33}^T (s_{11}^E + s_{12}^E)}$, elastic compliances s_{11}^E , s_{12}^E , piezoelectric charge coefficient d_{31} , dielectric constant ϵ_{33}^T , density ρ , frequency f , angular frequency $\omega = 2\pi f$ and plate dimension a .

Losses are supposed to be small and they combine in loss factor for electromechanical coupling factor, wave number and Poisson's ratio

$$\begin{aligned} k_p^* &= k_p (1 - j\Gamma), \eta^* = \eta (1 - j\Omega), \sigma^* \\ &= \sigma \left(1 - j \tan \phi'_{12} + j \tan \phi'_{11} \right) \end{aligned} \quad (3)$$

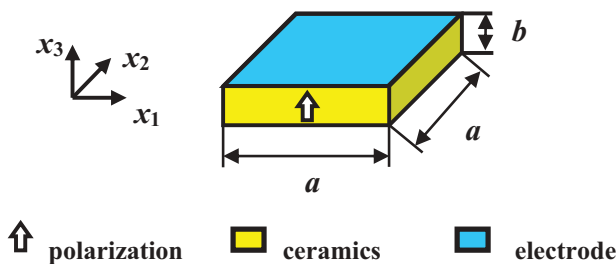


Figure 1. Square plate resonator.

where factors Ω and Γ could be expressed through loss factors of material coefficients

$$\Omega = \frac{1}{2(1 - \sigma^2)} \left[(1 + \sigma^2) \tan \phi'_{11} - 2\sigma^2 \tan \phi'_{12} \right] \quad (4a)$$

$$\begin{aligned} \Gamma &= \tan \theta'_{31} - \frac{1}{2} \tan \delta'_{33} \\ &\quad - \frac{1}{2(1 - \sigma)} \left(\tan \phi'_{11} - \sigma \tan \phi'_{12} \right) \end{aligned} \quad (4b)$$

Using losses up to first-order approximation, we can separate real and imaginary part of complex admittance Y^*

$$\text{Im} Y^* \cong \omega \left(\epsilon_{33}^T \frac{a^2}{b} \right) \left(1 - k_p^2 + k_p^2 \frac{\tan \eta}{\eta} \right) \quad (5a)$$

$$\begin{aligned} \text{Re} Y^* &\cong \omega \left(\epsilon_{33}^T \frac{a^2}{b} \right) \left\{ \tan \delta'_{33} \left(1 - k_p^2 + k_p^2 \frac{\tan \eta}{\eta} \right) \right. \\ &\quad \left. - 2\Gamma k_p^2 - k_p^2 \frac{\tan \eta}{\eta} \left(\Omega - 2\Gamma - \frac{\eta \Omega}{\tan \eta \cos^2 \eta} \right) \right\} \end{aligned} \quad (5b)$$

with $\tan \eta^* \cong \tan \eta - j \frac{\eta \Omega}{\cos^2 \eta}$. Motional part of admittance (2)

$$Y_m = j\omega \left(\epsilon_{33}^T \frac{a^2}{b} \right) k_p^2 \frac{\tan \eta}{\eta} \quad (6a)$$

is expressed in complex form by substitution of complex losses

$$\begin{aligned} Y_m^*(\eta) &\cong j\omega \left(\epsilon_{33}^T \frac{a^2}{b} \right) k_p^2 \frac{\tan \eta}{\eta} \\ &\quad \left(1 - j \tan \delta'_{33} - 2j\Gamma + j\Omega - j \frac{\eta \Omega}{\tan \eta \cos^2 \eta} \right) \end{aligned} \quad (6b)$$

Resonance condition for square resonator (fundamental resonance) is given by resonance wave number

$$\eta_r = \frac{\pi}{2} \quad (7)$$

Wave number in the vicinity of resonance could be calculated as $\eta = \eta_r + \Delta\eta$, where resonance means $\Delta\eta \rightarrow 0$. Approximation of $\tan \eta$ in the vicinity of resonance

$$\frac{1}{\tan \eta} \approx -\Delta\eta + j \frac{\pi}{2} \Omega \quad (8)$$

allows for the calculation of admittance maximum at resonance as

$$Y_{max} = |Y_m^*(\eta_r)| \cong j\omega \left(\epsilon_{33}^T \frac{a^2}{b} \right) k_p^2 \frac{4}{\pi^2 \Omega} \quad (9)$$

Mechanical quality factor at resonance Q_r , calculated by 3dB method (i.e. by the width of admittance curve at $|Y_m^*(\eta_r + \Delta\eta)| = \frac{1}{\sqrt{2}} Y_{max}$) is

$$Q_r = \frac{\eta_r}{2\Delta\eta} = \frac{1}{2\Omega}, \quad (10)$$

where $\Delta\eta = \pm \frac{\pi}{2} \Omega$ and $\eta_r = \frac{\pi}{2}$. Minimum of admittance at antiresonance Y_{min} is calculated from Equations (5a)

and (5b) by similar substitution of wave vector $\eta = \eta_a + \Delta\eta$, where antiresonance means $\Delta\eta \rightarrow 0$. Real and imaginary parts of admittance are

$$\text{Im}Y_{min}^* \cong \omega \left(\epsilon_{33}^T \frac{a^2}{b} \right) \left(1 - k_p^2 + k_p^2 \frac{\tan \eta_a}{\eta_a} \right) = 0 \quad (11a)$$

$$Y_{min} = \text{Re}Y_{min}^* \cong \omega \left(\epsilon_{33}^T \frac{a^2}{b} \right) \left\{ -2\Gamma + \Omega \left(1 - k_p^2 + \frac{k_p^2}{\cos \eta_a} \right) \right\}. \quad (11b)$$

Mechanical quality factor at antiresonance Q_a calculated by 3dB method (i.e. by the width of admittance curve at $|Y^*(\eta_a + \Delta\eta)| = \sqrt{2}Y_{min}$) needs to be solved from the condition

$$2Y_{min}^2 = |\text{Re}Y^*(\eta_a + \Delta\eta)|^2 + |\text{Im}Y^*(\eta_a + \Delta\eta)|^2 \quad (12)$$

where approximation up to the first power of $\Delta\eta$ is used

$$\tan \eta \approx \tan \eta_a \left(1 + \Delta\eta \frac{1}{\tan \eta_a \cos^2 \eta_a} \right) \quad (13)$$

Real and imaginary parts of admittance in the vicinity of antiresonance are

$$\text{Im}Y^* \cong \omega \left(\epsilon_{33}^T \frac{a^2}{b} \right) \left(\frac{\Delta\eta}{\eta_a} \right) \left(1 - k_p^2 + \frac{k_p^2}{\cos^2 \eta_a} \right) \quad (14a)$$

$$\begin{aligned} \text{Re}Y^* \cong \omega \left(\epsilon_{33}^T \frac{a^2}{b} \right) \left\{ \left(\frac{\Delta\eta}{\eta_a} \right) \left(1 - k_p^2 + \frac{k_p^2}{\cos^2 \eta_a} \right) \right. \\ \left. (\tan \delta'_{33} - \Omega + 2\Gamma) + \left(1 - k_p^2 + \frac{k_p^2}{\cos^2 \eta_a} \right) \Omega - 2\Gamma \right. \\ \left. - 2 \frac{(1 - k_p^2) \eta_a^2 \Omega}{\cos^2 \eta_a} \left(\frac{\Delta\eta}{\eta_a} \right) \right\} \end{aligned} \quad (14b)$$

Combination of Equations (12)–(14) results in equation

$$A^2 \left(\frac{\eta_a}{\Delta\eta} \right)^2 + 2AB \left(\frac{\eta_a}{\Delta\eta} \right) - (B^2 + C^2) = 0, \quad (15a)$$

where

$$A = -2\Gamma + \Omega \left(1 - k_p^2 + \frac{k_p^2}{\cos^2 \eta_a} \right), \quad (15b)$$

$$B = \left(-2\Gamma + \Omega - \tan \delta'_{33} \right) \left(1 - k_p^2 + \frac{k_p^2}{\cos^2 \eta_a} \right) + 2 \frac{(1 - k_p^2) \eta_a^2}{\cos^2 \eta_a} \Omega, \quad (15c)$$

$$C = 1 - k_p^2 + \frac{k_p^2}{\cos^2 \eta_a}. \quad (15d)$$

While it is $B \ll C$, Equation (15a) could be solved analytically

$$\Delta\eta \approx \pm \frac{C}{A} \eta_a \quad (16)$$

and mechanical quality factor at antiresonance is therefore

$$Q_a = \frac{\eta_a}{2\Delta\eta} = \frac{C}{2A}. \quad (17)$$

Relation between both mechanical quality factors at resonance Q_r and antiresonance Q_a could be written as

$$\frac{1}{Q_a} = \frac{1}{Q_r} - \frac{4\Gamma}{1 - k_p^2 + \frac{k_p^2}{\cos^2 \eta_a}}, \quad (18)$$

what may result or in $Q_a > Q_r$ for $\Gamma > 0$, or in $Q_a < Q_r$ for $\Gamma < 0$.

2.2. Width extensional mode of rectangular plate resonator

Let us assume resonator in shape of rectangular plate with electrodes deposited on major faces and poled in thickness direction. Dimensions and coordinate system are defined in Figure 2, where $l > w \gg b$. Such a resonator vibrates in transversal mode under AC voltage driving. Its vibration mode is width extensional mode with major transversal displacements along its width.

For the calculation of losses, we can start with the admittance of width extensional mode for the rectangular plate resonator [13]

$$Y = j\omega \left(\epsilon_{33}^T \frac{lw}{b} \right) \left(1 - k_p^2 \right) \left(1 + \frac{k_{31}'^2}{1 - k_{31}'^2} \frac{\tan \eta}{\eta} \right), \quad (19)$$

where all quantities are defined equally as for the square plate resonator, and only the electromechanical coupling factor differs

$$k_{31}'^2 = \frac{k_{31}^2}{1 - k_{31}^2} \frac{1 + \sigma}{1 - \sigma}, \quad k_{31}'^2 = \frac{d_{31}^2}{\epsilon_{33}^T s_{11}^E} \quad (20)$$

Losses are supposed to be small and they combine in loss factors for electromechanical coupling factors, wave number and Poisson's ratio

$$k_p^* = k_p(1 - j\Gamma), \quad k_{31}'^* = k_{31}'(1 - j\Theta) \quad (21a)$$

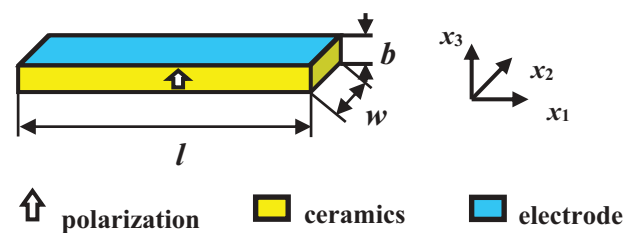


Figure 2. Rectangular plate resonator.

$$\eta^* = \eta(1 - j\Omega), \sigma^* = \sigma \left(1 - j \tan \phi'_{12} + j \tan \phi'_{11} \right) \quad (21b)$$

where factors Ω and Γ, Θ could be expressed through loss factors of material coefficients

$$\Omega = \frac{1}{2(1 - \sigma^2)} \left[(1 + \sigma^2) \tan \phi'_{11} - 2\sigma^2 \tan \phi'_{12} \right], \quad (22a)$$

$$\Gamma = \tan \theta'_{31} - \frac{1}{2} \tan \delta'_{33} - \frac{1}{2(1 - \sigma)} \left(\tan \phi'_{11} - \sigma \tan \phi'_{12} \right). \quad (22b)$$

$$\Theta = \frac{1}{1 - k_{31}^2} \left(\tan \theta'_{31} - \frac{1}{2} \tan \delta'_{33} - \frac{1}{2} \tan \phi'_{11} \right) + \frac{\sigma}{1 - \sigma^2} \left(\tan \phi'_{12} - \tan \phi'_{11} \right) \quad (22c)$$

Using losses up to first-order approximation, we can separate real and imaginary part of complex admittance Y^*

$$\text{Im}Y^* \cong \omega \left(\varepsilon_{33}^T \frac{l\omega}{b} \right) \left(1 - k_p^2 \right) \left(1 + \frac{k_{31}^2}{1 - k_{31}^2} \frac{\tan \eta}{\eta} \right), \quad (23a)$$

$$\text{Re}Y^* \cong \omega \left(\varepsilon_{33}^T \frac{l\omega}{b} \right) \left(1 - k_p^2 \right) \left\{ \left(\tan \delta'_{33} - 2 \frac{k_p^2}{1 - k_p^2} \Gamma \right) \left(1 + \frac{k_{31}^2}{1 - k_{31}^2} \frac{\tan \eta}{\eta} \right) + \frac{k_{31}^2}{1 - k_{31}^2} \frac{\tan \eta}{\eta} \left(2\Theta \frac{1}{1 - k_{31}^2} - \Omega + \frac{\eta\Omega}{\tan \eta \cos^2 \eta} \right) \right\} \quad (23b)$$

with $\tan \eta^* \cong \tan \eta - j \frac{\eta\Omega}{\cos^2 \eta}$. Motional part of admittance (19)

$$Y_m = j\omega \left(\varepsilon_{33}^T \frac{l\omega}{b} \right) \left(1 - k_p^2 \right) \frac{k_{31}^2}{1 - k_{31}^2} \frac{\tan \eta}{\eta} \quad (24a)$$

is expressed in complex form by substitution of complex losses

$$Y_m^*(\eta) \cong j\omega \left(\varepsilon_{33}^T \frac{l\omega}{b} \right) \left(1 - k_p^2 \right) \frac{k_{31}^2}{1 - k_{31}^2} \frac{\tan \eta}{\eta} \left(1 + 2j \frac{k_p^2}{1 - k_p^2} \Gamma - 2j\Theta \frac{1}{1 - k_{31}^2} + j\Omega - j \frac{\eta\Omega}{\tan \eta \cos^2 \eta} \right). \quad (24b)$$

Resonance condition for width extensional mode of rectangular resonator (fundamental resonance) is given by resonance wave number

$$\eta_r = \frac{\pi}{2} \quad (25)$$

Wave number in the vicinity of resonance could be calculated as $\eta = \eta_r + \Delta\eta$, where resonance means $\Delta\eta \rightarrow 0$. Approximation of $\tan \eta$ in the vicinity of resonance

$$\frac{1}{\tan \eta} \approx -\Delta\eta + j \frac{\pi}{2} \Omega \quad (26)$$

allows for the calculation of admittance maximum at resonance as

$$Y_{max} = |Y_m^*(\eta_r)| \cong j\omega \left(\varepsilon_{33}^T \frac{l\omega}{b} \right) \left(1 - k_p^2 \right) \frac{k_{31}^2}{1 - k_{31}^2} \frac{4}{\pi^2 \Omega} \quad (27)$$

Mechanical quality factor at resonance Q_r calculated by 3dB method (i.e. by the width of admittance curve at $Y_{max} = |Y_m^*(\eta_r + \Delta\eta)| = \frac{1}{\sqrt{2}} Y_{max}$)

is

$$Q_r = \frac{\eta_r}{2\Delta\eta} = \frac{1}{2\Omega} \quad (28)$$

where $\Delta\eta = \pm \frac{\pi}{2} \Omega$ and $\eta_r = \frac{\pi}{2}$. Minimum of admittance at antiresonance Y_{min} is calculated from Equations (23a) and (23b) by similar substitution of wave vector $\eta = \eta_a + \Delta\eta$, where antiresonance means $\Delta\eta \rightarrow 0$. Real and imaginary parts of admittance at antiresonance are

$$\text{Im}Y_{min}^* \cong \omega \left(\varepsilon_{33}^T \frac{l\omega}{b} \right) \left(1 - k_p^2 \right) \left(1 + \frac{k_{31}^2}{1 - k_{31}^2} \frac{\tan \eta_a}{\eta_a} \right) = 0 \quad (29a)$$

$$Y_{min} = \text{Re}Y_{min}^* \cong \left(\varepsilon_{33}^T \frac{l\omega}{b} \right) \left(1 - k_p^2 \right) \frac{k_{31}^2}{1 - k_{31}^2} \frac{\tan \eta_a}{\eta_a} \left(2\Theta \frac{1}{1 - k_{31}^2} - \Omega + \frac{\eta_a \Omega}{\tan \eta_a \cos^2 \eta_a} \right). \quad (29b)$$

Mechanical quality factor at antiresonance Q_a calculated by 3dB method (i.e. by the width of admittance curve at $|Y^*(\eta_a + \Delta\eta)| = \sqrt{2} Y_{min}$) needs to be solved from the condition

$$2Y_{min}^2 = |\text{Re}Y^*(\eta_a + \Delta\eta)|^2 + |\text{Im}Y^*(\eta_a + \Delta\eta)|^2 \quad (30)$$

where approximation up to the first power of $\Delta\eta$ is used

$$\tan \eta \approx \tan \eta_a \left(1 + \Delta\eta \frac{1}{\tan \eta_a \cos^2 \eta_a} \right) \quad (31)$$

Real and imaginary parts of admittance in the vicinity of antiresonance are

$$\text{Im}Y^* \cong \omega \left(\varepsilon_{33}^T \frac{l\omega}{b} \right) \frac{1 - k_p^2}{1 - k_{31}^2} \left(\frac{\Delta\eta}{\eta_a} \right) \left(1 - k_{31}^2 + \frac{k_{31}^2}{\cos^2 \eta_a} \right), \quad (32a)$$

$$\text{Re}Y^* \cong \omega \left(\epsilon_{33}^T \frac{lw}{b} \right) (1 - k_p^2) \left\{ \left[-2\Theta + \Omega \left(1 - k_{31}'^2 + \frac{k_{31}'^2}{\cos^2 \eta_a} \right) \right] \frac{1}{1 - k_{31}'^2} + \left(\frac{\Delta \eta}{\eta_a} \right) \frac{1}{1 - k_{31}'^2} \left(1 - k_{31}'^2 + \frac{k_{31}'^2}{\cos^2 \eta_a} \right) \right\} \quad (32b)$$

Combination of Equations (30)–(32) results in equation

$$A^2 \left(\frac{\eta_a}{\Delta \eta} \right)^2 + 2AB \left(\frac{\eta_a}{\Delta \eta} \right) - (B^2 + C^2) = 0, \quad (33a)$$

where

$$A = -2\Theta + \Omega \left(1 - k_{31}'^2 + \frac{k_{31}'^2}{\cos^2 \eta_a} \right), \quad (33b)$$

$$B = \left(2\Gamma \frac{k_p^2}{1 - k_p^2} - 2\Theta \frac{1}{1 - k_{31}'^2} + \Omega - \tan \delta_{33}' \right) \left(1 - k_{31}'^2 + \frac{k_{31}'^2}{\cos^2 \eta_a} \right) + 2 \frac{(1 - k_{31}'^2) \eta_a^2}{\cos^2 \eta_a} \Omega \quad (33c)$$

$$C = 1 - k_{31}'^2 + \frac{k_{31}'^2}{\cos^2 \eta_a}. \quad (33d)$$

While it is $B \ll C$, Equation (33a) could be solved analytically

$$\Delta \eta \approx \pm \frac{C}{A} \eta_a \quad (34)$$

and mechanical quality factor at antiresonance is therefore

$$Q_a = \frac{\eta_a}{2\Delta \eta} = \frac{C}{2A}. \quad (35)$$

Relation between both mechanical quality factors at resonance Q_r and antiresonance Q_a could be written as

$$\frac{1}{Q_a} = \frac{1}{Q_r} - \frac{4\Theta}{1 - k_{31}'^2 + \frac{k_{31}'^2}{\cos^2 \eta_a}}, \quad (36)$$

what may result or in $Q_a > Q_r$ for $\Theta > 0$, or in $Q_a < Q_r$ for $\Theta < 0$.

2.3. Length extensional mode of rectangular plate resonator

This type of resonator is a standard shape used for the measurement of material properties. Major displacement of vibrations is along the length l of resonator, where length is much bigger than other two dimensions, i.e. $l \gg w \gg b$. Solution of losses for this type of resonator was published previously in Ref [8]. Summary of results in used formalism is given by wave number

$$\eta = 2\pi f \frac{l}{2} \sqrt{\rho s_{11}^E}, \quad (37)$$

mechanical quality at resonance

$$Q_r = \frac{1}{2\Omega}, \quad (38a)$$

$$\Omega = \frac{1}{2} \tan \phi_{11}', \quad (38b)$$

and at antiresonance

$$\frac{1}{Q_a} = \frac{1}{Q_r} - \frac{4\Gamma}{1 - k_{31}'^2 + \frac{k_{31}'^2}{\cos^2 \eta_a}}, \quad (39a)$$

$$\Gamma = \tan \theta_{31}' - \frac{1}{2} \tan \delta_{33}' - \frac{1}{2} \tan \phi_{11}'. \quad (39b)$$

We may use combination of length extensional vibration mode (LE) of slender bar and either width extensional mode (WE) of rectangular plate or contour extensional mode (CE) of square plate to solve losses. Mechanical losses are calculated from Equations (38b), (4a) and (10), (39a, b)

$$\tan \phi_{11}' = \frac{1}{Q_{rL}}, \quad (40a)$$

$$\tan \theta_{31}' = \Gamma + \frac{1}{2} \tan \delta_{33}' + \frac{1}{2} \tan \phi_{11}', \quad (40b)$$

$$\Gamma = \frac{1}{4} \left(\frac{1}{Q_{rL}} - \frac{1}{Q_{aL}} \right) \left(1 - k_{31}'^2 + \frac{k_{31}'^2}{\cos^2 \eta_{aL}} \right), \quad (40c)$$

$$\tan \phi_{12}' = \frac{1 + \sigma^2}{2\sigma^2} \tan \phi_{11}' - \frac{1 - \sigma^2}{2\sigma^2} \frac{1}{Q_{rS}}, \quad (40d)$$

where subscript L means LE mode of bar and S means CE mode of square plate.

3. Experiment

Measurement of losses was performed on square plate resonators subsequently polished on one side to smaller width to the rectangular plate and finally to slender bar. One sample therefore covers all studied resonator geometries for the same material properties. Company poled and aged samples of soft PZT ceramics (types NCE51, NCE55) were supplied by CTS Ceramics Czech Republic s.r.o., Hradec Králové, Czech Republic. Original dimensions of samples were: NCE51 – 15 mm × 15mm/thickness 1 mm, NCE55 – 20 mm × 20mm/thickness 1 mm. Sample of lead-free ceramic BCZT (0.50Ba(Ti_{0.8}Zr_{0.2})O₃-0.50(Ba_{0.7}Ca_{0.3})TiO₃) has been re-

Table 1. Length extensional mode.

Material	w [mm]	f_r [kHz]	f_a [kHz]	Q_r [-]	Q_a [-]	k_{31} [-]	$\tan\phi'_{11}$ [10^{-4}]	$\tan\theta'_{31}$ [10^{-4}]	$\tan\delta'_{33}$ [10^{-4}]
NCE51	3	93.163	99.350	72	105	0.386	139	301	145
NCE55	3	68.755	72.525	64	81	0.354	156	318	188
BCZT	3.50	195.910	199.810	85	93	0.218	118	313	257

polished from disc shape into square plate with dimensions 9.40 mm $\times$ 9.40 mm/thickness 1.06 mm. One side of each sample was gradually polished by SiC400 in steps of 1 mm in sample width.

Measurement of impedance spectrum (i.e. f_r, f_a, Q_r, Q_a) and identification of vibration modes in it were performed by Precision Impedance Analyzer (Agilent 4294A) after each polishing step. The impedance spectra were measured at a low power signal (0.5 V oscillation level). No distortions of resonance/antiresonance peaks were observed. Dielectric loss ($\tan\delta'_{33}$) was measured directly by LCR meter (type LCR-821, GW Instek).

Results are summarized in Tables 1–3. Losses are calculated from Equations (40a–d), electromechanical coupling factors from resonance/antiresonance conditions for each specific resonator mode. Poisson's ratio for PZT ceramics was measured previously by the manufacturer. Poisson's ratio for BCZT ceramic was measured by the method of composite vibration mode according to Standard IEC483 [14]. Width of rectangular plate for WE mode was chosen according to the impedance spectrum, which was free of mode coupling at measured resonance/antiresonance frequencies. WE mode may be coupled with other modes and their overtones. Any resonance/antiresonance frequency for WE mode uncoupled to other modes in the impedance spectrum could be used for calculations of losses. As it is seen from Equations (4a) and (22a), the resonance mechanical quality Q_r is the same for WE and CE modes due to the same wave vector dependence on material coefficients. Frequencies for CE mode could be easily found in the impedance spectrum, because it is the lowest frequency mode, similarly to LE mode of slender bar. Square plate impedance spectrum was free of unwanted spurious modes for all studied materials.

4. Discussion and conclusions

Loss coefficients for all studied materials (soft PZT ceramic types, lead-free BaTiO₃-based ceramic) are of the order 10^{-2} and it satisfies the assumptions used in calculations. Mechanical quality factors at antiresonance are higher than at resonance for all vibration modes and all samples as seen in Equations (18), (36) and (39a), differences might be substantial. Measurement of mechanical quality factor by 3dB methods from impedance spectra is however sensitive to the mechanical clamping of the sample during measurement. Poisson's ratio σ must be available for the calculations, i.e. evaluated from other sample resonance (e.g. radial mode of thin disc resonator and composite mode of the same disc with divided electrode [14]). Neither CE nor WE mode resonances are able to provide such information.

The mechanical quality factor at resonance Q_r is exactly the same for CE and WE modes as demonstrated in Equations (4a) and (22a) and Tables 2 and 3. Loss factors $\tan\phi'_{12}$ calculated from WE and CE mode are similar for PZT ceramics but differ significantly for BCZT ceramic because of mode coupling observed in CE mode spectrum of square plate. Both CE and WE modes are however applicable for the loss coefficients determination in case the impedance spectrum is free of spurious or coupled modes. WE mode resonance is hidden between various modes in an impedance spectrum and it cannot be easily identified in it. It might be coupled also to other vibration modes or their overtones. LE and CE modes are the resonances with the smallest frequency observable in the spectrum and therefore easily identifiable. The mechanical quality factors satisfy the same relationship $Q_a > Q_r$ for all studied materials (soft PZT, type NCE51, NCE55 and BCZT ceramics), which does not hold generally as it was found for k_{31} mode [15].

Measured losses for PZT ceramics agree well in order of magnitude with previously published data [11] for soft PZT; type APC850 is similar in properties to NCE51 ceramic type. It is worth to mention that the smallest losses are for the mechanical properties, followed by the dielectric losses and the biggest one is observed for the piezoelectric losses, which are usually not taken into account at all. Losses for BCZT lead-free ceramic were not previously published. Our results show that BCZT has losses of the same magnitude as soft PZT ceramics.

Presented methods for the loss and material property measurement are applicable in the case of only single sample, but it requires sample re-polishing from square to rectangular plate and further to slender bar resonator. Only electrical measurement based on the impedance spectra is needed without any advanced technology for the strain

Table 2. Contour extensional mode.

Material	w [mm]	f_r [kHz]	f_a [kHz]	Q_r [-]	Q_a [-]	k_p [-]	$\tan\phi'_{12}$ [10^{-4}]
NCE51	15	108.844	130.213	77	143	0.612	178
NCE55	20	81.375	94.313	70	108	0.563	235
BCZT	9.40	235.200	251.500	105	192	0.394	188

Table 3. Width extensional mode.

Material	w [mm]	f_r [kHz]	f_a [kHz]	Q_r [-]	Q_a [-]	k'_{31} [-]	σ [-]	$\tan\phi'_{12}$ [10^{-4}]
NCE51	10	149.275	169.750	76	115	0.530	0.32	171
NCE55	15	99.938	113.100	66	93	0.521	0.28	184
BCZT	7.70	270.600	288.250	77	161	0.383	0.37	79

characterization like it is presented in Ref [7]. Care must be taken to the proper identification of WE mode and its coupling to other modes. Appropriate aspect ratios for thin square plate and slender bar must be satisfied.

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