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ÚSTAV AUTOMATIZACE A INFORMATIKY

Synchronization and Communication Using Deterministic Chaos

MASTER'S THESIS
DIPLOMOVÁ PRÁCE

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Assignment Master's Thesis

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Student:	**Bc. Anna Stolnikova**
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Supervisor:	**doc. Ing. Radomil Matoušek, Ph.D.**
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As provided for by the Act No. 111/98 Coll. on higher education institutions and the BUT Study and Examination Regulations, the director of the Institute hereby assigns the following topic of Master's Thesis:

Synchronization and communication using detereministic chaos

Brief Description:

The master thesis is focused on synchronization and chaos control. We introduce history and basic issue of this new science. We show synchronization of two chaos systems due to cybersecurity safe communication. We use already exist example for synchronization. And this example we programme with help of Matlab. Because theme is difficulty and complexity the master thesis is focused on basic information and principle of synchronization and chaos control.

Master's Thesis goals:

1/ Review and introduction to deterministic chaos and chaos synchronisation systems and methods.
2/ Principle of cryptograpgy with chaos systems, chaotic systems which can be use.
3/ Practical realisation – description of the design and implementation. Practical experiments in Matlab/Simulink.
4/ Presentation of results, conclusions and suggestions.

Recommended bibliography:

ŠENKEŘÍK, Roman. Optimal control of deterministic chaos: Optimální řízení pomocí deterministického chaosu : doctoral thesis summary. Zlín: Tomas Bata University in Zlín, 2008. ISBN 978-80-7318--81-1.

ČELIKOVSKÝ, Sergej. Chaos synchronization via nonlinear observers with application to secure encryption: Synchronizace chaotických oscilátorů pomocí nelineární rekonstrukce a její využití k bezpečnému šifrování. V Praze: České vysoké učení technické, 2007. ISBN 978-80-01-03723-2.

KOLOUCH, Jan a Pavel BAŠTA. CyberSecurity. Praha: CZ.NIC, z.s.p.o., 2019. CZ.NIC. ISBN 978-8-88168-31-7.

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ABSTRACT

The master thesis is focused on synchronization and chaos control. We introduce history and basic issue of this new science. We show synchronization of two chaos systems due to cybersecurity safe communication. We use already exist example for synchronization. And this example we implement in Simulink software. Because theme is difficult and complex the master thesis is focused on basic information and principle of synchronization and chaos control.

KEYWORDS

Deterministic chaos, chaos synchronization, secure communication

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DECLARATION

I declare that this thesis is my original work, I prepared it independently under the guidance of doc. Ing. Radomil Matousek, Ph.D. and using the literature listed in the bibliography.

Brno, 01. 10. 2020 ………………………………………………

Anna Stolnikova

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1 INTRODUCTION

Deterministic chaos and its application for the transmission of information have received a lot of attention lately. Methods of direct chaotic information transmission using pulses received from a dynamic chaos signal generator have been developed and patented; there are corresponding prototypes of devices.

When we talk about **determinacy**, we mean an unambiguous relationship between cause and effect. If a certain initial state of the system is given at $t = t_0$, then it uniquely determines the state of the system at any moment of time $t > t_0$.

If we talk about **chaos**, we mean that the change in the state of the system over time is random (it cannot be unambiguously predicted) and irreproducible (the process cannot be repeated). We come to the conviction that the concepts of determinism and chaos are directly opposite in meaning. Determinism is associated with complete predictability and reproducibility, chaos with complete unpredictability and non-reproducibility. The picture will change fundamentally if we consider a dynamical system, the state of which is characterized by three independent variables (phase coordinates).

The trajectory unwinds in three-dimensional space, moving away from point O in a spiral. Having reached some values and experiencing the action of the nonlinear limiting mechanism, the trajectory will again return to the vicinity of the initial state. Further, due to instability, the process will be repeated. If the trajectory reproduces a certain aperiodic process, then at $t \to \infty$ no closure will occur. This case corresponds to the regime of **deterministic chaos**.

Numerous theoretical and experimental works have shown that the phenomenon of dynamic chaos can be widely used in various fields of science and technology, in particular, by creating new technologies based on it. Chaos theory has been widely used in cryptography, robotics (in order to build a predictive model), biology, chemistry.

Possibilities of hidden transmission of information signal based on the effect of complete synchronization of chaotic systems with the same or similar parameters are investigated. In this case, the information is encoded by modulating (changing) one or more parameters of the corresponding differential equations.

One of the promising areas of using deterministic chaos is its application in communication technologies. It has a number of properties that can be useful in the transmission and processing of information. For example, the use of dynamic chaos makes it possible to obtain complex oscillations using devices that are simple in structure, while a large number of different chaotic modes can be realized in one device; control of chaotic regimes by small changes in system parameters; increasing the modulation rate in relation to the modulation of regular signals due to the sensitivity of the chaotic system to external disturbances. Chaotic signals make it possible to use a variety of methods for inputting an information signal into a chaotic one; increase the level of confidentiality when transferring messages. An important feature of chaotic systems is the ability to self-synchronize the transmitter and receiver. Finally, in communication systems on chaotic signals, it is possible to implement unconventional methods of multiplexing and demultiplexing.

Chapter 2 of this work includes the Introduction to chaos and review of known synchronization methods. Chapter 3 offers review of chaos synchronization systems, including simulation of abstract equations using Simulink software. Chapter 4 shows the review of using the chaos synchronization in digital data transmission.

2 INTRODUCTION TO CHAOS AND CHAOS SYNCHRONIZATION SYSTEMS AND MODELS

2.1 Introduction to Deterministic Chaos – Definitions and Terms

The existence of chaos in any system is not necessarily related to an action of some random forces. The nature of chaos behaviour of fully determined systems lies in the capability of getting exponential instability of trajectory while having specific parameters. Fundamental meaning of research in this field is to find out the nature of chaos.

That is why it is important to start with review of what the chaos is. Dynamic chaos (also called deterministic chaos) is a phenomenon in the theory of dynamical systems in which the behaviour of a nonlinear system looks random, despite the fact that it is determined by deterministic laws. The name deterministic chaos is often used as a synonym; both terms are completely equivalent and are used to indicate a significant difference between chaos as a subject of scientific study in synergetics from chaos in the ordinary sense. [1]

The reason for the appearance of chaos is instability (sensitivity) with respect to the initial conditions and parameters: a small change in the initial condition with time leads to arbitrarily large changes in the dynamics of the system.

Dynamics, which is sensitive to the slightest changes in the initial conditions of the system, from which its development, change begins, and in which these slightest deviations multiply over time, making it difficult to predict future states of the system, is often called chaotic.

For example, we know the trajectory of a mechanical system if the initial conditions are given. If the system were stable, not chaotic, then with small changes in the initial conditions, the new trajectory would not differ much from the previous one, it is even possible that the new trajectory would coincide with the previous one with time. But if the system were chaotic, unstable, then at first the old and new trajectories could be close, but over time the trajectories would become completely different, that is, the system would show high sensitivity to the initial data of the motion problem.

Since the initial state of a physical system cannot be specified absolutely accurately (for example, due to the limitations of measuring instruments), it is always necessary to consider a certain (albeit very small) region of initial conditions. When moving in a limited region of space, the exponential divergence of close orbits over time leads to mixing of the initial points throughout the region. After such mixing, it practically makes no sense to talk about the coordinate of a particular particle; it is more expedient to go over to a statistical description of the process, that is, to determine the probability of finding a particle at a certain point.

2.2 Chaos Synchronization

The emergence of synchronous generation in networks consisting of elements demonstrating complex dynamics is one of the most important nonlinear phenomena attracting wide attention of researchers. As a result of studying this phenomenon, which is of interest from both a theoretical and a practical point of view, various types of synchronous behaviour of chaotic subsystems were discovered. Let us dwell in more detail on the description of the features of some of them, namely: phase, generalized, complete synchronization, as well as synchronization with delay (the so-called lag synchronization).

The definition of **phase synchronization** is based on the concept of the instantaneous phase of the signal under investigation.

In view of the fact that the instantaneous phase should uniquely characterize the state of the system, its determination is possible only for systems with a fairly simple topology of a chaotic attractor (systems with a phase-coherent attractor). For attractors of this type, the projection of the phase trajectory onto a certain plane of states rotates all the time around a certain centre.

The phenomenon of **generalized synchronization** was first discovered in the mid-1990s [2] for a system of two unidirectionally coupled chaotic oscillators:

$$\dot{x_d} = f(x_d)$$
$$\dot{x_r} = g(x_r, h(x_d)), \tag{1}$$

Where $x_d \in \mathbb{R}^m, x_r \in \mathbb{R}^n$, function $h(x_d) : \mathbb{R}^m \to \mathbb{R}^n$ defines the relationship between

master x_d (drive) and slave x_r (response) subsystems.

The generalized synchronization mode means that after the completion of the transition process between the states of interacting chaotic oscillators, a functional relationship is established

$$x_r(t) = \Psi[x_d(t)]. \tag{2}$$

With a certain interaction of identical chaotic subsystems, one can observe the phenomenon of **complete synchronization** of chaos, when, after the completion of the transition process between the states of interacting chaotic oscillators, the evolution in time of the states of the subsystems occurs in the same way. It is easy to see that complete synchronization is a special case of generalized synchronization if we take the dependence $x_1(t) = \Psi[x_2(t)] \equiv x_2(t)$.

Synchronization of self-oscillatory processes is one of the fundamental nonlinear phenomena, which for several centuries since Huygens [3] (who first described this phenomenon as an example of coupled mechanical systems (pendulum clocks))

In recent decades, the centre of research in this field has shifted to the study of synchronization of chaotic self-oscillations, which is due to the great interest in nonlinear physics in the problem of deterministic chaos and various applications of chaos theory. Therefore, the study of chaotic synchronization has become a natural development of the theory of dynamic chaos, which is due to both the great fundamental importance of the study of chaotic synchronization and its wide practical applications, for example, in the hidden transmission of information, in biological, physiological and chemical problems, when controlling chaos, including in microwave systems, microwave electronics, etc.

Recently, the attention of researchers has been increasingly attracted not only by radio physical models and systems for which the main results in this field were obtained, but also by wildlife systems (in particular, the effect of an external stimulus on the brain electroencephalograms, the interaction of the rhythms of the respiratory and cardiovascular systems, synchronization of the dynamics of neural ensembles of various parts of the brain of a person with epilepsy, etc.). These applications are very important and are increasingly used in physiology and medicine, in the processing of experimental data. However, the possibilities of using chaotic synchronization are not limited to physiological and medical applications. One of the interesting, important, and rapidly developing areas is the application of the chaotic effect of synchronization in telecommunication tasks, first of all, when creating systems for covert information transmission. At the same time, there is very little review work on the use of chaotic synchronization in information and telecommunication systems. So, over the past ten years alone, according to ISI Web of Knowledge, the number of publications on this topic has increased by more than 50 times. Finally, it is important to note that in recent years there has been a transition from a theoretical consideration of the problem to the creation of practical models that made it possible to transmit information based on chaotic synchronization for several tens of kilometres using previously created telecommunication systems. All of the above indicates the importance and relevance of the considered direction and the need for a review of this topic, which would summarize, generalize and, no less important, allow you to compare the results obtained in this direction.

Most methods of covert information transmission using chaos synchronization are based primarily on the complete chaotic synchronization mode, which entails the requirement for a high degree of identity of generators located on different sides of the communication channel. In connection with the discovery and intensive study of other types of synchronous behaviour of coupled chaotic systems, like phase synchronization. Generalized synchronization, delay synchronization, noise-induced synchronization, improvement of methods for covert data transmission based on them has become one of the important research tasks in the field of creating information and telecommunication systems based on dynamic chaos.

One of the important problems, from the point of view of information transmission, is the effect of noise and signal distortion on the performance of information transmission schemes. It is known that noise almost always affects the dynamics of systems, and this effect can lead to significant changes in the behaviour of systems, which with respect to information transmission schemes based on the phenomenon of chaotic synchronization, can negatively affect their performance. Nonlinear distortions can also lead to reduced performance of such circuits. Meanwhile, the consideration of information transfer schemes based on the use of chaotic synchronization phenomenon modes is carried out in the vast majority of cases under the assumption of no noise and distortion, which leaves a number of possibilities for the practical application of these schemes and their effectiveness open.

2.3 Methods of Secure Communication Based on the Phenomenon of Complete Chaotic Synchronization

We will be considering the complete chaotic synchronization as most of known methods and devices are based specifically on this type of synchronized behaviour. The use of complete chaotic synchronization for the covert transmission of information implies the presence of at least two unidirectionally coupled identical generators.

2.3.1 Chaotic Masking

Chaotic masking is one of the first and simplest methods for covert data transmission [4].

The principal implementation scheme of this method is shown in Fig. 1 [17]. On the transmitting side, the information signal $m(t)$ is mixed in the adder to the carrier signal generated by the chaotic transmitting system $x(t)$, and then transmitted through the communication channel. The receiver performs complete chaotic synchronization of the chaotic generator $u(t)$ located in it using the received signal, as a result of which the dynamics of the receiving generator becomes identical to the dynamics of the transmitting one. The detected signal $\widetilde{m}(t)$ is obtained after passing through a subtractor as the difference between the received signal and the synchronous response of the chaos generator in the receiver [5].

Such a hidden data transmission scheme works quite efficiently (i.e., it allows to qualitatively transmit information and detect it at the output) in the absence of noise in the communication channel in the case when the signal power generated by the transmitting system exceeds the power of the information signal by 35–65 dB [6]. Adding noise to the communication channel leads to a sharp deterioration in the quality of the transmitted information, and therefore to high signal-to-noise ratios, at which the circuit remains operational. In addition, the introduction of mismatch of control parameters between identical chaotic oscillators (located on different sides of the communication channel) also leads to the appearance of additional desynchronization noises and makes the transmission of information difficult to implement. Moreover, there is a problem of confidentiality of transmission of information 2. Despite the low level of the information signal compared to the level of the carrier, there are methods and approaches that allow you to restore the original chaotic signal from the signal transmitted through the communication channel, and therefore, highlight useful information [7].

All the above drawbacks make the schemes of covert information transmission based on chaotic masking inapplicable in practice.

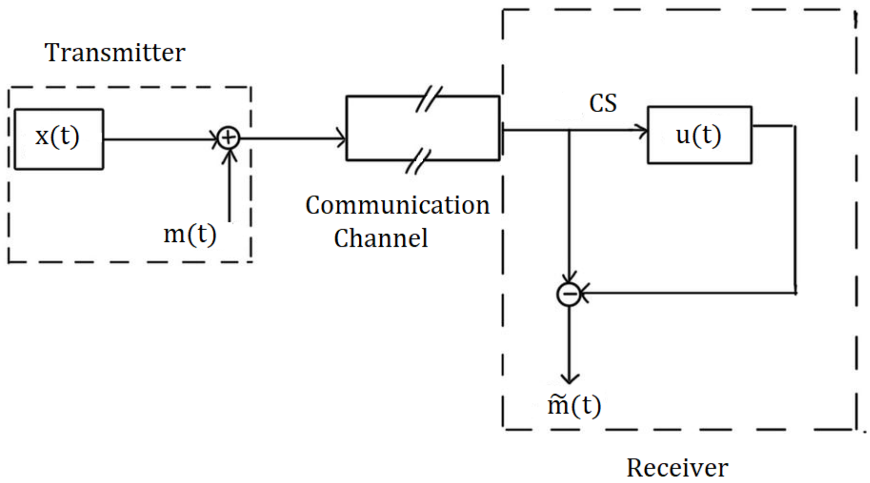

Figure 1. The scheme of hidden information transmission using chaotic masking (CS – complete synchronization) [18]

2.3.2 Switching Chaotic Models

In addition to chaotic masking, several other methods of covert data transmission were proposed, united under the general name "switching of chaotic modes" [8]. One of the schemes for switching chaotic modes is shown in Fig. 2 [17]. The transmitting device contains two chaotic generators, $x_1(t)$ and $x_2(t)$, which can be different or the same, but with different parameters, however, in the interest of confidentiality of data transmission, it is preferable to use the latter; moreover, the signals generated by these systems must have similar spectral and statistical properties. The useful digital signal $m(t)$, represented by a sequence of binary bits 0/1, is used to switch the transmitted signal, i.e. the signal produced by the first chaotic generator encodes, for example, binary bit 0, and the signal from the second random bit 1. The chaos receiver, thus corresponding signal is transmitted via the communication channel to the receiving device. Depending on the number of generators located on the receiving side of the communication channel, several schemes of covert data transmission based on switching of chaotic modes are distinguished. In the circuit shown in Fig. 2 [17], the receiving device comprises one chaotic generator $x(t)$. identical to any of the transmitters, for example the first. The parameters of the generators should be chosen in such a way that the signals generated by them lead to the appearance of complete chaotic synchronization only if only binary bit 0 (or only binary bit 1) is transmitted. As in the case of chaotic masking, the reconstructed signal $\tilde{m}(t)$ is obtained after passing through the subtractor a signal transmitted through the communication channel and the synchronous response of the chaotic generator of the receiving device.

Other schemes of covert information transmission using switching that are based on the same idea differ from the above scheme only in the structure and operation of the receiving device. Such data transmission schemes are more resistant to noise in the communication channel than schemes with chaotic masking, but their noise immunity, however, remains very limited. The fundamental drawback of such schemes is the occurrence of transient processes during switching (the duration of which can be very long) [9], which manifests itself in the time delay of switching on the receiving generator in the synchronous mode. Therefore, such schemes are rather slow. In addition, the degree of secrecy (confidentiality) of such schemes remains rather low [10].

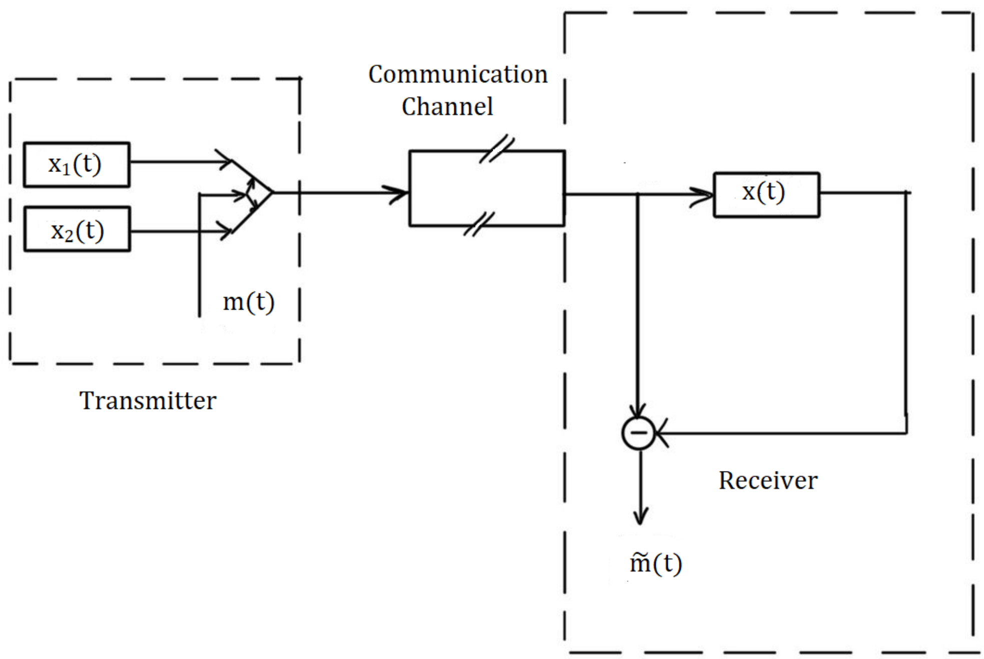

Figure 2. The scheme of hidden information transmission using chaotic switching [18]

2.3.3 Nonlinear Mixing of Information Signal with Chaotic

Improvements in the method of chaotic masking were aimed at increasing the secrecy and confidentiality of information transmission. As a result, several methods were proposed that can be combined under the general name “nonlinear mixing of an information signal with a chaotic”. A feature of the operation of such schemes is the direct input of the information signal into the transmitting system and its participation in the formation of the output signal [11].

Among the schemes in which various operations are used ("addition - subtraction", "division - multiplication", "addition modulo with base 2", "voltage-current conversion", etc.), the most widespread now are schemes using “addition - subtraction”. In such schemes, the information signal is mixed with the chaotic one and thereby participates in the formation of the complex behaviour of the system. The simplest and most technically feasible way of providing "nonlinear mixing" is to install an additional chaotic generator on the transmitting side of the communication channel that is identical to the first transmitting one and mutually connected with it. A schematic diagram of the implementation of this method of covert data transfer is shown in Fig. 3 [17].

So, the transmitting side contains two chaotic generators identical in control parameters, $x_1(t)$ and $x_2(t)$. The information signal $m(t)$ is mixed with the signal produced by one of the generators of the transmitting device (or both signals at the same time). The signal (provided by the interconnection of the generators of the transmitting device) undergoes non-linear changes. Thus, a signal obtained by non-linear mixing of an information signal with a chaotic one will be transmitted through a communication channel. The receiving device, as in the above schemes, contains a chaotic generator $x(t)$, identical in terms of control parameters to the transmitting generators. The signal transmitted via the communication channel to the receiving device synchronizes the receiving generator in the case of transmission of binary bit 0 (and does not synchronize when transmitting binary bit 1). After passing through the subtractive device the signals from the transmitting and receiving generators, the reconstructed signal $\widetilde{m}(t)$ is detected.

An important advantage of such schemes over schemes based on chaotic masking is the possibility of varying the level of the input information message, which makes it possible to control the quality of information transmission (that is, varying the accuracy of decryption of the initial information message by the receiving party). However, an increase in the quality of information transmission entails a loss of confidentiality, which is a significant drawback [5]. In addition, such schemes are characterized by the low noise immunity in the communication channel and the mismatch of the control parameters of initially identical chaotic generators. The need to ensure the identity of the three chaos generators, two of which are on different sides of the communication channel, is a difficult technical task, and therefore, is one more disadvantage of such a scheme.

In addition, the dependence of the transmitted signal on the information one, since the transmitting generator is essentially a non-autonomous system, which does not guarantee that it forms a chaotic signal when changing certain circuit parameters, can lead to loss of confidentiality.

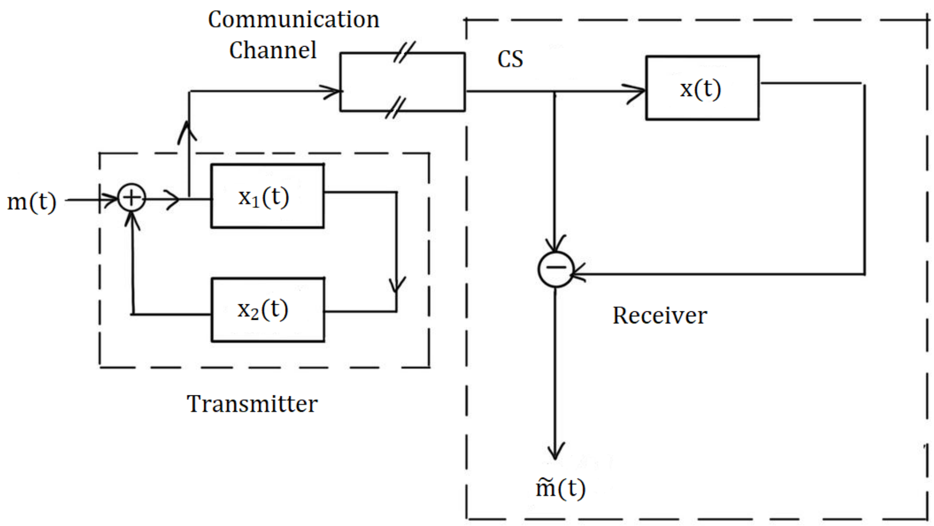

Figure 3. The scheme of hidden information transmission using nonlinear mixing of information signal with chaotic [18]

2.3.4 Modulation of Control Parameters of a Transmitting Generator with an Information Signal

Schemes based on modulating control parameters are a natural step in the process, or adaptive methods, of switching from discrete modulation of the control parameter of a transmitting generator in a circuit with switching chaotic modes to modulation with a continuous signal [12]. The role of the modulating signal is played by the information signal. An indispensable condition for the implementation of such schemes is a preliminary determination of the allowable range of parameter variation and rationing of the modulating information signal. A special case is the use of a binary digital signal as an information signal and its modulation of the control parameter of the transmitting generator. The scheme of the hidden transmission of information in this way is shown in Fig. 4 [17]. The principle of its operation is similar to the principle of operation of the circuit based on the switching of chaotic modes described in section 2.4.2. The useful digital signal $m(t)$ modulates one of the parameters of the transmitting generator $x(t)$ so that, depending on the transmitted binary bit 0 (1), between the transmitting $x(t)$ and the receiving $u(t)$ generators exists (absent) the regime of complete chaotic synchronization. Then, after passing through the subtractive device the signals of the transmitting and receiving devices, the reconstructed signal $\widetilde{m}(t)$ is detected. In order to be able to realize the full synchronization mode, the control parameters of the receiving generator must be chosen identical to the control parameters of the transmitting (more precisely, one of the sets of transmitting parameters generator, corresponding, for example, to a binary bit 0).

The operation features, advantages and disadvantages of circuits based on modulating control parameters are the same as in the case of switching circuits. However, for the considered circuit the technical implementation is somewhat simplified due to the presence of only one generator on the transmitting side of the communication channel.

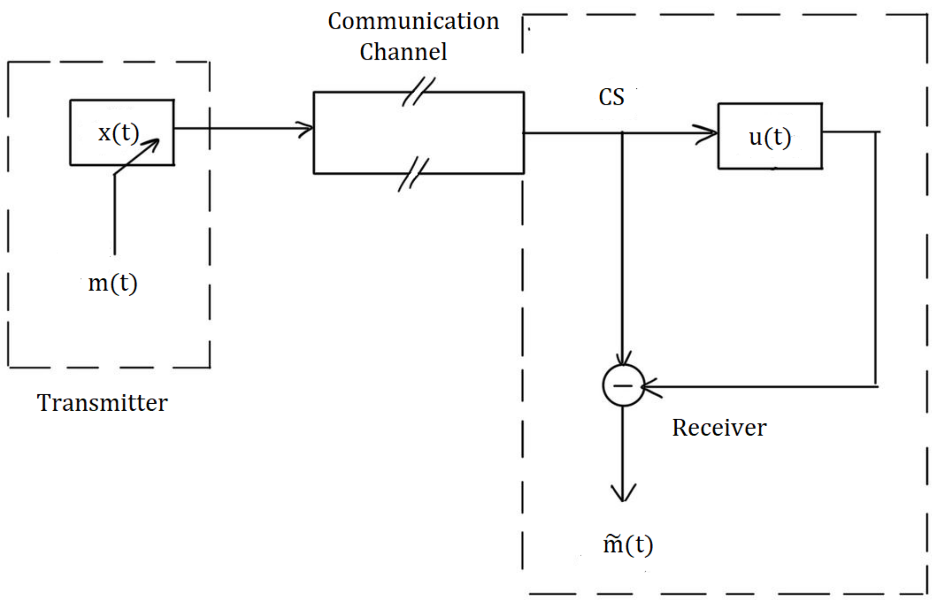

Figure 4. The scheme of hidden information transmission using modulation of control parameters of a transmitting generator with an information signal [18]

2.4 Using Other Types of Chaotic Synchronization for Covert Information Transfer

In Section 2.3, we examined the main types of schemes for covert information transmission based on complete chaotic synchronization. There are other schemes, but they are varieties of already known schemes and are not of fundamental interest, reflecting rather these or those features of the technical scheme. The schemes considered in Section 2.3.1-2.3.4 are the simplest systems that provide the basis for using chaotic synchronization for covert transmission data. It is clear that none of them is not without flaws, which we mentioned in section 2.3 so far. Further research goes towards creating an implementation of new circuits, in which attempts are made to eliminate these drawbacks, increasing in some cases the confidentiality of circuits, in some – noise immunity, in some generators identity and thereby providing the possibility of a simpler technical implementation of circuits. The natural way in this case is the transition from complete chaotic synchronization to other types of synchronous behaviour.

2.4.1 Method of Covert Information Transmission Based on Generalized Synchronization

One of the few works that use the generalized synchronization mode for covert information transmission is work by Terry J R, Chaos Solitons Fractals [13]. A schematic diagram of the implementation of this method of covert data transmission is shown in Fig. 5 [17]. The transmitting side contains two chaotic generators, the leading $x(t)$ and the driven $u(t)$, which may be non-identical. The signal from the master oscillator is transmitted to the slave, and its intensity is modulated by useful digital signal $m(t)$ in its way: if binary bit 0 is transmitted, then the mode of generalized synchronization is set between the master and slave generators, and if binary bit 1 is transmitted, the mode of generalized synchronization between them is destroyed. On the receiving side of the communication channel is the so-called auxiliary chaotic generator $v(t)$, which is identical to the slave in terms of control parameters. The signal from the master oscillator through the communication channel is transmitted to the auxiliary one, which ensures the appearance of a generalized synchronization mode between them, and the intensity of the signal transmitted through the communication channel must coincide with the intensity of the signal received by the slave system when transmitting binary bit 0. The signal from the slave generator is already transmitted to the receiving side through another communication channel. As in the methods of covert data transmission based on the full chaotic synchronization mode, the receiving side has at its disposal both a chaotic signal containing useful information and a signal without it. Therefore, it is easy to isolate a useful digital signal $\widetilde{m}(t)$ by simply subtracting one signal from another.

It is easy to see that in such a scheme for covert information transfer, the auxiliary system method is actively used, which requires two chaotic generators identical in control parameters. As in schemes based on the complete chaotic synchronization mode, these generators are located on different sides of the communication channel, which is a significant problem from the point of view of the technical implementation of this method. A slight mismatch in the values of the control parameters in these systems leads to the appearance of desynchronization noise, making such a circuit inoperative. In addition, the implementation of two communication channels is a significant drawback, not only due to additional costs during implementation, but also because the presence of two channels contributes to the appearance of additional noise in the communication channel (possibly even of a completely different nature), distorting the transmitted signal. Therefore, such a scheme for covert data transmission is characterized by rather low noise immunity in the communication channel and is difficult to implement in practice.

There are also problems with the confidentiality of the transmission of information. It is clear that the use of another type of synchronous behaviour, as well as the presence of an additional communication channel, from this point of view, play a positive role. However, just as in schemes based on the nonlinear mixing of an information signal with a chaotic one (see Section 2.3.3), an increase in the quality of the transmitted information entails a loss of confidentiality. But this problem here is less significant in comparison with a similar problem for circuits based on the regime of complete chaotic synchronization.

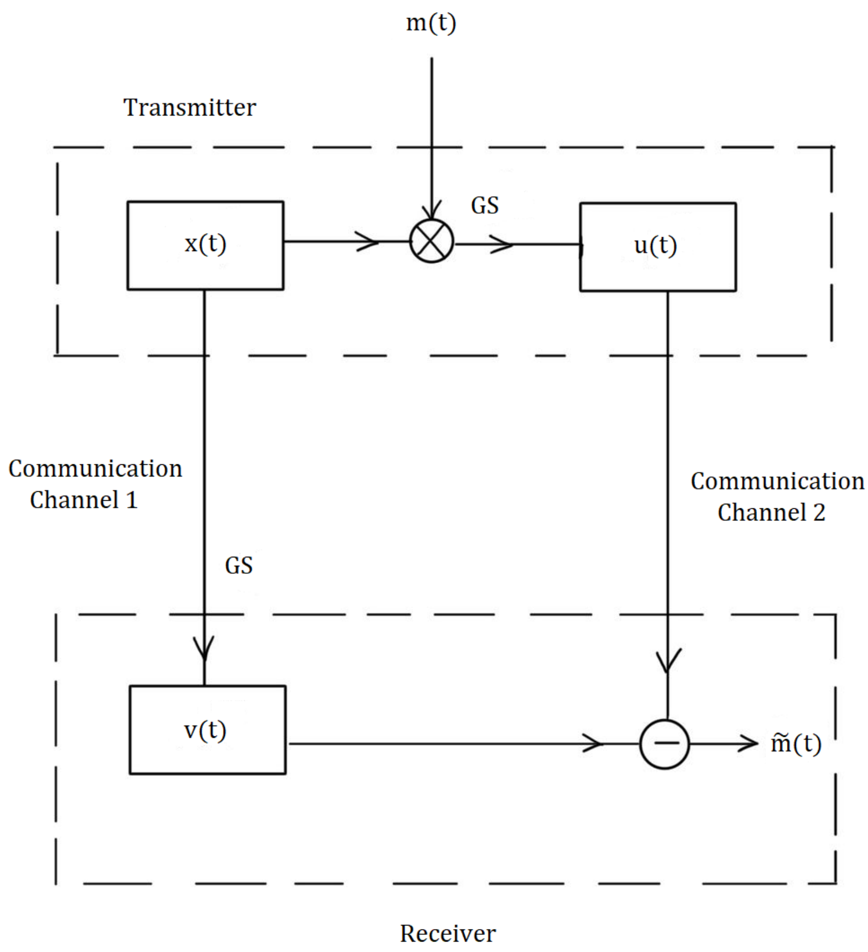

Figure 5. Scheme of hidden information transmission based on generalized synchronization (GS - generalized synchronization) [18]

2.4.2 Using Several Types of Synchronous Behaviour for Covert Information Transfer

You can increase the confidentiality of information transfer by using several types of synchronous behaviour simultaneously. For example, in methods of covert data transmission were proposed, using simultaneously the generalized and complete chaotic synchronization modes [14].

The circuit proposed in (Fig. 6 [18]) is a modification of the circuit considered in Section 2.4.1. The principle of operation of the transmitting device is similar to the principle of operation of the transmitting device of the circuit described in Section 2.4.1. The modification consists in the fact that on the receiving side of the communication channels there is an additional chaotic generator $x_2(t)$, which is identical to the leading $x_1(t)$ in terms of control parameters (hereinafter the second leading generator). The signal generated by the host system is transmitted through the first communication channel, putting the second lead generator in full synchronization mode. Confidentiality can be increased due to the fact that the signals arriving at the slave and second master generators can be different (for example, a signal representing the x-coordinate of the master system and the second master signal representing y-coordinate). On the receiving side of the communication channel signal from the second leading generator, acting on the auxiliary, provides the emergence of the signal mode, which is a generalized synchronization between them. The signal from the slave generator is supplied via the second communication channel to the receiving side. Due to the identity of the signals acting on the slave and auxiliary generators, as in the previous case, the receiving side has at its disposal both a signal containing useful information and a signal without it. After passing through the subtractor, the useful signal can be easily detected.

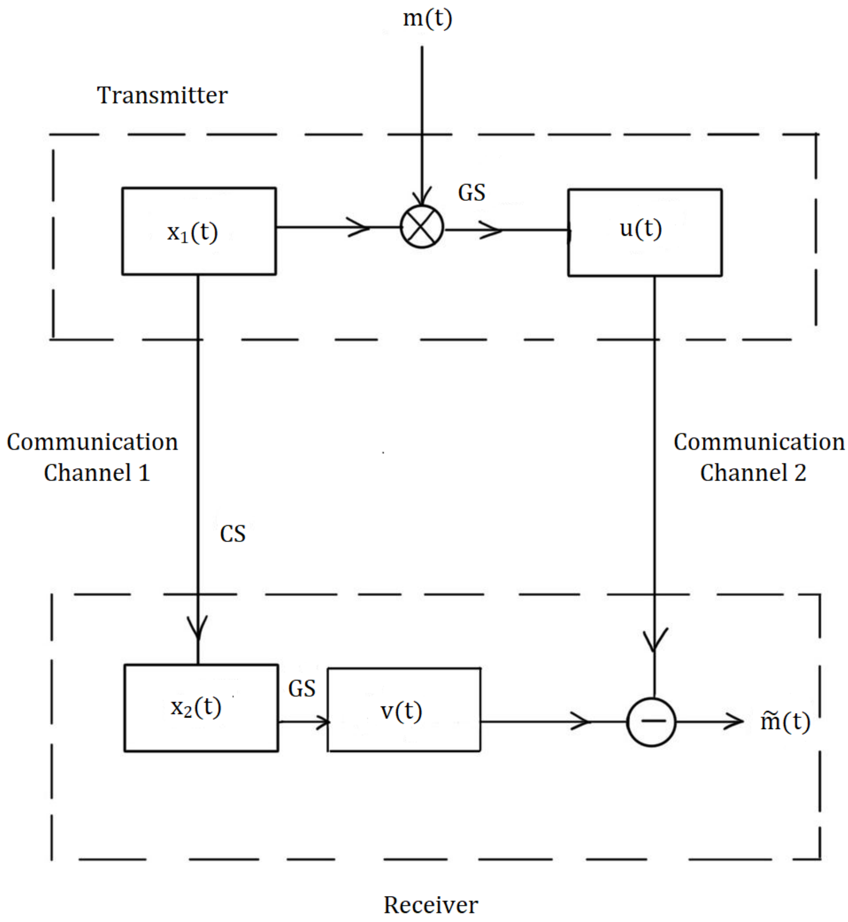

Figure 6. The scheme of hidden information transmission using generalized and complete synchronization [18]

It is clear that such a scheme is more effective in terms of confidentiality, i.e. the probability of detecting an information message by a third party is reduced. However, a number of other problems still remain unresolved. The presence of identical generators in the transmitting and receiving devices (now they are already two pairs of identical generators), the implementation of two communication channels, low noise immunity in the communication channel, which becomes even lower due to the destruction of complete chaotic synchronization, all these disadvantages make similar covert data transmission schemes are of little practical use.

Another method for covert information transmission was proposed, which also uses two types of synchronous behaviour – generalized and full chaotic synchronization, but circuit is a modification of one of the schemes for covert data transmission based on nonlinear mixing of the information signal to chaotic (see section 2.3.3). [14]

A schematic diagram of the implementation of this method of covert data transfer is shown in Fig. 7 [18]. The device, as in the scheme based on the nonlinear mixing of the information signal with a chaotic signal (see Section 2.3.3), contains two interconnected identical chaotic generators $x(t)$ and $y(t)$ (hereinafter the first and second). The information signal $m(t)$ is mixed with the signals produced by these generators, and thus undergoes nonlinear changes. In addition, on the transmitting side of the communication channel there is another generator $x(t)$ (we will call it third), which is not identical to the first and second in the control parameters and unidirectionally connected with the second. The values of the control parameters of the transmitting device generators should be chosen so that the second and third generators are in the mode of generalized chaotic synchronization, while the first and second would be in fully synchronized mode, i.e. were in full synchronization mode. The third generator is used to increase confidentiality: it generates a signal, which in the simplest case is simply added to the signal containing useful information, which forms an already combined signal, thereby creating an additional masking.

This method of transmitting information in the original work was called "secure communication using a compound signal from generalized synchronizable chaotic systems" The combined signal is transmitted via a communication channel to a receiving device containing two generators: the fourth $y_r(t)$, identical to the first and second in control parameters, and the fifth - $z_r(t)$, identical in the same sense. The third, the fourth and fifth generators must be in the mode of generalized synchronization. Then, according to the method of the auxiliary system, due to the identity of the fourth and second systems, the third and fifth generators will make identical oscillations. The signals from the communication channel and the fifth generator are fed to the subtractor. In case of an impact on the fourth generator, this signal synchronizes when transmitting the binary bit 0 and does not synchronize when transmitting the binary bit 1. Signals free of additional components will already be sent to the fourth generator and the second subtractor. Reconstructed signal $\widetilde{m}(t)$, which is a sequence of sections with synchronous (binary bit 0) and non-synchronous (binary bit 1) behaviour.

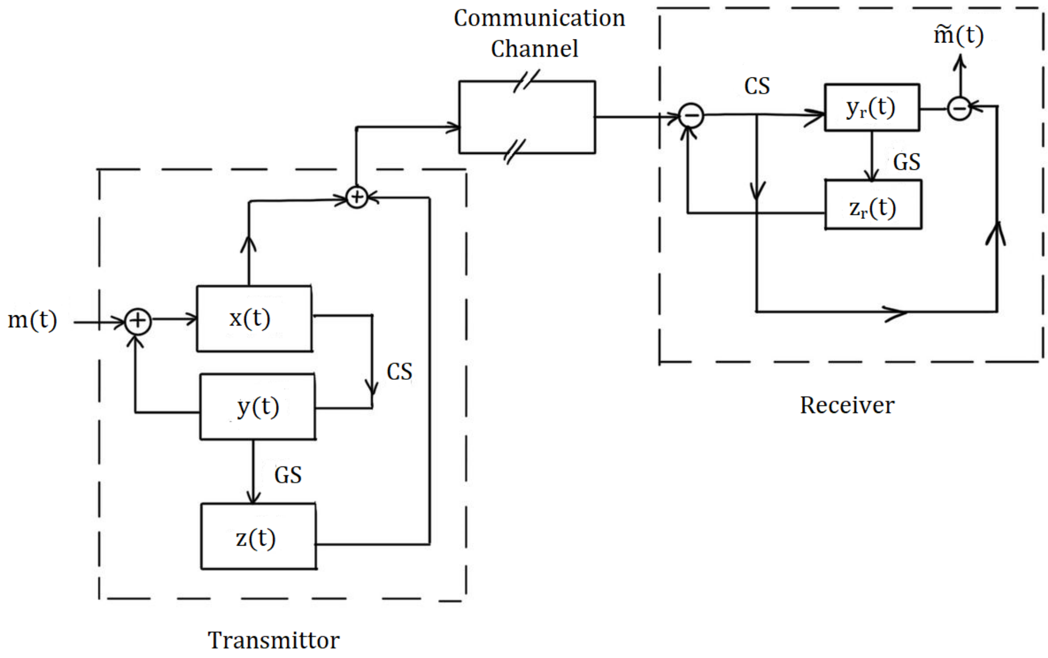

Figure 7. The scheme of hidden information transmission using combined chaotic signal [18]

From the above discussion it follows that this scheme is quite confidential: it is not possible to diagnose an information message by a third party using a combined signal transmitted through a communication channel, in most cases even in the absence of noise. However, as in schemes based on nonlinear mixing, the quality of information transfer (and, therefore, the ability to restore high-quality information) strongly depends on confidentiality, namely: the higher the confidentiality, the lower the quality. At the same time, it is clear that due to the creation of a combined signal, this dependence will not be so sharp, which is some kind of advantage of this circuit in front of others. However, one advantage does not close a number of disadvantages. The creation of five generators, three and two of which must be identical with each other, is a practically unsolvable technical problem, especially if the generators are located on different sides of the communication channel. The introduction of a sufficiently small detuning of the control parameters of these generators immediately makes the circuit inoperative. In addition, noise in the communication channel will undoubtedly lead to a distortion of the transmitted signal, and therefore, to the destruction of the modes of complete synchronization between the second and fourth generators and generalized synchronization between the fourth and fifth. The signals on different sides of the communication channel will become non-identical, and the detection of an informational message on the receiving side of the communication channel will be impossible.

Thus, the partial elimination of some shortcomings in more cases leads to the aggravation of others. Due to the low immunity to noise and the disruption of control parameters, the technical implementation of such schemes, which have a fairly high confidentiality, is very difficult. Therefore, the “extensive” way of improving methods for covert data transmission of synchronous behaviour for transmitting information is, apparently, not optimal.

2.5 The Stability of the Generalized Synchronization Mode to Noise

The generalized synchronization regime arising in system can be considered as a result of two interconnected processes occurring simultaneously: an increase in dissipation in the modified system and an increase in the amplitude of the external (chaotic and noise) signal. Both processes are interconnected via a parameter ε and cannot be implemented separately in the slave system. However, an increase in dissipation in the modified system leads to a simplification of its behaviour and a transition from chaotic oscillations to periodic (or to a stationary state). External influence, on the contrary, seeks to complicate the behaviour of the modified system and impose its own dynamics on it. Obviously, the emergence of a generalized synchronization regime is possible only when the intrinsic chaotic dynamics in the driven system is suppressed due to dissipation.

Thus, the stability of the generalized synchronization mode is determined primarily by the properties of the modified system itself. Therefore, the threshold for the emergence of the generalized synchronization regime should not strongly depend on the intensity of the noise acting on unidirectionally coupled chaotic systems. If the noise does not change the characteristics of the modified system, then it should not affect the threshold for the appearance of the generalized synchronization mode.

Let us review the description of the noise-resistant method of covert information transmission. The information signal $m(t)$ is encoded in the form of a binary code. One or more control parameters of the transmitting generator $x(t)$ is modulated by a binary signal so that the characteristics of the transmitted signal change insignificantly. The signal thus obtained is transmitted through the communication channel. Here it is distorted by noise. The receiver, which is located on the other side of the communication channel, is two identical generators $u(t)$ and $v(t)$, capable of being in generalized synchronization mode with this way transmitting generator. The principle of operation of the receiver is based on the diagnostics of the generalized synchronization mode using the auxiliary system method. The signal from the communication channel goes to the generators of the receiver. The signals obtained at the output pass through a subtractor, and then the reconstructed useful signal $m(t)$ is detected.

Modulation of the control parameters of the transmitting generator must be implemented in such a way that, depending on the transmitted binary bit 0 (1), the generalized synchronization mode exists (being absent) between the transmitting and receiving generators. For example, if the generalized synchronization mode is observed if the binary bit 0 is transmitted, then both receiving oscillators will exhibit identical oscillations, and after passing through the subtraction device, there will be no chaotic oscillations, i.e. binary bit 0. On the contrary, when transmitting binary bit 1, generalized synchronization is not observed, and the oscillations of the receiving oscillators are not identical. Then, after passing through the subtractive device, a nonzero amplitude of chaotic oscillations will be observed, i.e. binary bit 1.

2.6 Methods of Chaos Stabilization

The effect of synchronization of periodic oscillations was discovered by Huygens as early as the 17th century. Currently, the problem of synchronization of regular (periodic and quasiperiodic) oscillations is quite well studied. In the spectrum of regular oscillations, one can distinguish the main frequencies that are uniquely associated with characteristic times (period, quasiperiodic) and phases of oscillations. When capturing frequencies, the phase shift between the interacting modes is stabilized. In the case of regular oscillations of interacting oscillators, phase locking corresponds to a saddle-node bifurcation of cycles on a two-dimensional torus, as a result of which the attractor undergoes a qualitative restructuring at the synchronization boundary: instead of ergodic motion on a two-dimensional torus, a stable limit cycle arises. Several concepts of chaos synchronization are currently known. One of the first was the concept according to which the synchronization of chaos is understood as the phenomenon of the emergence of a periodic regime under the influence of influence on chaotic self-oscillations or as a result of the interaction of chaotic oscillators.

According to the concept often found in the literature, chaos synchronization taking place during the interaction of identical oscillators consists in the fact that as the connection grows, the temporary realizations of the corresponding dynamic variables of the partial systems completely repeat each other without any time shift. That is, the oscillators oscillate in phase.

Recently, effects similar to synchronization have been discovered in systems with more complex chaotic dynamics, including irregular transitions of a trajectory from a neighbourhood of one saddle-focus to a neighbourhood of another.

The effects of synchronization in chaotic switching systems are similar to the discovered phenomenon of stochastic synchronization in multistable systems with switching caused by random forces.

Thus, the concept of synchronization can be extended to a wide range of phenomena observed not only in dynamic, but also in stochastic modes. The ability to use noise to control the behavior of a nonlinear system is attracting increasing interest from researchers in many branches of science. A number of recently discovered phenomena, such as stochastic resonance, coherent resonance, stochastic synchronization in bistable and excitable oscillators, show the constructive role of noise, which can lead to an increase in order in the behavior of a system. On the other hand, noise can introduce a certain degree of disorder into the system and even induce the transition of the system itself to chaotic dynamics, which manifests itself in exponential instability of trajectories. Chaotic synchronization is one of the fundamental nonlinear phenomena that have been actively studied recently.

Stabilization of the chaos behaviour can be done by two different methods. First way is to make the system stable by using external disturbances without a feedback. This method is not taking into consideration the current state of dynamic values of the system. The opposite method is done by correcting the action based on the intended values of dynamic variables and, therefore, engaging the feedback as a necessary component of dynamic system. The first method of stabilization of the chaotic dynamic system is called chaos suppression without a feedback, the second method – chaos control with feedback.

One of the known methods would be OGY method. E. Ott, C. Grebogi and J. A. Yorke were the first to make the key observation that the infinite number of unstable periodic orbits typically embedded in a chaotic attractor could be taken advantage of for the purpose of achieving control by means of applying only very small perturbations. After making this general point, they illustrated it with a specific method, since called the OGY method (Ott, Grebogi and Yorke) of achieving stabilization of a chosen unstable periodic orbit. In the OGY method, small, wisely chosen, kicks are applied to the system once per cycle, to maintain it near the desired unstable periodic orbit. [19]

Another common method would ETDAS or Extended Time Delay Auto Synchronization which can be presented in a following form:

$$\begin{gathered} \mathrm{x}_{\mathrm{n+1}} = a - \mathrm{x}_{\mathrm{n}}^2 + b\mathrm{y}_{\mathrm{n}} + \mathrm{F}_{\mathrm{n}}, \\ \mathrm{F}_{\mathrm{n}} = K[(1-R)\mathrm{S}_{\text{n-m}} - \mathrm{x}_{\mathrm{n}}], \\ \mathrm{S}_{\mathrm{n}} = \mathrm{x}_{\mathrm{n}} + R\mathrm{S}_{\text{n-m}}, \end{gathered} \tag{3}$$

With *K*, *R* being constants, *F* – the perturbation, *S* – delay equation and *m* – the period. [20]

The method of resonance activation is used to control the behaviour of chaotic dynamic systems [15]. It is based on the fact that the system will now show periodic behaviour due to non-linear mode interactions. In order to receive the given motion, it is needed to disturb in a specific way. The key of this method is the know the equation of movement for the system after the disturbance.

To achieve control of the dynamic system, the external disturbance needs to be added:

$$\dot{x} = v(x, \alpha) + F(T),\ \alpha\epsilon R. \tag{4}$$

Let us present the needed dynamic with a function *v(t)* that satisfies the so-called equation of prescribed motion:

$$\dot{y} = g(y) \tag{5}$$

Now presenting disturbance as *F* = *g*(*y*(*t*))-*v*(*y*(*t*),*a*) and adding it to (3), we get the equation of control:

$$\dot{x} = v(x, \alpha) + g(y) - v(y, \alpha) \tag{6}$$

Where $x \to y$ with $t \to \infty$, the final dynamic is represented by the equation (1.3).

Artificial creation of excitations in a system of stable oscillations through external multiplicative requires consideration of a dynamic system of the form:

$$\dot{x} = \mathsf{V}(x, a), \text{where } x = \{x_1, x_2, \dots, x_n\}, \mathsf{V} = \{v_1, v_2, \dots, v_n\}, \alpha\epsilon R, x(t_0) \equiv x0 \tag{7}$$

The multiplicative control consists in modifying the function V in relation to (7) so that the new system $\dot{x} = \mathsf{V}'(x, a', t)$ has the desired (pre-selected) behaviour.

Here

$$\mathsf{V}'(x, a', t) = \mathsf{V}(x, a_0 + a_1 t) \tag{8}$$

where the parameter $a_1(t)$ is a periodic function.

Sometimes the introduction of multiplicative excitations into the system is impossible. Then the phase flow $F^t(x, G)$ decomposes into two components: the part corresponding to the unperturbed phase flow, $F^t(x)$ and the component $F^t(G)$ which is initiated by perturbations:

$$F^t(x, G) = F^t(x) + F^t(G) \tag{9}$$

In this case, an additive perturbation takes place, meaning

$$\mathrm{V}'(x, a', t) = \mathrm{V}(x, a) + g(t) \tag{10}$$

Where $g(t)$– external impact.

Hence, the control of the dynamics of the system implies the application of the force component to a vector function. Therefore, this type of control of the behaviour of a dynamic system is called force. In turn, if feedback is taken into account in force control, the function V is modified as

$$v_i' = v_i(x, a) + g_k(x_i(t)), i = 1,2, \ldots, n, 1 \leq k \leq n \tag{11}$$

For control problems, the property of trajectories of chaotic processes, called recurrence, is essential: over time, these trajectories fall into an arbitrarily small neighbourhood of their position in the past.

Function $x: R^1 \to R^n$ *is called recurrent if with any* ε *>0 there is* $T_\varepsilon > 0, then\ with\ any\ t \geq 0\ there\ is\ T(t, \varepsilon), 0 < T(t, \varepsilon) < T_\varepsilon\ the\ way\ that\ \|x(t + T(t, \varepsilon)) - x(t)\| < \varepsilon$

Recurrent trajectories have two important qualities that are given by lemma (C.C. Pugh and Anosov).

These lemmas show that a chaotic attractor is the closure of all periodic trajectories contained in it. The concept of attractor is also associated with the following recurrence criterion formulated by G. Birkhoff in 1927.

Theorem (Birkhoff). Any trajectory belonging to a compact minimal invariant set is recurrent. Any compact invariant minimal set is the closure of some recurrent trajectory.

It follows from this theorem that any solution starting from its ω-limit set is recurrent. Under the additional assumption that ω is a limit set $\bar{x}(t)$ is an attractor, it follows that any chaotic trajectory starting in its ω-limit set is recurrent.

The description of the system uncertainty by means of **fuzzy models** leads to specific versions of control algorithms.

The most convenient for control synthesis is the description in the form of fuzzy Takagi-Sugeno systems (T-S-fuzzy systems), presented as a set of rules for fuzzy conditional inference

$$IFz_{li}(t) \in F_{li}AND \ldots ANDz_p(t) \in F_{lp}THEN$$
$$\dot{x} = Ax_i + B_i u, y = C_i x + D_i u, i = \overline{l, r} \tag{12}$$

Matrices A_i, B_i can depend on variables $z_j(t)$ that allows us to describe non linear systems using the form (12).

In a number of works, fuzzy models of nonlinear systems are combined with a network neural-like structure of regulators.

3 SYNCHRONIZATION SYSTEMS

Starting with Newton, the traditional way for physics to describe dynamical systems is to use differential equations. However, in many cases it turns out to be just as natural and convenient to work with mappings - difference equations that determine dynamics in discrete time.

Let us first consider chaotic systems described in terms of differential equations.

3.1 Lorenz System

In 1963, the American weather forecast researcher Edward Lorenz published an article entitled "Deterministic Nonperiodic Flow" in the Journal of Atmospheric Sciences.

The physical process underlying the Lorenz model is two-dimensional thermal convection. The fluid motion is described by the Navier - Stokes equation. Using a number of simplifying assumptions, from the partial differential equations for the perturbed flow, Lorenz obtained a system of three ordinary differential equations of the form

$$\begin{gathered} \dot{x} = \sigma(y - x) \\ \dot{y} = rx - y - x\mathrm{z} \\ \dot{\mathrm{z}} = -b\mathrm{z} + xy \end{gathered} \tag{13}$$

where the value of x is proportional to the convection rate; y is the temperature difference between the ascending and descending flows, z is the deviation of the vertical temperature profile from the linear parameter; σ - Prandl number; r is the reduced Rayleigh number; b is a constant characterizing the size of the physical system.

The Lorenz equations were the first dynamical system in which the presence of a strange attractor was proved. The Lorenz attractor in a certain range of values of control parameters is hyperbolic and in experiments demonstrates the main properties of strange attractors.

Then, removing the variable y from the equation, we get

$$\begin{gathered} \ddot{x} + (1+\sigma)\dot{x} + \sigma(1 - r + \mathrm{z})x = 0, \\ \dot{\mathrm{z}} = -b\mathrm{z} + x^2 + x\dot{x}/\sigma \\ y = \frac{\varepsilon x}{\sqrt{2\sigma}}, z = \frac{\varepsilon}{\sigma}\left(\sigma\mathrm{z} - \frac{x^2}{2}\right), r = \frac{\sqrt{\sigma t}}{\varepsilon}, \varepsilon = \frac{1}{\sqrt{r-1}} \end{gathered} \tag{14}$$

Finally, we get a system

$$\begin{gathered} \ddot{y} + \varepsilon h\dot{y} + y^3 + (z-1)y = 0, \\ \dot{z} = \varepsilon\alpha z + \varepsilon\beta y^2 \end{gathered} \tag{15}$$

Where $h = (1+\sigma)/\sqrt{\sigma}, a = b/\sqrt{\sigma}, \beta = (2\sigma - b)/\sqrt{\sigma}$

The last equations describe a nonlinear dissipative parametrically excited oscillator whose frequency is inertially controlled by the oscillation amplitude

The Lorenz attractor is calculated based on three degrees of freedom. Therefore, it is described by three ordinary differential equations, each containing three constants and initial conditions. Nevertheless, despite its simplicity, the Lorenz system has chaotic properties and behaves in a pseudo-random way.

An attractor representing the behavior of a gas at any given time, and its state at a certain moment, depends on its states preceding this moment. If we change the initial data even to very small values, checking the state of the attractor will show completely different numbers. This follows from the fact that minor differences increase as a result of the recursion.

Despite this, the graphs of the attractors will not differ much. Both systems can have completely different values at any given moment in time, but the attractor's graph will remain the same, since it describes the overall behaviour of the system.

Having simulated his system on a computer, Lorenz identified the reason for its chaotic behaviour - the difference in the initial conditions. Even a microscopic deviation of the two systems at the very beginning in the process of evolution led to an exponential accumulation of errors and, accordingly, their stochastic divergence.

However, any attractor has boundary dimensions, from which it follows that the exponential divergence of the trajectories of two different systems cannot continue indefinitely. From a certain point in time, the orbits will converge again and pass close to each other or coincide, although the probability of a complete coincidence is unlikely. It is worth noting that the coincidence of trajectories is a rule of behaviour for simple predictable attractors.

The Simulink model of Lorenz system looks as follows:

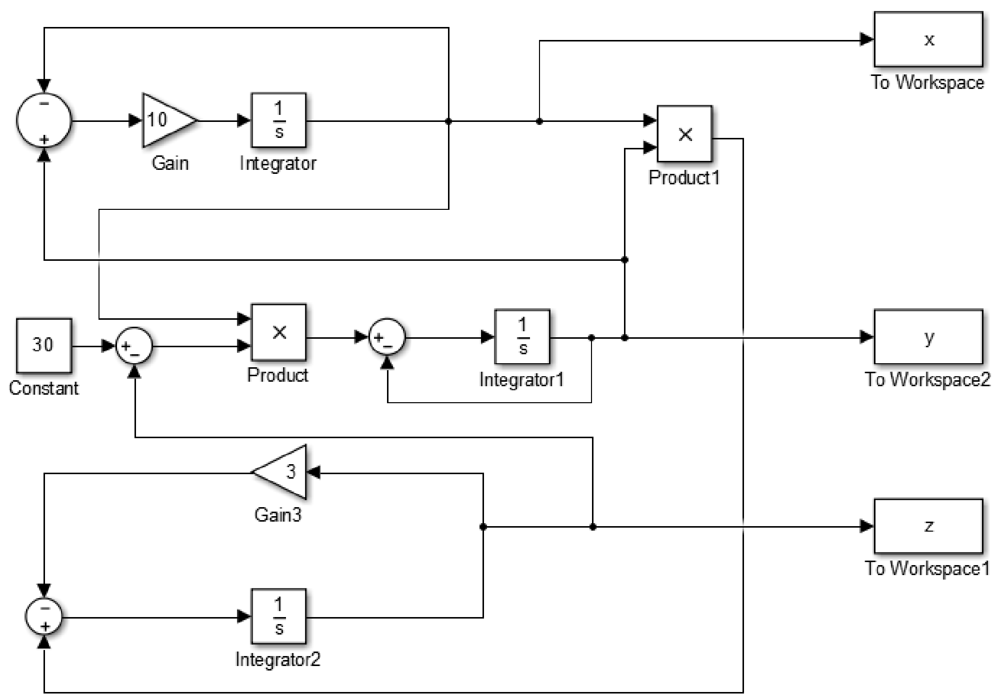

Figure 8. Simulink model of Lorenz system

The result of the simulation on different axis will look like that:

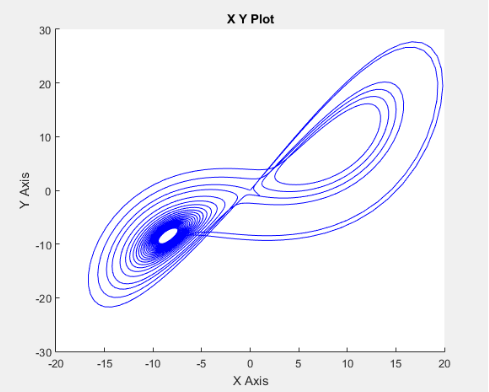

Figure 9. Lorenz attractor in two dimensions, x-y plane

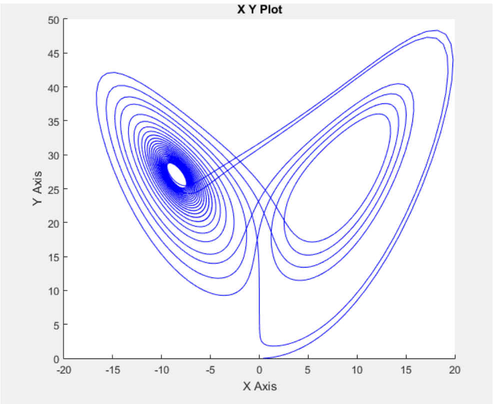

Figure 10. Lorenz attractor in two dimensions, x-z plane

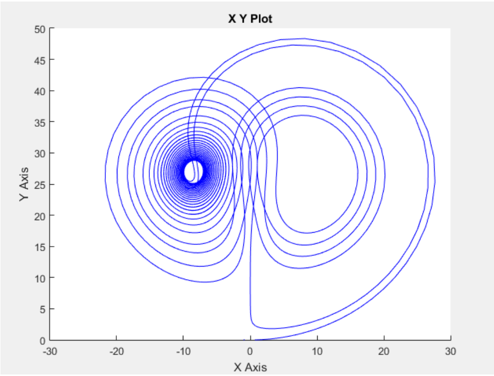

Figure 11. Lorenz attractor in two dimensions, y-z plane

3.2 Rössler system

Now let us consider a well-known differential system - the Rössler model:

$$\begin{aligned} \dot{x} &= -y - z, \\ \dot{y} &= x + ay, \\ \dot{z} &= b - cz + xz, \end{aligned} \tag{16}$$

Where a, b and c are system parameters.

Initially, these equations were proposed by Rössler as the simplest model of some chemical reactions taking place in a stirred tank. It is also a three-dimensional system in which a special trajectory such as a loop of the saddle-focus separatrix is realized with all the ensuing consequences.

By differentiating the equation and removing the variable y, we can bring the equation to a general form:

$$\begin{aligned} \ddot{x} - a\dot{x} + (1 + z)x &= (a + c)z - b, \\ \dot{z} &= b - cz + xz. \end{aligned} \tag{17}$$

In this form, the system is interpreted as an oscillator under parametric and force influences, the intensity of which in an inertial manner depends on the oscillation amplitude of the oscillator.

The Rössler system with variation of control parameters demonstrates a universal transition to chaos through a sequence of period doubling bifurcations.

The Rössler system allows an approximate transition from a three-dimensional flow to a one-dimensional irreversible mapping, which is due to the strong compression of the phase volume in one of the eigen directions.

The Simulink model of the system represents these differential equations:

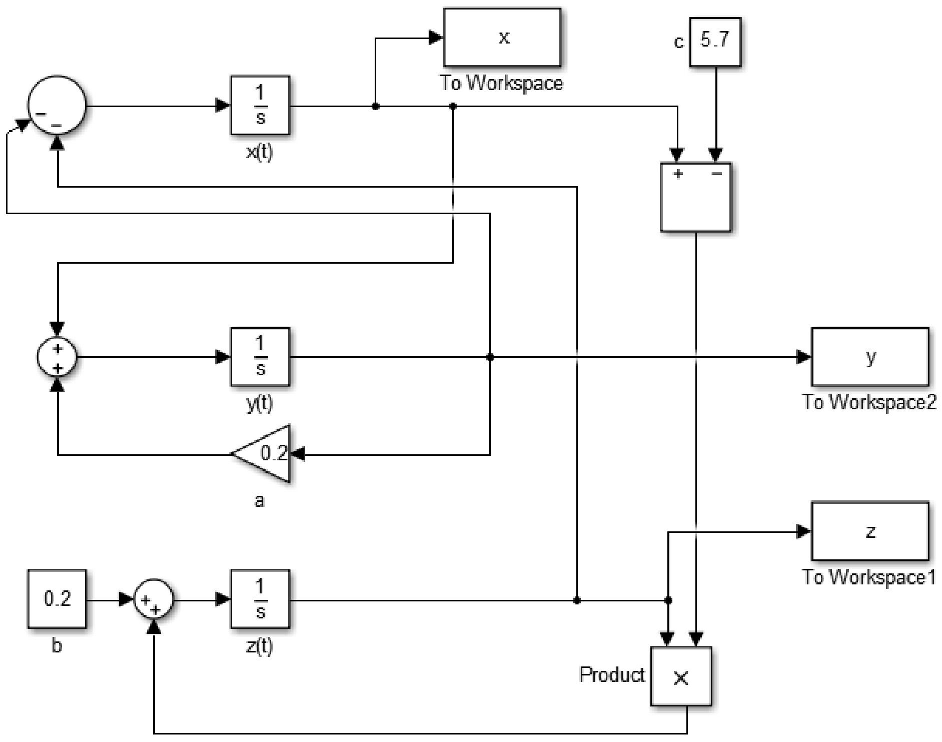

Figure 12. Simulink model of Rössler system

With the outcome being:

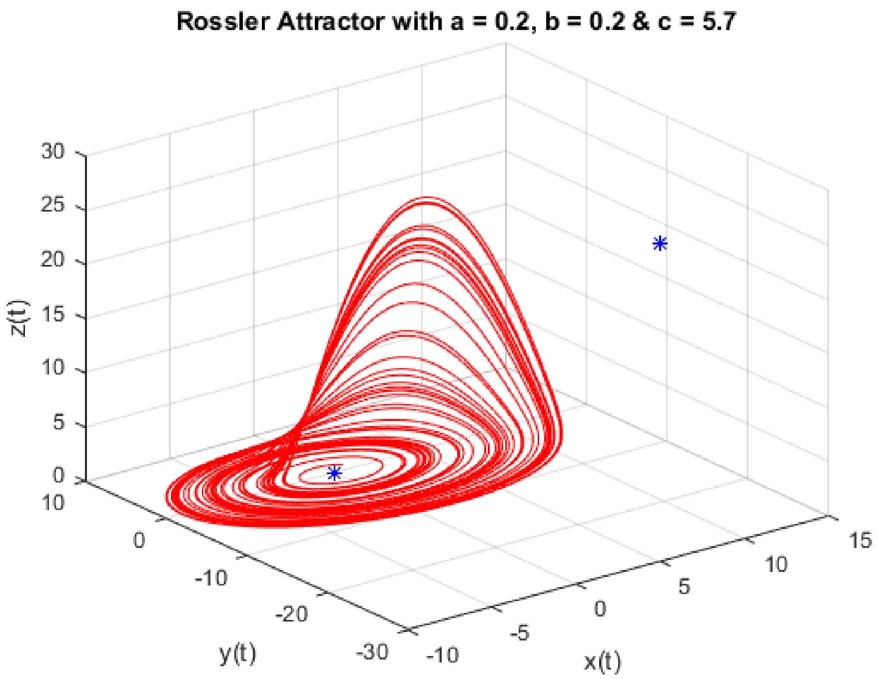

Figure 13. Rössler attractor in 3D space

3.3 Chua System

In a number of problems related to the study of the interaction of attractors, stochastic resonance, intermittency "chaos - chaos" and others, there is a need to have more complex special models at your disposal. The difficulty here is associated with the presence of not one, but three states of equilibrium, symmetry, and more complex types of special trajectories. From this point of view, the so-called Chua circuit is interesting, the circuit of which is shown in the figure below, which includes only one nonlinear element (Chua diode).

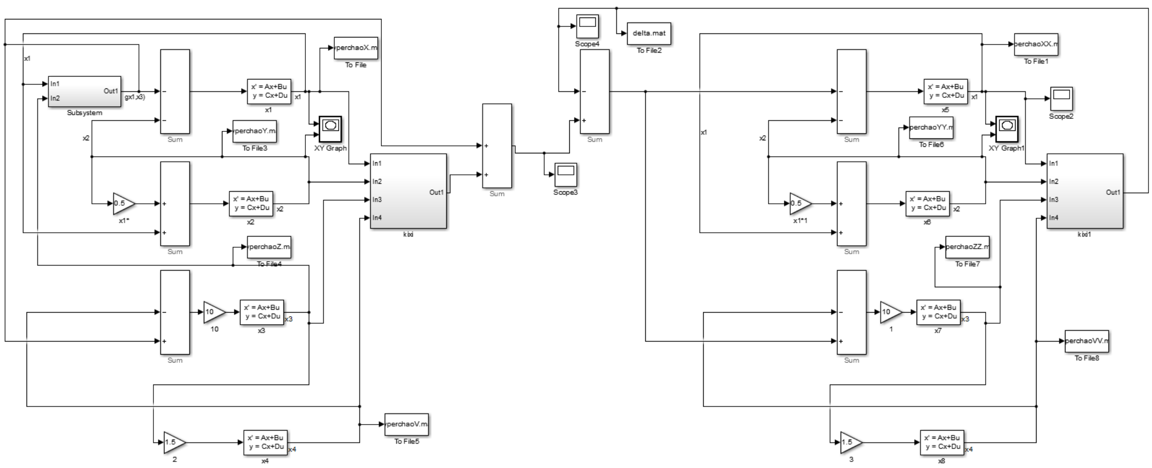

Figure 14. Simulink model of Chua system

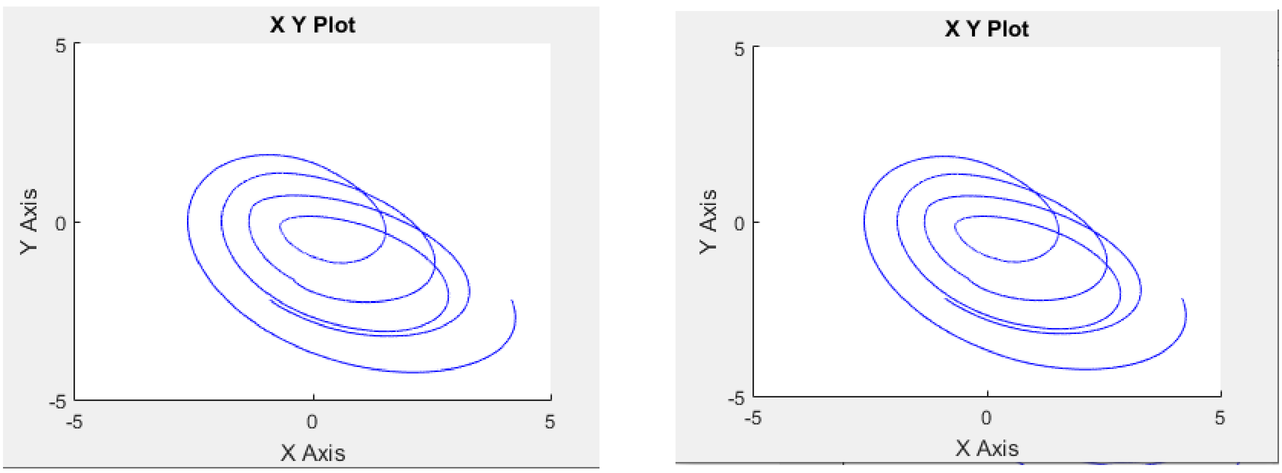

Figure 15. Chua System

The equations of Chua System are:

$$\dot{x} = \alpha[y - h(x)],$$
$$\dot{y} = x - y + z, \quad (18)$$
$$\dot{z} = -\beta y,$$

Where $x = V_1/E, y = V_2/E, z = i/(EG),$

$$\alpha = C_2/C_1, \beta = C_2/(LG^2), \tau = tG/C_2, h(x) = bx + 0.5(a-b)(|x+1| - |x-1|), a = 1 + G_a/C, b = 1 + C_b/G. \tag{19}$$

The system equation is also reduced to the generalized equations of the inertial oscillator:

$$\ddot{z} + \dot{z} + \beta z = \beta x,$$
$$\dot{x} = -\frac{\alpha}{\beta}\dot{z} - \alpha h(x). \tag{20}$$

Due to symmetry, the system implements a double loop of the saddle-focus separatrix, a more complex figure that generates a more complex type of chaotic attractor.

It should be noted that the Chua system is one of the basic models used in the study of various fundamental and applied issues of dynamic chaos.

It can also be noted that depending on the parameters, the Chua scheme can demonstrate various regular and chaotic regimes.

3.4 Other Known Systems

3.4.1 Chen System

Chen's system equations are described as:

$$\begin{cases} \dot{x} = a(y-x) \\ \dot{y} = (c-a)x + cy = cz \\ \dot{z} = -bz + xy \end{cases} \tag{21}$$

Where *a>0*, *b>0* and parameter *c*(*2c>a*)

The chaotic nature of the Lorenz system in a non-chaotic regime demonstrates a new chaotic system - the Chen system. The Lorenz and Chen systems are mathematically similar structures, but topologically they are not equivalent, since the Chen system has more complex behavior. All three equilibrium positions of the Chen system in its chaotic state are unstable and the dynamics of this system is completely irregular.

3.4.2 Henon Attractor

The Henon attractor is a reversible two-dimensional two-parameter mapping of the form

$$x_{n+1} = 1 - \alpha x_n^2 + y_n,$$
$$y_{n+1} = \beta x_n \tag{22}$$

Where x_n, y_n – dynamic variables and α, β are system parameters.

The Henon map is dissipative under the condition $0 < \beta < 1$. In the case $\beta = 1$, the mapping is a conservative system. As β decreases, the compression of the phase volume in one iteration begins to increase. As a result, for $\beta = 0$, an invertible two-dimensional mapping becomes a one-dimensional irreversible logistic mapping.

This two-dimensional mapping, constructed by Henon as the simplest model of the Lorenz system, was intended to simplify the mathematical analysis and conduct more detailed studies of the structure of the Lorenz attractor. However, it turned out that this mapping has a much more general character. Such mappings arise in a very natural way in the study of systems close to a system with a non-rough homoclinic curve. For three-dimensional differential systems with chaos generated by the saddle - focus singularities, the Henon map is a canonical model. It reflects all the fundamental properties of a whole class of differential systems.

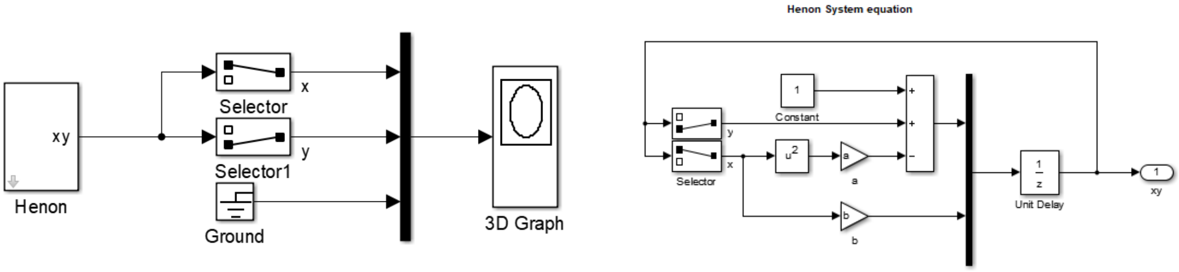

Figure 16. Simulink model of Henon system

3.4.3 Rabinovich-Fabrikant System

In 1979, Rabinovich and Fabrikant presented and investigated a physical model describing stochasticity arising from modulation instability in a nonequilibrium dissipative medium. This finite-dimensional three-mode model can describe various physical systems, such as Tollmien – Schlichting waves in hydrodynamic flows, wind waves on water, waves in chemical media with diffusion, Langmuir waves in plasma, etc.

The system is described by following equations:

$$\begin{aligned} \dot{x} &= y(z-1+x^2)+\gamma x \\ \dot{y} &= x(3z+1-x^2)+\gamma y \\ \dot{z} &= -2z(\nu+xy) \end{aligned} \tag{23}$$

Where, x, y, z are dynamic variables and γ, ν are parameters.

The Rabinovich-Fabrikant system has rich dynamics. First, a transition to chaos is observed for it through a sequence of limit cycle doubling bifurcations. Second, the Rabinovich - Fabricant system demonstrates multistability. Attractors of different types coexist in the phase space. Depending on the values of the parameters, several combinations of coexisting attractors can be distinguished, for example: a stable fixed point and a limit cycle, the period of the cycle can be any (or a chaotic attractor); two limit cycles of different types and periods; limit cycle and chaotic attractor, etc. In addition, since the Rabinovich - Fabrikant system has symmetry under the change of variables x to $–x$ and y to $–y$, all attractors have a symmetric pair or have the corresponding symmetry.

3.4.4 Van der Pol System

The main models for the analysis of periodic self-oscillations are the van der Pol equations:

$$\ddot{x} - (\lambda - x^2)\dot{x} + x - 0 \tag{24}$$

And Rayleigh equations:

$$\ddot{y} - \left(\lambda - \dot{y^2}\right) + y = 0 \tag{25}$$

Self-excitation of self-oscillations occurs at $\lambda > 0$. As λ increases, a gradual transition from weakly nonlinear quasi-harmonic vibrations to relaxation ones takes place.

The Van der Pol oscillator was proposed by the Dutch engineer Baltazarov Van der Pol, who found stable (relaxation) oscillations - "limit cycles". In 1927 he discovered that at certain frequencies, noises were recorded, always located near the natural frequencies of the waves. This is one of the first observations of detective chaos.

The system of van der Pol equations is:

$$\frac{d^2 x_n}{dt^2} + \omega_k^2 = \mu f_k(x_1, \dots, x_n,\ \dot{x_1}, \dots, \dot{x_n})$$
$$x(0) = x_0,\ \dot{x}(0) = \dot{x_0}, \quad k = 1, \dots, n \tag{26}$$

Here $f_k(x, \dot{x})$ – analytic functions of variables $x, \dot{x}$ in a 2n-dimensional open ball of radius R:

$$\sum_{k=1}^{n}(x_k^2 + \dot{x_k^2}) < R^2 \tag{27}$$

With *k* being a scalar index.

3.4.5 Ikeda map

The Ikeda image is a model of a laser-excited ring resonator with a nonlinear medium. In 1980, Japanese researchers Ikeda, Daido, and Akimoto proposed a mechanism for the emergence of complex dynamics in a nonlinear optical system - a ring resonator containing a medium with phase nonlinearity (İkeda, Daido, Akimoto, 1980). The resonator is excited by a laser beam through a semitransparent mirror.

Non-trivial oscillatory modes can be realized due to the interference of a monochromatic signal at the input and a phase-modulated signal that has passed through a nonlinear medium.

If we neglect the relaxation time of the response of the medium to a change in the light intensity, then the values of the complex amplitude of the field E at the entrance to the nonlinear medium at the *n*-th and *(n+1)*-th pass through the ring resonator are related by the following relation:

$$E_{n+1} = A + BE_n \exp(i|E_n|^2 + i\varphi) \tag{28}$$

Here *A* is the parameter of the intensity of light from the laser, *B* is the parameter of the field dissipation in the cavity. The quantity appearing in the exponent corresponds to the phase incursion during the round trip of the cavity: the parameter φ characterizes the detuning of the laser radiation frequency from the cavity eigenmode, and the addition

$|E^2|$ is due to a nonlinear phase shift due to the dependence of the refractive index on the field amplitude.

If the radiation intensity parameter A is increased at a fixed level of the loss parameter B in the cavity AB, then the transition to chaos occurs, as a rule, through a cascade of period doubling bifurcations.

However, the global structure of the chaos border is arranged in a complex and non-trivial manner. In particular, there are narrow bands of regular dynamics extending far into the region occupied by chaos.

4 DIGITAL TRANSMITTION OF INFORMATION

Today there are a number of models that model associative memory. Among these models, the most popular is a neural network. It describes it as a dynamic system, the objects for memorizing and recognizing which are the simplest attractors, that is, stable equilibrium positions. According to experimental data, both simple and complex attractors, including chaotic ones, play an important role in information processing. Therefore, the study of information processing processes using complex nonlinear dynamics is of genuine interest.

4.1 Attractors in One-dimensional Mappings

Consider a transformation of a one-dimensional logistic map, which can be written as follows:

$$\mathrm{x}_{n+1} = f(\mathrm{x}_n, \alpha) \tag{29}$$

Where α is a parameter vector.

This model, at first glance, seems to be very limited, not only in terms of the initial model intended for information processing, but in general as a model of a chaotic system. However, this is not the case; in fact, the display has a fairly wide list of phenomena inherent in complex dynamics and inherent in chaotic systems. It serves as a full-fledged basis in modelling a variety of information processing operations using chaos.

Due to the instability in chaotic mappings, uncertainties arise in the position of the phase trajectories. The dynamic equations connecting the previous and subsequent states of the system limit the degree of this uncertainty. During the transition from the previous state to the next, there is a loss of information about the initial state of the system, given with a certain accuracy. From which we can conclude about the formation of information in such systems.

The formation of new information is equal to zero if the display behaviour demonstrates the property of regularity (let us assume periodic oscillations). Using any initial conditions that form the basin of attraction of a regular attractor, the trajectory of the mapping will be attracted to the given attractor (limit cycle or a stable equilibrium position). In this case, the loss of information about initial conditions, due to the fact that the trajectories emerging from different starting points become indistinguishable after a certain time. Based on the above, regular attractors can be called a kind of "sinks" of information.

Nonlinear one-dimensional maps are capable of both producing and “destroying” information. Any trajectory of a dynamic system can be considered as some information signal. Thus, the complex of display trajectories is a kind of "storage" of information in the form of a set of trajectories of the system. This "storage" has a number of properties, the manifestation of which depends on the presence of attractors in the dynamic system and their type.

Let's look at some of these properties. If at an arbitrary moment of time $t = t_0$ set the initial conditions, then formally the trajectory of the mapping will be known and uniquely determined for all subsequent moments of time. If we associate it with an information signal along its length, then it is reproduced by iterating the mapping with known initial conditions, that is, by solving the evolutionary equation, the process of extracting information from memory will take place.

If the display contains only one attractor, such as a fixed point, then the trajectory, given by arbitrary initial conditions from the domain of the mapping, converges to a fixed point, and the corresponding information signal converges to a constant value. As the convergence progresses, the content of such an information signal will decrease. In the case of a one-dimensional display, the information content of the most fixed stable point is the value of a single constant. The process of increasing the amount of recorded information can be carried out only by improving the accuracy of specifying the coordinates of the attractor, that is, by improving the resolution.

For the above reasons, limit cycles of mappings should be considered as the main objects for recording and storing information in chaotic mappings.

Let the system have a single chaotic attractor. Then the trajectory will lose information about the initial conditions, and the mapping will produce its own. The type of the received information signal can be very complex. The attractor remains well localized in the phase space if the rate of production of new information is low. These chaotic attractors, as a rule, can store significant amounts of information. The chaotic attractor loses its attractiveness as a storage of information and becomes an object with great uncertainty if the occurrence of information is large enough.

Consider the case when the mapping has two or more regular attractors. In this case, the trajectory will be attracted to one of the attractors, depending on the choice of the initial condition. Thus, the mapping recognizes the initial conditions and distributes them into several types according to the number of attractors. It unites the set of all initial conditions into several classes, that is, the ability to play the role of a classifier is expressed. In this display, information is generalized, without the formation of information.

The unification of attractors is often preceded by their crises and intermittency. With intermittency, the trajectory is most of the time on attractors that have lost stability, sometimes making irregular transitions between them. If we consider attractors as images of information objects, intermittency processes can be interpreted as a memory scan. It should be noted that this way of viewing is very economical. First, the trajectory rarely leaves the former attractor, which means that it only visits those sets of the phase space where information is available. Secondly, no additional information is required, except for the equations of the dynamical system itself.

Consider the process of recording and restoring information based on one-dimensional dynamical systems with a limit cycle. We represent the recorded information as finite sequences of the form

$$a_1, a_2, \dots, a_n$$

Where each element of the a_i block belongs to a finite ordered set $A = \{a_j, j = 1...N\}$ which we can call the alphabet.

It should be noted that in the process of recording on a closed trajectory, the information block is folded into a ring, that is, after the last element a_n, the entire block is repeated, starting from the first a_1, therefore, a fragment of the information block will be called a simply connected segment of this ring.

A number of requirements are imposed on the synthesized function $f(x)$ of one-dimensional mapping $x_{n+1} = f(x_n, \alpha)$, on which information is recorded: the nature of the stability of the limit cycle is constantly monitored; there is a one-to-one correspondence between the points of the cycle and the elements of the corresponding information block; a limit cycle carrying information about an information block of length *n* passes sequentially through *n* points.

The creation of a one-dimensional chaotic mapping begins with the fact that in a one-dimensional phase space (unit segment), for each information block $a_1, a_2, \dots, a_m$ a closed trajectory is constructed - a cycle $\gamma_n = \{x_1, \dots, x_n\}$, each point of which is in one-to-one correspondence with a fragment of an information block. After constructing in the phase space the skeleton of closed trajectories-cycles corresponding to the written information blocks, the function *f*(*x*) itself is constructed. For this, on the plane (X_m, X_{m+1}) we put pairs of consecutive points of all cycles (x_i, x_{i+1})

Any curve that passes through these points represents a function *f*(*x*) of a one-dimensional dynamical system that satisfies two requirements for the mapping function. While the stability control of cycles is carried out using short segments with a fixed slope s (hereinafter we will call them informative segments). As is known, the stability of a limit cycle depends on its multiplier μ. In the case of a one-dimensional mapping for the cycle s, it is equal to $\mu = f'(x_1) \cdot \ldots \cdot f'(x_n)$ and in the case under consideration $\mu = s^n$. For $|\mu| < 1$ and $(|s| < 1)$ the cycle is stable, for $|\mu| > 1$ $(|s| > 1)$ - unstable.

The completion of the synthesis of the function *f*(*x*), the desired one-dimensional mapping, occurs by connecting informative sections with straight lines with the ends of the unit segment

4.2 Sequential Recording of Information

The need for sequential recording of images can arise when the system is operating in the "online" mode, that is, when sequentially arriving images are written to the display. First, a mapping is built that corresponds to a block of information, then the next block is added, and a mapping is built for two blocks of information, and so on. However, the technical implementation of these systems (and their computer modelling) is not the most efficient approach since the appearance of a new block of information will require a complete reorganization of the display. On the other hand, the new display will contain fragments of the old one, and with the addition of more and more new blocks, the corresponding changes in the display structure will become more and more local.

Therefore, it is advisable to use algorithms for sequential recording of blocks of information that would make the necessary local changes to the display, not changing the global structure.

Consider one of the approaches to sequential recording of information. Let the mapping already contain i blocks of information and it is required to write $(i+1)$-th block. In order to implement the recording of a new block of information, it is necessary to calculate the positions of the cycle points through which the new display will pass. It is assumed that the extreme points of each segment are given by a pair of numbers and the mapping in our model is piecewise linear. In order to write a new block of information into the original display, in fact, it is necessary to determine the position of all points of the cycle, corresponding to the new block of information, relative to the array of points j h that set the abscissas of the ends of the segments. And also, calculate the abscissas and ordinates of points that correspond to the ends of the line segment containing this cycle point. Next, you should order and rewrite the array h_j into the array h_{j+1}, increasing its dimension by $2n_{j+1}$ points (where n_{j+1} is the length of $(i+1)$-th block of information), and connect the points h_i^j with the left boundary point of the new informative segment, which is connected to the right point of the new informative segment, and this point - to the point with the abscissa h_i^{j+1}.

If, before this, the points of the cycle corresponding to the block of information with the number $(i+1)$ are ordered by increasing the abscissas, then the indicated operation

can be reduced to a single reordering of all points to the right of the leftmost point in the cycle corresponding to a new block of information. Moreover, this reordering comes down to simply adding the appropriate numbers to the indices. Thus, in the considered approach, the problem is reduced to ordering elements of the cycle corresponding with $(i+1)$-th block of information and one-time reindexing of display elements.

4.3 Encoding Information Blocks

Analysis of information sequences (texts, images, music files, etc.) shows that great difficulties in recording can arise with long homogeneous sections, although they contain a minimum of information. These considerations suggest the idea of recording information previously compressed in a special way to ensure recording at the q -level. One of the algorithms used is the coding of information sequences with the construction of an alphabet of repeated fragments. The idea is as follows: if a repeating fragment of length q appears in information blocks, then the alphabet is increased by one character represented by this fragment, and in sequences it is encoded with a new character. New information blocks of shorter length appear, described by an alphabet of length (N+1). The procedure can be repeated until the blocks no longer contain identical fragments of length q.

As a result of applying this algorithm, orthogonal, shorter encoded information sequences and an enlarged alphabet are obtained, which consists of an initial dimension N and an additional alphabet, the elements of which are fragments of length q and symbols encoding this fragment in sequence.

Thus, the essence of orthogonalization is to eliminate redundant information presented in the form of repeating fragments, by encoding them with shorter symbols. Unlike the approach discussed above, where the recording was provided by increasing its level, in this case the level is fixed and only the alphabet is increased. That makes it possible in principle to record any set of information blocks at any level, starting from the second.

This method of encoding information blocks is reversible, and there is no loss of information. Decoding of information occurs by replacing characters of the additional alphabet in the encoded sequence with fragments of length q from the alphabet.

This encoding procedure is ambiguous. The final form of the encoded sequence, the extended alphabet, their length is determined by the order of replenishing the alphabet with the same fragments, that is, by the procedure for finding them [16].

4.4 Associative Information Extraction Using Encoding

The implementation of associative memory, that is, the restoration of a recorded image from its arbitrary fragment occurs by setting an initial condition corresponding to its own attractor. To do this, you need to know a fragment $\left(a_j a_{j+1} \dots a_{j+q-1}\right)$of length *q* of the information block and calculate the starting point x_o.

$$x_o = \sum_{j=1}^{q} \frac{a_{j+k-1}}{N^k} \qquad (30)$$

When recording at the *q* level, it is first necessary to orthogonalize the information blocks, and only then the coded blocks are written to the limit display cycles. If a fragment of an encoded information block of length *q* is known, then according to (30), the starting point x_0 can be obtained, the encoded information block can be restored, and the original information block can be decoded. However, much more interesting is the case of restoring a sequence from a fragment of the original pattern in the alphabet.

To implement associative access in this case, you first need to get from the provided pattern fragment the corresponding fragment of the encoded information sequence. First, you need to encode it using the available additional alphabet - a table of fragments, obtained by orthogonalization of information blocks.

If the entire encoded fragment was restored, but it was not possible to find the starting point on the attractor, then the fragment length was insufficient - after encoding, it did not find the correct fragment of length *q*.

The associative memory system, built on the described principles, almost instantly forms one of two responses to any information block provided to it, or any part of it. Or it returns the initial condition x_o corresponding to the attractor, by which it is possible to completely restore the block. Or it answers that the provided information is not enough to form the initial condition on the attractor, the same happens when the presented block is not recorded on the display. When a certain fragment of a block is provided, the associative memory system either unmistakably restores the information block, or reports the inability to do so. The error is eliminated due to the unambiguity of the encoding procedure of the provided fragment, the orthogonality of the recorded information blocks, the uniqueness of the display itself, and also due to the use of the initial condition of the above procedure, comparing the encoded fragment with the information stream generated by it.

It should be noted that the formation of the initial condition on the attractor and, accordingly, the restoration of the original pattern from its arbitrary fragment occurs without comparing the presented fragment with each of the recorded images. After encoding the presented fragment, to compute the starting point x_o, it may be necessary to perform only a few iterations to check if it hits the display attractor.

5 CONCLUSION

In this work we have presented one of the possible uses of deterministic chaos – a phenomenon of nonlinear dynamical systems in both continuous and discrete form. We have reviewed various ways of covert information transmission with the use of chaotic signals based on different types of chaotic behaviour: phase, generalized and complete synchronization as well as a combination of two types. Each scheme has its own features and principles, advantages and disadvantages. There is a common difficulty to a technical realization of such schemes. Mainly, the need to have almost identical generators on both sides of communication channel, high resistance to noise in the communication channel and confidentiality.

We have reviewed common chaos systems that may be used for data transmition and built up a simulation using Simulink software, as well as researched the process of digital transmittion of information.

A continuation of this work could be a simulation experiment with the transmission of a visual message, as presented by many researchers. This domain appears to be very promising for secure communication (cryptographic security) using chaotic modulation. Another interest in the topic is related to a comparison of different chaotic systems depending on their purpose.

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7 LIST OF ABBREVIATIONS, SYMBOLS, FIGURES AND TABLES