



3rd International Conference on Material and Component Performance  
under Variable Amplitude Loading, VAL2015

## Influence of threshold values on residual fatigue lifetime of railway axles under variable amplitude loading

Pokorný, P.<sup>a,b,\*</sup>, Náhlík, L.<sup>a</sup>, Hutař, P.<sup>a</sup>

<sup>a</sup>Institute of Physics of Materials of the Academy of Sciences of the Czech Republic, v.v.i., Žitkova 22, Brno 616 62, Czech Republic

<sup>b</sup>Faculty of Mechanical Engineering, Brno University of Technology, Technická 2896/2, Brno 616 69, Czech Republic

---

### Abstract

The paper deals with an estimation of residual fatigue lifetime of railway axles under real loading spectrum. The residual fatigue lifetime is given by magnitude of fatigue crack propagation rate. This rate depends predominantly on load, geometry and material of the axle. Standard steel EA4T for manufacturing of railway axles is considered in this paper. The scatter of data in v-K curve could be caused by inaccuracy of experimental measurement or by local change of material properties. The paper shows important differences between obtained residual fatigue lifetime estimations considering scatter in measured material data, especially near the threshold region.

© 2015 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

Peer-review under responsibility of the Czech Society for Mechanics

*Keywords:* railway axle; residual fatigue lifetime; fatigue crack; EA4T

---

### 1. Introduction

The failure of railway axle could be dangerous and is undesirable during operation of trains. Therefore, it is desirable to know residual fatigue lifetime of the railway axle and define regular inspection intervals based on the damage tolerance methodology [1]. For this reason the railway axle with crack of certain size is considered. In this work the crack is assumed at the most loaded spot on the axle surface, in so-called U-notch, i.e. in the location near press-fitted wheel, see Fig. 1a. The numerical model of the railway axle contained crack with a semi-elliptical shape

---

\* Corresponding author. Tel.: +420 532 290 347; fax: +420 541 218 657.

E-mail address: [pokorny@ipm.cz](mailto:pokorny@ipm.cz)

of the crack front. The length of one semi-axes was equal to the crack length  $a$ . The length of the second semi-axes  $b$  was changed in dependence on actual length  $a$  (crack depth) during crack growth according to [2], see Fig. 1a. The crack could theoretically initiate anywhere at the axle surface, but the fastest fatigue crack propagation due to stress concentration and influence of press-fitted wheel is expected at the U-notch. This fact is supported by numerical calculations and experimental observations. International standard EN 13 103 and reference [1] show, that substantial loading of the railway axles is caused by bending moment. The bending moment is caused predominantly by weight of a train. During train movement the bending moment causes cyclic loading, which is dangerous for fatigue failure. The Fig. 1b shows histogram of sorted railway axle load spectrum [3], which acts at the U-notch. It is assumed, that mode of the load stress amplitude 55 MPa corresponds to the bending moment caused by static load without effect of dynamic forces. However, the load spectrum includes different load stress levels caused by different service regimes (influence of curved track, switches, crossovers, etc.). The maximal load stress amplitude in load spectrum at place of considered crack (U-notch) is 162 MPa.

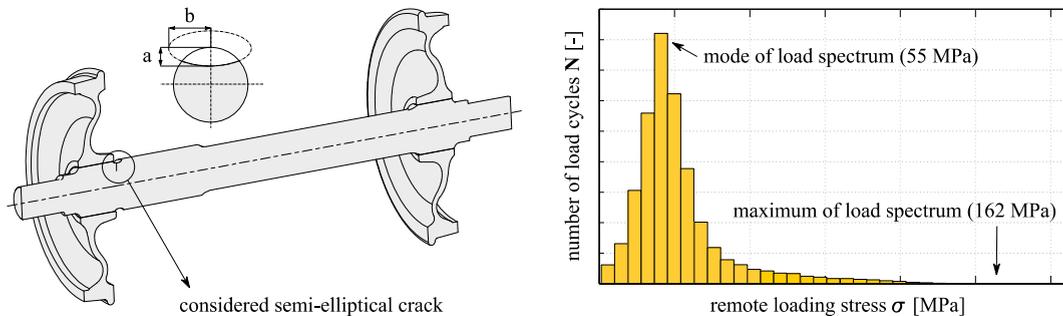


Fig. 1. (a) railway axle with considered position of semi-elliptical crack; (b) histogram of load spectrum at location of considered crack [3]

## Nomenclature

$a$	crack length (semi-elliptical surface crack)
$b$	parameter describing the width of the crack front according to Fig. 1a
$C, m$	material constants used in Paris-Erdogan law
$K$	stress intensity factor
$K_{max}$	maximal value of stress intensity factor at load cycle
$K_{I,PF}$	additional stress intensity factor caused by press-fitted wheels
$K_{IC}$	critical value of the stress intensity factor
$K_{th}$	threshold value of the stress intensity factor
$v$	fatigue crack propagation rate ( $v=da/dN$ )
$Y_I$	shape function taking into account the geometry of the crack and railway axle
$\sigma_{max}$	remote loading stress (maximal longitudinal stress in the position of considered crack at given load cycle)
$R$	loading stress ratio
$RFL$	residual fatigue lifetime
$U\text{-notch}$	geometric transition on the railway axle

## 2. Fatigue crack growth description

For description of the crack behaviour linear elastic fracture mechanics approach based on the stress intensity factor  $K$  was used. Besides bending of the axle the growing crack is influenced by existence of press-fitted wheel. The maximal stress intensity factor during one cycle  $K_{max}$  is given by relation:

$$K_{max} = \sigma_{max} \sqrt{\pi a} Y_I(a) + K_{I,PF}(a), \quad (1)$$

where  $\sigma_{max}$  is loading stress given by the load spectrum and  $Y_I(a)$  is shape function obtained numerically. For numerical modelling finite element method (FEM) was used. Three-dimensional model of the axle with wheel (wheelset) was prepared and  $K$ -calibration with semi-elliptical crack was done to obtain function  $Y_I(a)$ . Quantity  $K_{I,PF}(a)$  in eq. 1 is

stress intensity factor caused by acting residual stresses due to presence of press-fitted wheel in the vicinity of growing fatigue crack.  $K_{I,PF}(a)$  values were determined numerically for given railway axle geometry, see [3] for details. The effect of the press-fitted wheel was taken into account, and maximum value of the stress intensity factor was determined using eq.1.

Dependence between fatigue crack propagation rate  $v$  ( $da/dN$ ) and fracture parameter (the stress intensity factor  $K$  is very often used in this case) is essential for the estimation of residual fatigue lifetime. Fig. 2 shows typical dependence (in log-log coordinates) of fatigue crack growth rate  $v$  on the stress intensity factor  $K$ . This  $v$ - $K$  curve is obtained by experimental measurement for particular loading stress ratio  $R$ . The rotary bending load during train movement causes stress ratio circa  $R=-1$ , however the existence of the press-fitted wheels slightly increase the stress ratio  $-1 < R < 0$ .

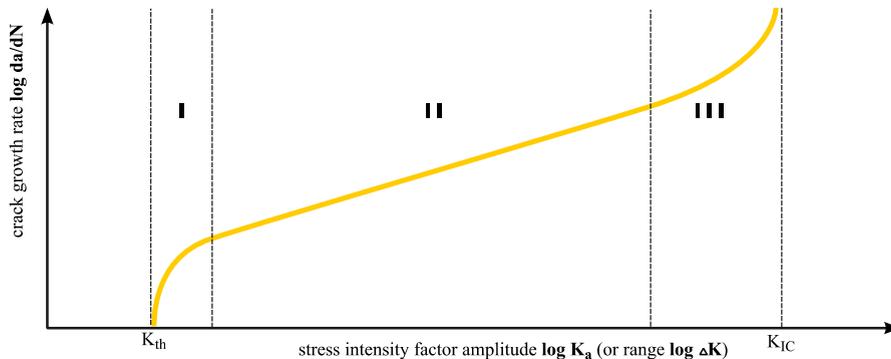


Fig. 2. Typical fatigue crack propagation rate dependence on the stress intensity factor ( $v$ - $K$  curve)

The  $v$ - $K$  curve can be separated into 3 parts. First part represents bending of the curve (knee) near the threshold value of the stress intensity factor  $K_{th}$ . The second part is approximately linear. Paris-Erdogan relationship [4] is valid there. Last part of the curve represents the bending near the critical value  $K_{IC}$ . The third region is not important from the point of view of the residual fatigue lifetime, because the crack propagation in this region is very rapid and contribution of this region to the total residual fatigue lifetime could be neglected [4].

The shape of  $v$ - $K$  curve depends on material properties. This paper considers the standard steel EA4T which is often used for manufacturing of the railway axles. The experimental data of EA4T steel are depicted in Fig. 3. Sampling period and other conditions of measurement were used according to ASTM standard [5]. The experimental data contain some scatter naturally, which could be caused by inaccuracies of measurement method, inhomogeneity of material, etc.

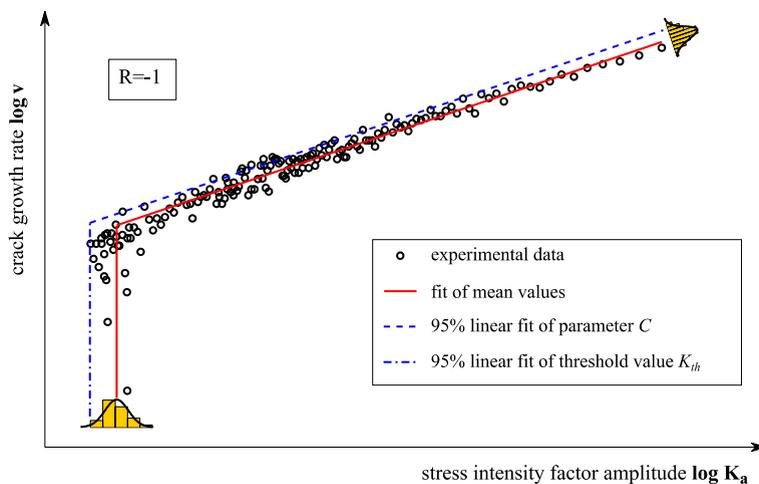


Fig. 3. Experimentally obtained  $v$ - $K$  data for EA4T steel and stress ratio  $R=-1$

This paper mainly focuses on impact of experimental data scatter on the residual fatigue lifetime estimation of the railway axle. The third region of  $\nu$ - $K$  curve is neglected in following considerations and the first region is simplified to the threshold value only. The second region of  $\nu$ - $K$  curve can be described by Paris-Erdogan law [4]:

$$\frac{da}{dN} = C(K_{\max})^m \quad \text{for } K_{\max} \geq K_{th} \quad (2)$$

Parameters  $C$  and  $m$  are material constants determined by experimental measurement. This relationship is valid if the stress intensity factor  $K$  is higher than threshold value  $K_{th}$ . In the case that the stress intensity factor is below the threshold value the fatigue crack propagation rate  $\nu$  is considered as zero. By adopting Paris-Erdogan relationship the  $\nu$ - $K$  curve was divided into two different parts (lines) and the scatter was determined for each of them. The first is a scatter of data determining the threshold value  $K_{th}$ , see Fig. 4a. It was assumed, that this scatter belongs to the normal distribution. From the mathematical point of view the material constant  $m$  represents the slope of the line and  $C$  is its intercept on  $\nu$  axis of  $\nu$ - $K$  curve plotted in log-log coordinates for  $\log(K_{th})=0$ . According to [6] the parameter  $m$  is assumed to be constant and parameter  $C$  is assumed as belonging to the normal distribution (in log-log representation), see Fig. 4b. The mean value of the constant  $C$  was obtained by least squares fit of experimental data in the second region of  $\nu$ - $K$  curve. For each scatter the distribution function was determined and then linear fits of certain percentage were created, see e.g. 95% linear fit of data in Fig. 3.

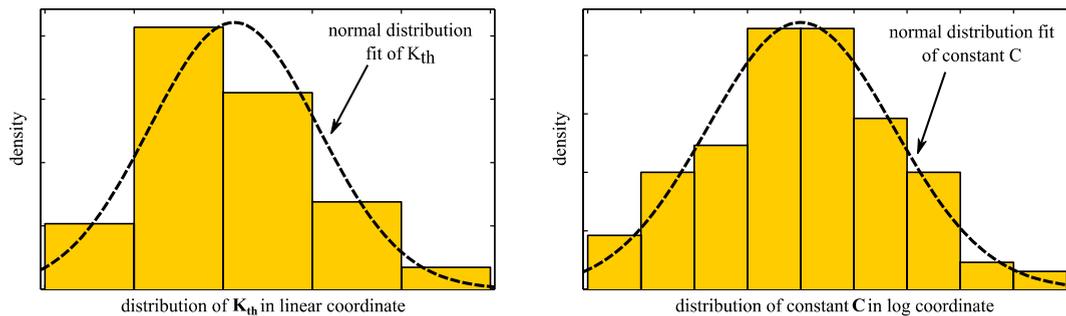


Fig. 4. (a) Histogram of threshold value distribution (b) Histogram of material constant  $C$  distribution

The residual fatigue lifetime of the railway axle is considered as number of load cycles (or load blocks) necessary for the crack growth from initial crack length 1 or 2 mm to the critical length considered here as 55 mm. The critical length 55 mm is shorter than length corresponding to the critical value of the stress intensity factor  $K_{IC}$ , but as was mentioned formerly, the final stage of the fatigue crack propagation is very rapid and this stage has unimportant contribution to the whole residual fatigue lifetime. Material data obtained under loading stress ratio  $R=-1$  were used for the estimation of residual fatigue lifetime of the railway axle. The fatigue crack increments were computed by relationship (2) for all load amplitudes of the load spectrum, see Fig. 1b. Whole process was repeated until the crack length was shorter than the considered critical length.

### 3. Results obtained

The estimated residual fatigue lifetimes of the railway axles are calculated for certain probability of constant  $C$  and probability of the stress intensity factor threshold value  $K_{th}$ . The table 1 shows calculated residual fatigue lifetimes in “load blocks” determined for the mean value of  $K_{th}$  and five different linear fits determining value of material constant  $C$ . Each linear fit represents different probability that measured crack propagation rate value is lower than the one described by given  $C$ . Thus, for 99% probability almost all measured data lies below the linear fit describing crack propagation rate, i.e. the real crack propagates by lower rate with 99% probability than is considered in the residual fatigue lifetime calculations. The 50% linear fit represents mean value (obtained by least square method) of crack propagation rate for given stress intensity factor  $K$ . It is evident that the 99% linear fit used in numerical estimations provides conservative results. On the other side the 50% linear fit could provide better correlation with real crack propagation, however in some cases, the residual fatigue lifetime estimation can be non-conservative.

The ratio of obtained residual fatigue lifetimes for mean value (50%) and 99% linear fit of constant  $C$  is 1.4 for both considered initial crack lengths 1 mm and 2 mm.

Table 1. RFL estimations in “load blocks“ for mean threshold value of  $K_{th}$  in combination with different linear fits of constant  $C$

initial crack length	mean value of $K_{th}$ + variation of linear fit of constant $C$ :					ratio
	99%	95%	90%	80%	50%	50%/99%
$a_0=1\text{mm}$	319	350	367	390	437	~1.4
$a_0=2\text{mm}$	70	76	80	85	96	~1.4

In the following, the constant  $C$  was considered as mean value of the measured data and varying parameter was the threshold value of the stress intensity factor  $K_{th}$ . Residual fatigue lifetime estimations for different probability of  $K_{th}$  are summarized in Table 2. The ratio between obtained residual fatigue lifetime estimations for the mean value (50%) and 99% linear fit of threshold value  $K_{th}$  is 2.3 for the initial crack length 1 mm and 1.2 for the initial considered crack length 2 mm. It follows from this result that the scatter of threshold value data is more important in RFL estimations with shorter initial crack lengths.

Table 2. RFL estimations in “load blocks“ for mean value of constant  $C$  in combination with different linear fits of  $K_{th}$

initial crack length	mean value of $C$ + variation of linear fit of $K_{th}$ :					ratio
	99%	95%	90%	80%	50%	50%/99%
$a_0=1\text{mm}$	188	222	251	297	437	~2.3
$a_0=2\text{mm}$	81	84	87	89	96	~1.2

Table 3 shows resultant residual fatigue lifetime estimations of the railway axle for the same percentage linear fit of  $C$  and  $K_{th}$  parameters. In other words, material constant  $C$  and the threshold value of the stress intensity factor  $K_{th}$  had the same probability in one calculation of the residual fatigue lifetime (see e.g. 95% linear fits of both parameters in Fig. 3). The ratio of obtained residual fatigue lifetime estimations for mean values (50%) and 99% linear fits of both parameters ( $C$  and  $K_{th}$ ) is 3.2 for initial crack length 1 mm. This ratio is 1.6 for initial crack length 2 mm.

Table 3. RFL estimations in “load blocks“ for the same percentage linear fit of both material parameters ( $C$ ,  $K_{th}$ )

initial crack length	coincident linear fits for both regions					ratio
	99%	95%	90%	80%	50%	50%/99%
$a_0=1\text{mm}$	137	177	211	265	437	~3.2
$a_0=2\text{mm}$	59	67	72	80	96	~1.6

#### 4. Discussion

It should be noticed here that the resultant values of estimated lifetime were determined for  $\nu$ - $K$  curve with relatively low scatter. Specifically, the stress intensity factor threshold values were established by measurements on 2 experimental samples. It could be expected that for more samples the scatter could be greater. The greater scatter is associated with higher differences of determined residual fatigue lifetimes for various probability curves of  $\nu$ - $K$  curve.

Above depicted tables show that the residual fatigue lifetime estimation of the railway axle is more influenced by the scatter of parameter  $K_{th}$  in the case of shorter initial crack lengths (around 1 mm) than by scatter of material constant  $C$ . Similar conclusion was published e.g. in the work [7]. Higher sensitivity of given application to the scatter of the stress intensity factor threshold value  $K_{th}$  follows from the fact that this parameter determines which load amplitude contributes to the crack elongation and which not, see equation (2). A small change of the threshold value could lead to the addition or remove of acting damaging amplitudes from the load spectrum in the numerical estimations of the residual fatigue lifetime. For this reason the relatively high scatter of obtained residual fatigue lifetimes occurs. The damaging load amplitudes for particular crack lengths are yellow depicted in Fig. 5. The grey depicted load amplitudes are below the threshold value and for particular crack length they do not contribute to the crack elongation. The Fig. 5 shows also a typical fatigue crack evolution with initial fatigue crack length 1 mm. The figure shows, that fatigue crack growth from length 1 mm up to 2 mm represents approximately 75% of the whole

residual fatigue lifetime of the railway axle (in the case of initial crack length 1 mm). This confirms the fact that the most residual fatigue lifetime is concentrated in early stages of fatigue crack growth.

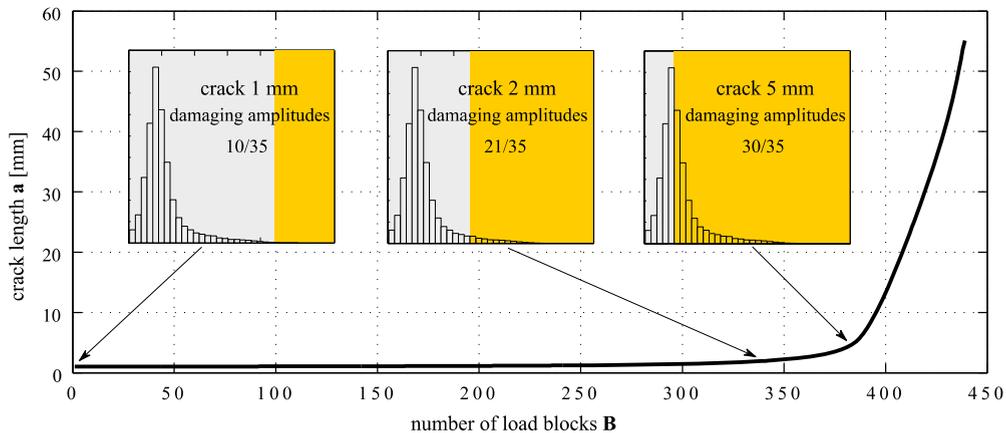


Fig. 5. Fatigue crack length evolution with scheme of damaging amplitudes for certain crack lengths (marked by yellow colour)

## 5. Conclusions

The paper deals with establishing of the residual fatigue lifetime of the railway axle with crack. Standard steel EA4T commonly used for manufacturing of the railway axles is considered in the work. The scatter of data of measured  $v$ - $K$  curve occurs mainly due to inherent non-ideal material properties of the considered steel (experimental samples). The Paris-Erdogan relationship was used for the description of  $v$ - $K$  curve. The scatter of whole  $v$ - $K$  curve was separated, simplified and described by scatters of two material parameters. First parameter was the scatter of material constant  $C$ . The second parameter was scatter of the stress intensity factor threshold value  $K_{th}$ . Both scatters were described by normal distribution and certain probability linear fits of measured data were carried out. The ratio of mean values and 99% linear fits of measured data were chosen for the comparison. The obtained estimations of residual fatigue lifetime show great sensitivity to the threshold value for the short initial crack lengths (around 1 mm). The sensitivity of obtained results on the scatter of material constant  $C$  is the same for both considered initial crack lengths (1 mm and 2 mm). In the case of initial crack length 1 mm the consideration of 99% linear fits for both scatter parameters provides approximately 3.2 times lower residual fatigue lifetime estimation in comparison to the one considering mean values. In the case of initial crack length 2 mm the difference of obtained lifetime estimations was 60%, see Table 3. This shows that sensitivity of the residual fatigue lifetime estimations of the railway axle on  $v$ - $K$  curve scatter is more pronounced for shorter initial crack lengths. Obtained results can be used for safer operation of the railway axles in service.

## Acknowledgements

This work was supported through the specific academic research grant No. FSI-S-14-2311 provided to Brno University of Technology, Faculty of Mechanical Engineering and grant No. CZ.1.07/2.3.00/20.0214 of the Ministry of Education, Youth and Sports of the Czech Republic.

## References

- [1] U. Zerbst et al., Safe life and damage tolerance aspects of railway axles – A review, *Engng Fract Mech* 98 (2012) 214-271.
- [2] M. Ševčík et al., Numerical estimation of the fatigue crack front shape for a specimen with finite thickness, *International Journal of Fatigue* 39 (2012) 75-80.
- [3] Residual lifetime of the railway axle, Technical report for Bonatrans Group, a.s., Institute of Physics of Materials ASCR (2012)
- [4] J. Schijve, *Fatigue of structures and materials*. New York: Springer (2008)
- [5] ASTM E647, Standard Test Method for Measurement of Fatigue Crack Growth Rates (2013)
- [6] S. Beretta, M. Carboni, Experiments and stochastic model for propagation lifetime of railway axles, *Engng Fract Mech* 73 (2006) 2627-2641
- [7] P. Pokorný, L. Náhlik, P. Hutař, The effect of threshold value on the residual fatigue lifetime of railway axles, *Engineering mechanics* 2014, Svratka, Czech Republic (2014)