

# Bootstrap Estimation of Expected Risk and Return of Strategy Equity Indices

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## Abstract

The aim of this article is to present the results of research associated with the ex-post estimation of expected risk, return and other characteristics of strategy equity indices and capital-weighted equity indices partially and to determine credible methods for a transparent comparison. The data sources are the MSCI and STOXX equity index providers. Suitable statistical methods and a computation-intensive method for estimating selected characteristics have been used and compared to one another. For the measurement of excess return per unit of risk a modified Sortino ratio was used, which takes into account only the downside size and frequency of returns, measuring the return to negative volatility trade-off. Based on our results, it is apparent that some strategic equity indices outperform capital-weighted equity indices in a long-term investment perspective (1997-2018). A suitable combination of strategic equity indices, namely the mix of dividend strategy and momentum strategy may lead to the highest yield / risk ratio expressed by the Sortino ratio. The outperformance path of a mix of dividends and momentum strategy indices is much more stable than either the performance of the individual strategy equity indices or capital-weighted equity indices alone.

## Keywords

risk, return, equity indexes, semideviation, bootstrap, real-world data

## 1 Introduction

A capitalization-weighted index is the most common way to gain access to broad equity market performance. These portfolios are generally concentrated in few equities and lack diversification. In order to avoid this drawback or to simply diversify market exposure, alternative indexation methods have recently generated considerable interest among both academic researchers and market practitioners. Traditionally, beta was defined as the return from broad asset class exposure, while alpha represented additional return from active portfolio management. More recently, however, investors have started to recognize that many sources of return that were considered added value (alpha) actually represent systematic risk premia (beta) (Anson, 2008; Berger et al., 2008). As a result, there has been a proliferation of new indices aiming to capture various sources of systematic return.

Strategy indices are designed on the basis of quantitative models and rule-based investment schemes to provide a single value for the aggregate performance of a number of companies. In contrast to broad-based market and

sector indices that group companies by size or industry, strategy indices reflect the performance of a rule-based investment strategy.

All equity strategies carry risks, which can be divided into two categories: systematic risks and specific risks. Systematic risks stem from the fact that strategy indices can be exposed to systematic risk factors (such as value and small cap risk). In contrast with systematic risks that may be common among Smart Beta investments, specific risks are related to the characteristics of a given portfolio construction methodology. As shown in Amenc et al. (2012), alternative weighting schemes yield different performance results depending on market conditions.

The objective of this paper is to make an ex-post estimation of expected risk, return and other characteristics of strategy equity indices and capital-weighted equity indices partially over the last 21 years and to determine credible methods and compare them in a transparent manner. The paper formulates the measurement of risk-adjusted return of equity indices based on below-target semivariance,

computed by using a computationally-intensive method—bootstrap. The objective is to verify the solvability of the task. Matlab R2018b and Statgraphic 18 were used for the computation and presentation of graphical results.

## 2 Literature review

Traditionally, equity returns were attributed to passive market exposure (the equity risk premium) expressed by means of capitalization-weighted indices and active portfolio management. More recently, various measures have been taken to go beyond cap-weighted equity indices and to address some of their known shortcomings, such as the issue of high concentration in larger capitalisation equity or their lack of risk/return efficiency (Ferson et al., 1987; Goltz and Le Sourd, 2011). It is now commonly accepted that moving away from cap-weighting tends to enhance diversification and increase risk-adjusted performance in the long run (Choueifaty and Coignard, 2008; DeMiguel et al., 2009; Maillard et al., 2010; Meucci, 2009; Small and Hsieh, 2017). The empirical evidence presented by Chen et al. (1986) suggests that equity dynamics are best characterized by a multi-factor representation of equity index returns. Research into systematic sources of strategy indices return has a relatively long history in equity investing. Since the publication of a seminal paper on common factors in equity returns by Fama and French in 1993, equity investors have been aware of factor-based approaches that reflect either systematic exposures to themes such as valuation (as measured, for example, by book-to-market ratios), quality (as measured, for example, by the stability of earnings and dividend policies), size (as measured by market capitalization), momentum (as measured by relative past price performance), or risk anomalies (as measured by the outperformance of low-risk portfolios by high-risk portfolios) (Fama and French, 1993). See, for example, Melas and Kang (2010), for the discussion of the application of systematic equity indices in the investment process. Belimam et al. (2018), and Choi and Choi (2018) followed this line by publishing their research. Belimam et al. (2018) evaluate and compare the performance of three-asset pricing models or seek empirical evidence to confirm this, and suggest individual trading weight as a proxy for noise trader risk. Choi and Choi (2018) gathered empirical evidence to support the employment of individual trading weight as a proxy for noise trader risk. According to Gonzalez and Thaboult (2013), equity underlying strategies can be broadly divided into:

- Indices based on Economic Size, which weigh equities according to intrinsic accounting measures of

the size of the firm such as revenue, profits, book value, cash flows and dividends;

- Risk-weighted indices which consider stand-alone risk properties or equity;
- Innovation indices which incorporate measures of the attractiveness of a firm's intellectual property;
- Dividend-based indices that focus on companies with the most attractive dividend features (Gonzalez and Thaboult, 2013).

In general, the performance of equity underlying strategies has been cyclical in nature. Individual strategies have been shown to perform better than others in different macroeconomic environments. In recent years, many stock indices have been built on different strategies, and the question is whether some strategies are more profitable than others. However, it is not possible to only compare the yield; risk must also be considered.

For many years, there has been a widespread assumption amongst practitioners in many markets that standard deviations of returns are a simple and appropriate measure of risk. Much of the work on this issue has been empirical and it has shown through the years that the standard deviation of returns may be an oversimplified and inappropriate measure of risk. These observations are related to the assumption that the distribution of returns is symmetric. Greater reliability of computations should only be applied when this underlying distribution is normal.

Meanwhile, further risk measurement techniques in the financial industry have been developed, which aim to provide a more transparent expression of risk. In 1952, two authors published fundamental papers about the financial industry; the first was Markowitz (1952), who identified that risk was related to varying financial outcomes and adopted the standard deviation of residual assets as a tool for risk measurement. The second was Roy (1952), who introduced the “Safety First” criterion, which involved the introduction of a downside risk measurement principle.

A few years later, Markowitz (1959) published a significant discussion on risk, and introduced alternative measurements tools, such as semivariance, expected value of loss, expected absolute deviation, probability of loss and maximum loss. Markowitz (1956) and Markowitz (1959) also introduced his idea of downside-risk. Downside-risk means a semivariance computed using below-target semivariance ( $SV\bar{r}$ ). This metric calculates variance using only those returns under a target return ( $SV\bar{r}$ ). Markowitz called these measures partial or semi-variances, because only a subset of the return distribution is used (Nawrocki, 1999).

Below-target semivariance ( $SVt$ ) is calculated as follows (Eq. (1)):

$$SVt = \frac{1}{K} \sum_{T=1}^K \max[0, (t - R_T)]^2, \quad (1)$$

where  $R_T$  is the asset return over time period  $T$ ,  $K$  is the number of observations, and  $t$  is the target rate of return of the asset's return. A maximizing function, denoted as  $\max$ , indicates that the formula will square the larger of two values, i.e. 0 and  $(t - R_T)$ .

Even after the semivariance measure was proposed, most researchers kept following the variance measure because it was computationally simpler. The semivariance optimization models using a cosemivariance matrix (or semicovariance, if that is your preference) require twice the number of data inputs as the variance model. Given the lack of cost-effective computer power and the fact that the variance model was already mathematically very complex as it belonged in the class of quadratic programs, this was a dominant consideration in practical applications until the advent of the microcomputer in the 1980s (Nawrocki, 1999). Markowitz et al. (1987) also developed this approach further, in order to define a measure of downside risk. According to findings by Kahneman and Tversky (1979), loss aversion preferences imply that investors who dislike downside losses will demand greater compensation in the form of higher expected returns for holding shares with high downside risk.

The Sortino ratio (Sortino and Van der Meer, 1991) was used to measure risk-adjusted return of equity indices. It is a modification of the Sharpe ratio, but only penalizes those returns which fall below a specified target (positive outliers should be regarded as a bonus and not as a risk), while the Sharpe ratio penalizes both upside and downside volatility equally. Thus, it is a measure of risk-adjusted returns that treats risk more realistically than the Sharpe ratio.

Sortino ratio is calculated as follows (Eq. (2)):

$$\text{Sortino} = \frac{R_T - t}{SVt}. \quad (2)$$

To quantify below-target semivariance ( $SVt$ ) and Sortino ratio, it is necessary to use computationally-intensive methods. The most popular technique is Monte Carlo simulation, utilized to estimate future market performance. With basic assumptions about the natural behaviour of markets and about the distribution of market returns, these simulations use computing power and random number generation to help predict the future. One of the most useful

simulation techniques for investment planning is the bootstrap method, formulated in 1979 by Brad Efron, professor at Stanford University (Efron, 1979). The idea behind the bootstrap is quite clear. Analysts are interested in finding out information about a population but they only observe a sample of data from that population. The bootstrap treats the available sample as a population, using a computer to generate repeated random samples with replacement (resamples) from that sample data, in turn calculating statistics of interest from these resamples. Over thousands of iterations, the distribution of such statistics can be considered as satisfactory. Rather than dealing with arduous and often overly theoretical assumptions to make statistical inferences, in practice, an analyst prefers to deal with a multitude of bootstrap samples using the plentiful computer power available.

Moreover, in many applications, the bootstrap method can often yield higher-order accurate estimates of distribution, which result in more precise asymptotic approximation. Thanks to these advantages, it is not surprising to find applications of the bootstrap method in finance. For example, Ferson and Foerster (1994) and Kothari and Shanken (1997) applied it to asset pricing and Lyon et al., (1999), among others, have used it in corporate finance, while Shaik and Maheswaran (2018) implement the bootstrap technique in their work on robust volatility ratio, and Liu (2018) examines extreme behaviours at both the lower 5 % and 1 % quantile levels of the three exchange rates series.

To use the bootstrap or any other statistical methodology effectively, one has to be aware of its limitations. The bootstrap is of value in any situation in which the sample can serve as a surrogate for the population. If the sample is not representative of the population because the sample is too small, biased, or not selected in a random way, or its constituents are not independent, then the bootstrap-based techniques will fail. Canty et al. (2006) and Chernick (2008) also list data outliers, the inconsistency of the bootstrap method, incorrect resampling of a model, wrong or inappropriate choice of statistics, non-pivotal test statistics, nonlinearity of the test statistics, and discreteness of the resample statistic as potential sources of error. The pitfalls of using the bootstrap method have also been shown by Davison and Hinkley (1997), and Terpstra and McKean (2005), and Salibian-Barrera and Zamar (2002).

### 3 Data and methods

For the purpose of our research, we focus on comparing and determining the important characteristics (e.g. return,

risk, Sharpe ratio, Sortino ratio and correlations) of selected investment strategies of equity indices from renowned providers of these indices.

- MSCI World Index (Net Return) captures large and mid-cap representation across 23 Developed Markets. With 1643 constituents, the index covers approximately 85 % of the free float-adjusted market capitalization in each country. The index is free-float market capitalizations weights (MSCI WORLD INDEX Fact Sheet (MSCI Indices, 2018a)).
- MSCI World High Dividend Yield Index (Net Return) is based on the MSCI World Index, its parent index, and includes large and mid-cap equity across 23 Developed Markets (DM) countries. The index is designed to reflect the performance of equities in the parent index (excluding REITs) with higher dividend income and quality characteristics. Securities are screened based on certain “quality” factors, such as return on equity (ROE), earnings variability, debt to equity (D/E), and on the recent 12-month price performance. The goal is to exclude equity with potentially deteriorating fundamentals that could be forced to cut or reduce dividends. From the list of eligible companies, only those with higher than average dividend yields are selected for inclusion in the index (MSCI WORLD HIGH DIVIDEND YIELD INDEX Fact Sheet, (MSCI Indices, 2018b)).
- MSCI World Minimum Volatility Index (Net Return) aims to reflect the performance characteristics of a minimum variance strategy applied to the MSCI large and mid-cap equity universe across 23 Developed Markets countries. The index is calculated by optimizing the MSCI World Index, its parent index, for the lowest absolute risk (within a given set of constraints) (MSCI WORLD MINIMUM VOLATILITY INDEX Fact Sheet, (MSCI Indices, 2018c)).
- MSCI World Momentum Index (Net Return) is based on MSCI World Index, its parent index, which includes large and mid-cap equity across 23 Developed Markets (DM) countries. It is designed to reflect the performance of an equity momentum strategy by emphasizing equity with high price momentum, while maintaining reasonably high trading liquidity, investment capacity and moderate index turnover. A momentum value is determined for each equity in the MSCI parent index by combining the equity’s recent 12-month and 6-month local price performance. This momentum

value is then risk-adjusted to determine the equity’s momentum score. A fixed number of securities with the highest momentum scores are included in each MSCI Momentum Index, generally covering about 30 % of the parent index markets capitalization. Constituents are weighted by the product of their momentum score and their market capitalization (MSCI WORLD MOMENTUM INDEX Fact Sheet, (MSCI Indices, 2018d)).

- MSCI World Enhanced Value Index captures large and mid-cap representation across 23 Developed Markets (DM) countries exhibiting overall value style characteristics. The index is designed to represent the performance of securities that exhibit higher value characteristics relative to their peers within the corresponding GICS® sector. The value investment style characteristics for index construction are defined using three variables: Price-to-Book Value, Price-to-Forward Earnings and Enterprise Value-to-Cash flow from Operations (MSCI WORLD ENHANCED VALUE INDEX Fact Sheet, (MSCI Indices, 2018e)).
- STOXX Global Select Dividend 100 (Net Return) is designed to measure the performance of the highest dividend-paying equity relative to their home markets. Equity is screened by defined historical non-negative dividend-per-share growth rates and dividend to earnings-per-share (EPS) ratios. The index is derived from their respective benchmark index, such as the STOXX North America 600, STOXX Asia/Pacific 600, STOXX Europe 600, EURO STOXX and STOXX EU Enlarged TMI. The components are weighted by their indicated annual net dividend yield, i.e. the largest dividend-yielding companies have the highest weight in the index (STOXX Global Select Dividend 100 Fact Sheet, (STOXX Indices, 2018)).

According to Fitzherbert (2001), when an investor is making decisions on the basis of mean rates of return, the only definition of ‘mean return’ that makes any sense is mean continuously compounded return or something that is equivalent., therefore the compound annual growth rate (CAGR) has to be used in Eq. (3).

$$R_n = \ln \left( \frac{C_n}{C_{n-1}} \right) \quad (3)$$

$C_n$  = closing price

$C_{n-1}$  = previous day’s closing price

Similarly, volatility is measured with a statistic known as the Geometric Standard Deviation (GSD), which is defined as the exponential of the annual volatility (Eqs. (4) and (5)):

$$\text{GSD} = \exp[\sigma] \tag{4}$$

$$\text{GSD} = e^{sd} (\log X_i), \tag{5}$$

where *sd* is the sample standard deviation.

All equity indices returns are quarterly net returns (CAGR) denominated in Euro. The data was collected over a 20-year period from July 1997 until June 2018, 84 quarterly data were obtained per each equity index. We used a robust approach to data analysis as regards outlier-resistant interpretation. This means that the statistical methods aim at constructing statistical procedures that are stable (robust) even when the underlying model is not perfectly supported by the available dataset. A typical example of this is the presence of outliers – observations that are very different from the rest of the data. Outliers are “bad” data in the sense that they deviate from the pattern set by the majority of the data (Hall 1985; Huber and Ronchetti, 2009; Hampel et al., 2011). Hence, they tend to obscure its generic flow and may lack explanatory and predictive power regarding the generic portion of the data. Robust models focus on the statistical properties of the bulk of the data without being distracted by outliers, while in classical models all data equally contribute to the analysis. Classical estimators that assign equal importance to all available data are highly sensitive to outliers. Therefore, in the presence of just a few extreme losses, classical analysis can produce arbitrarily large estimates of mean, variance, and other statistics.

Bassett et al. (2004) investigate the performance of portfolio return distribution using robust and quantile-based methods, and conclude that the resulting forecasts outperform those under a conventional classical analysis. Perret-Gentil and Victoria-Feser (2005) used robust estimates for the mean and the covariance matrix in the mean-variance portfolio selection problem. They showed that robust portfolios outperform classical models, as the outlying observations (that account for 12.5 % of the dataset) can have a serious influence on portfolio selection when the classical approach is employed. Firstly, the individual index data was analyzed. The results are presented in Table 1.

Table 1 shows the summary statistics for each of the selected data variables. It includes measures of central tendency, measures of variability, and measures of shape. Of particular interest are the standardized skewness and standardized kurtosis, which can be used to determine whether or not the sample is part of the normal distribution. Values of these statistics that are outside the range of -2 to +2 indicate significant departures from normality, which would tend to invalidate many of the statistical procedures normally applied to these data.

The graphical results of central tendency (and position indicators), scattering and extreme values of the indices are shown in Box-and-Whiskers Plot, see Fig. 1. The graphical results of the central tendency (and position indicators), scattering and the extreme values of the indices are shown in a Box-and-Whiskers Plot, see Fig. 1.

Based on the results of Table 1 and Fig. 1, it can be stated that:

**Table 1** Quarterly summary statistics of equity indices

|                     | MSCI EValue     | MSCI HDY        | MSCI Momentum  | MSCI MV         | MSCI World     | STOXX 100       |
|---------------------|-----------------|-----------------|----------------|-----------------|----------------|-----------------|
| Count               | 84              | 84              | 84             | 84              | 84             | 84              |
| Mean                | 1.98095         | 1.27024         | 1.92619        | 1.44524         | 1.13452        | 2.24286         |
| Median              | 3.25            | 2.45            | 3.2            | 2.0             | 3.25           | 3.35            |
| Median-Mean Diff.   | 1.27            | 1.18            | 1.27           | 0.55            | 2.12           | 1.11            |
| 12% Trimmed mean    | 2.6209          | 2.01723         | 2.4833         | 1.84311         | 1.91729        | 2.92594         |
| Standard deviation  | 9.9952          | 8.05734         | 9.42088        | 6.66338         | 8.4723         | 8.59852         |
| Minimum             | -25.9           | -23.4           | -20.8          | -16.9           | -22.8          | -33.8           |
| Maximum             | 24.2            | 16.9            | 35.9           | 14.7            | 21.1           | 19.9            |
| Lower quartile      | -3.7            | -2.15           | -2.75          | -2.95           | -3.2           | -2.15           |
| Upper quartile      | 8.15            | 5.95            | 6.6            | 5.9             | 5.9            | 7.65            |
| Interquartile range | 11.85           | 8.1             | 9.35           | 8.85            | 9.1            | 9.8             |
| Std. skewness       | <b>-2.03316</b> | <b>-3.27136</b> | -0.170439      | <b>-2.21243</b> | <b>-3.0738</b> | <b>-4.36202</b> |
| Std. kurtosis       | 0.839592        | <b>2.0253</b>   | <b>3.37053</b> | 1.28582         | 1.91748        | <b>6.26492</b>  |

Source: authors' own calculation

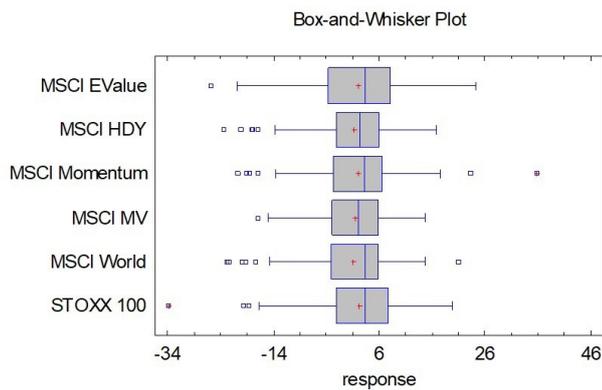


Fig. 1 Quarterly Box-and-Whiskers Plot of equity indices

- All equity indices are asymmetric, with extreme values. This will have a significant impact on the quantification of risk (depending on how it is measured). The lowest value (and the largest negative extreme value) was from the STOXX Global Select Dividend 100 Index. The highest value (and the largest positive extreme value) was from the MSCI World Momentum index.
- There are differences between median and mean values in all indices. The highest difference was observed for the MSCI World Index, where the median is nearly three times as high as the mean value.
- Based on the findings above, we compare the performance of individual indices with a trimmed mean (12 %), the highest yield being achieved by the STOXX Global Select Dividend 100 Index (2.92 p.q.), then MSCI World Enhanced Value (2.62 p.q.), and MSCI World Momentum (2.48 p.q.).

- The lowest volatility, expressed by the standard deviation, is achieved by the MSCI World Minimum Volatility Index, which supports its index construction strategy. In this concept, index fluctuations are roughly 20 % lower than the parental index MSCI World, followed by the MSCI High Dividend Yield index and STOXX Global Select Dividend 100 (with nearly the same volatility as the previous index).
- A common graphical presentation of data is a scatter plot chart of individual indices, where scattering is captured in individual time slots. Fig. 2 shows large disparities, especially in periods of dramatic declines (September 1998, September 2001, September 2008) or strong growth (December 1999, June 2009).
- This suggests that individual indices respond very differently, especially in extreme market situations.

A Spearman rank correlation was used to compare returns over time between equity indices, see Table 2. A Spearman rank correlation is somewhat more robust than a Pearson product-moment correlation. A Spearman rank correlation is less sensitive to non-normality in distributions.

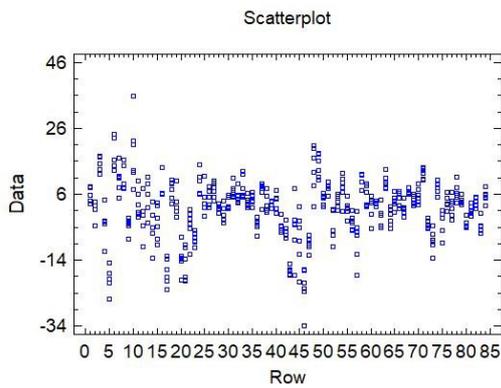
There is a strong correlation between all of indices except MSCI World Momentum and STOXX Global Select Dividend 100 Indices. The value of Spearman rank correlation reached 0.6536. This finding may indicate a diversification potential, in particular in relation to both of these indices. Therefore, a new “mixed index” of 50 % MSCI World Momentum + 50 % STOXX Global Select Dividend 100 Indices was created. Quarterly summary statistics of this “mixed” index are shown in the Table 3 and on a Box-and-Whiskers Plot, see Fig. 3.

Table 2 Spearman rank correlation of equity indices

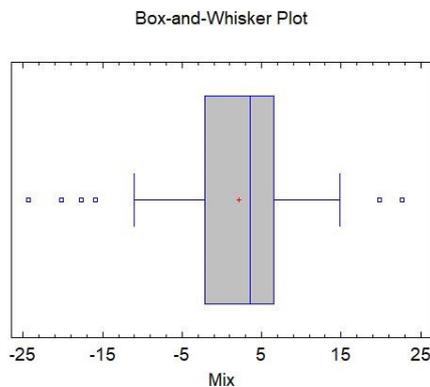
|               | MSCI EValue      | MSCI HDY                | MSCI Momentum           | MSCI MV                 | MSCI World              | STOXX 100               |
|---------------|------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| MSCI EValue   |                  | 0.8823<br><b>0.0000</b> | 0.7561<br><b>0.0000</b> | 0.7570<br><b>0.0000</b> | 0.9095<br><b>0.0000</b> | 0.8253<br><b>0.0000</b> |
| MSCI HDY      | 0.8823<br>0.0000 |                         | 0.7834<br><b>0.0000</b> | 0.8784<br><b>0.0000</b> | 0.9131<br><b>0.0000</b> | 0.8601<br><b>0.0000</b> |
| MSCI Momentum | 0.7561<br>0.0000 | 0.7834<br>0.0000        |                         | 0.7799<br>0.0000        | 0.8666<br>0.0000        | 0.6536<br>0.0000        |
| MSCI MV       | 0.7570<br>0.0000 | 0.8784<br>0.0000        | 0.7799<br>0.0000        |                         | 0.8399<br><b>0.0000</b> | 0.7861<br><b>0.0000</b> |
| MSCI World    | 0.9095<br>0.0000 | 0.9131<br>0.0000        | 0.8666<br>0.0000        | 0.8399<br>0.0000        |                         | 0.7860<br><b>0.0000</b> |
| STOXX 100     | 0.8253<br>0.0000 | 0.8601<br>0.0000        | 0.6536<br>0.0000        | 0.7861<br>0.0000        | 0.7860<br>0.0000        |                         |

Correlation. P-Value (P-values below 0.05 indicate statistically significant non-zero correlations at the 95.0 % confidence level.)

Source: authors' own calculation



**Fig. 2** Scatterplots of equity indices  
 Source: authors' own calculation



**Fig. 3** Quarterly Box-and-Whiskers Plot of “mixed” index  
 Source: authors' calculation

Based on the findings from a survey data analysis and partial correlations, the bootstrap method was used, applying robust statistics to quantify the estimates of yields, risk, and other stock index indicators.

The estimates thus obtained are suitable input values for the purpose of comparing equity indices with each

**Table 3** Quarterly summary statistics of “mixed” equity index

|                        |                 |
|------------------------|-----------------|
| Count                  | 84              |
| Mean                   | 2.16548         |
| Median                 | 3.6             |
| Median-Mean Difference | 1.44            |
| 12% Trimmed mean       | 2.72644         |
| Standard deviation     | 8.12873         |
| Minimum                | -24.8           |
| Maximum                | 22.7            |
| Lower quartile         | -2.15           |
| Upper quartile         | 6.55            |
| Interquartile range    | 8.7             |
| Std. skewness          | <b>-2.47447</b> |
| Std. kurtosis          | <b>2.69693</b>  |

Source: authors' own calculation

other, since they do not contain distortions due to extreme values or strongly skewed data distribution.

For such a comparison, the Sortino ratio was chosen, which only uses the below-target semivariance, which is decisive for the investor.

Moreover, this below-target semivariance risk metric yields markedly different results from the metric commonly used to express the risk level, i.e. standard deviation.

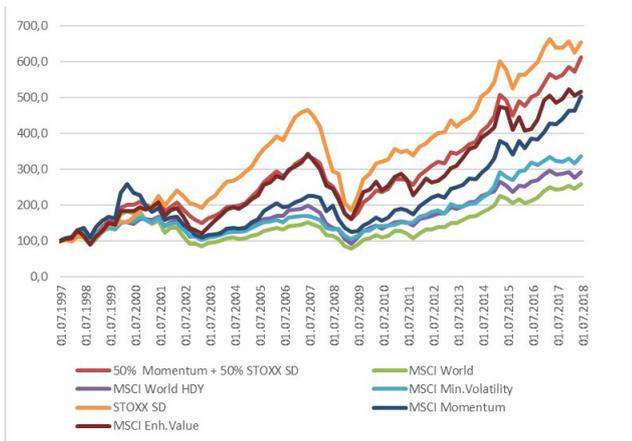
Equity indices statistics were calculated using the bootstrap method; therefore, 10 times 10,000 bootstrap samples of each index were carried out. It means that each set of index statistics was estimated 10 times and for a “final” enumeration, a trimmed mean (20 %) was used with the values rounded to 0.05. Annualized statistics and characteristic were obtained from the quarterly data, which simulated partial withdrawals made up of four random quarterly values. These annual values (annualized return p.a.) were entered as input variables in the bootstrap procedure (see Table 4).

The development of individual indices in the period 3q1997-2q2018 is shown in Fig. 4.

#### 4 Discussion

Based on the results quantified by the bootstrap method, we can state the following:

- Over the past 21 years, the STOXX Global Select Dividend 100 Index has achieved the highest appreciation, which is more than 80% compared to the MSCI World High Dividend Yield Index, which also focuses on companies paying above-average dividends at a comparable risk level.
- The smallest risk, expressed by standard deviation or Below Target Semi-variance, was achieved by the MSCI World Minimum Volatility Index, which confirms the strategy it is built on. Both risk metrics are approximately 25 % lower than the parent index MSCI World Index. Appreciation of the MSCI World Minimum Volatility Index was 1 % p.a. higher than the parent index MSCI World Index.
- Of the MSCI index “family”, the MSCI World Enhanced Value Index achieved the highest value for that period, but it also incurred the highest level of risk of any index.
- The least value of Spearman rank correlation was observed between STOXX Global Select Dividend 100 Index and MSCI World Momentum Index. This finding indicates a diversification potential, in particular in relation to both of these indices. Therefore, a new “mixed index” of 50 % MSCI World Momentum



**Fig. 4** Strategy equity indices development  
 Source: authors' own calculation

+ 50 % STOXX Global Select Dividend 100 Indices was created. The index thus formed attained the second highest yield of all the evaluation indexes and the second smallest value of Below Target Semivariance.

- Excess return per unit of risk was measured by means of the Sortino ratio, which uses Below Target Semivariance, instead of total risk (the standard deviation), as used by the Sharpe ratio. Since the Sortino ratio only considers the downside size and frequency of returns, it measures the return to negative volatility trade-off. According to the Sortino ratio (at the target level set to 3.75 % p.a.) the best strategy is an index comprising 50 % MSCI World Momentum + 50 % STOXX Global Select Dividend 100 Indices.
- Empirical results show that is possible to participate in the long-term outperformance of investment

strategy equity indices at considerably lower levels of short-term risk compared to investment in a single strategy equity index only. The outperformance path of a diversified mix of dividend and momentum strategy is much more stable than the performance of the individual factors.

### 5 Results

Strategy indices are designed on the basis of quantitative models and rule-based investment schemes to provide a single value for the aggregate performance of a number of companies. In contrast to broad-based market and sector indices that group companies by size or industry, strategy indices reflect the performance of a rule-based investment strategy.

For deeper risk analysis of strategy equity indices, we used a robust statistical approach and a computer intensive method - a bootstrap method. Using downside risk measurement is revealing as it reveals the “true” risk of investing in equity markets. The bootstrap method with downside risk metric can evaluate risk in a more appropriate way, and it is also more suitable if statistical characteristics do not conform to a normal distribution assumption (mostly because of fat tails or outliers). In general, the Sortino ratio is much more useful than the Sharpe ratio, because the Sortino ratio uses a below-target semivariance that only penalizes those returns which fall below a specified target (positive outliers should be regarded as a bonus and not as a risk), while the Sharpe ratio penalizes both upside and downside volatility equally. Thus, it is a measure of risk-adjusted returns that treats risk more realistically than the Sharpe ratio.

**Table 4** Bootstrapped equity indices' annualized characteristics

|                                     | MSCI EValue | MSCI HDY | M S C I<br>Momentum | MSCI MV | MSCI World | STOXX 100 | Mix   |
|-------------------------------------|-------------|----------|---------------------|---------|------------|-----------|-------|
| Return                              | 7.95        | 5.2      | 7.5                 | 5.8     | 4.75       | 9.3       | 8.90  |
| Median                              | 8.65        | 6.1      | 7.8                 | 6.3     | 5.7        | 10.4      | 9.4   |
| Standard Deviation                  | 19.65       | 15.8     | 18.45               | 13.1    | 16.65      | 16.9      | 15.95 |
| Below Target Semivariance (T=3.75%) | 12.25       | 11.2     | 11.25               | 8.6     | 12.05      | 10.1      | 9.3   |
| Min                                 | -31.9       | -28.15   | -28.6               | -20.8   | -30.1      | -27.9     | -24.4 |
| Prc25%                              | -5.1        | -5.3     | -4.65               | -2.8    | -6.3       | -1.1      | -1.5  |
| Prc75%                              | 21.3        | 15.85    | 19.45               | 14.6    | 16.1       | 20.4      | 19.4  |
| Max                                 | 42.55       | 31.65    | 43.1                | 28.85   | 32.8       | 37.3      | 36.9  |
| Sharpe (3.75%)+                     | 0.214       | 0.092    | 0.203               | 0.156   | 0.060      | 0.328     | 0.323 |
| Sortino (3.75%)+                    | 0.343       | 0.129    | 0.333               | 0.238   | 0.083      | 0.550     | 0.554 |

As a risk-free rate, the average value of the benchmark iBoxx € Eurozone 3-5 years Government Bond index was used, the average rate was 3.75 % over last 21 years. (Source: ECB)

Source: authors' own calculation

The downside risk indicator will be particularly useful for comparing the risk levels of individual asset classes for the loss averse investor. This downside risk metric also yields markedly different results from the metric commonly used to express the risk level, i.e. standard deviation.

To quantify downside risk, it is recommended to use computationally-intensive methods e.g. bootstrap method, which can also be used to determine the Sortino ratio.

The methodology mentioned herein of estimating yields and risk in particular, based on robust statistics and the

bootstrap method, may be very useful for stock indices which use different weighting mechanisms.

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