Estimation of the Behavioral Equilibrium Real Exchange Rate of the Czech Koruna

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Abstract

Purpose of the article The paper examines the behavior of the real exchange rate in the Czech Republic. It focuses on the analysis of its driving forces with the emphasis on the turbulences which have been lately seen in the financial and real sector of the economy.

Methodology/methods Real equilibrium exchange rate can be estimated using various approaches ranging from purely statistical to fully structural models. In this paper it is estimated using the BEER methodology, i.e. behavioral equilibrium exchange rate. The BEER approach as applied here rests on building vector error correction models which relate the behavior of the actual real exchange rate to various economic fundamentals from both the real and financial sector of the economy.

Scientific aim The estimated behavioral equilibrium exchange rate serves as a benchmark to which the actual behavior of real exchange rate is compared. The paper also points to various problems that are faced when estimating the real equilibrium exchange rate in a posttransitive economy.

Findings Three variants of the model, which differ in the respective fundamental variables included in the estimation, are estimated in the paper. The gap between the estimated real equilibrium exchange rate and real exchange rate as well as the key determinants of the real equilibrium exchange rate are analyzed and compared. The models show that the misalignment between the real exchange rate and fundamentals have narrowed in the recession and post recession period. The key drivers of the real equilibrium exchange rate are the productivity differential, real interest rate differential and net foreign assets.

Conclusions (limits, implications etc) The relatively short time series for the Czech economy, especially for some of the variables, do not enable to make reliable estimation of models which would include all of the variables discussed in this paper.

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JEL Classification: E44, F31, F37
Introduction

The world economy has been going through severe turbulences over the last few years and so have the post-transitive economies of the Middle and Eastern Europe. The behavior of real exchange rate can bear some important information on the state of the economy; in the current context it is especially the information whether or not it has gone off its rails.

There is a vast amount of both purely theoretical and empirical literature focused on the analysis and estimation of some equilibrium value of exchange rate. Rogoff (1996) and Taylor and Taylor (2004) present an overview of the behavior of exchange rate especially with respect to purchasing power parity condition where they also hint at the Balassa-Samuelson effect which is put to use in some of the models for the estimation of real equilibrium exchange rate as it is shown further on. Dornbusch (1989) and Sarno (2002), among others, paint a broader picture of theoretical approaches taken to exchange rate modelling, the latter covering dynamic intertemporal models. However, the approaches presented in those papers deal traditionally with the modelling and analysis of nominal exchange rates.

The analysis of real exchange rate and, especially, the estimation of real equilibrium exchange rate have gained popularity since the 90’s. Driver and Westaway (2004) give a full taxanomy of the various real equilibrium exchange rate models. The fact is that the most frequent modelling strategies include behavioral equilibrium exchange rate (BEER), fundamental equilibrium exchange rate (FEER), Smidkova (1998) presents one of the earliest estimates of real equilibrium exchange rates in a post-transitive economy using this modelling strategy, and natural real exchange rate (NATREX). Egert and Halpern (2005) give an exhaustive account of the respective modelling strategies and discuss their implementation in detail.

This paper exploits the BEER approach to modelling the real equilibrium exchange rate. This approach has one clear setback which is that it is not a structural model which would be explicitely built on derived relationships among the included variables, let alone microfundations as do the modern intertemporal models. The NATREX approach, see Stein (2001) and Bello (2007) for the special treatment of the „rest of the world“ part of the model, is the one with clear structural features, however, it is often estimated in reduced form using vector autoregression methods which wipes out almost all the positive features of the structural approach. Detken (2001) comes up with a mixture of structural and VAR approaches. On the other hand, the BEER approach has one advantage over the other two modelling strategies: it is the least sensitive to shorter time series which is a fact that must be taken into account considering the economy which will be used in the analysis. Komárk and Melecký (2005) present a BEER estimate of the equilibrium exchange rate for the Czech economy.

The estimation of real equilibrium exchange rate serves two basic purposes regarding the case of post-transitive economies and current state of the world economy. First, it can detect possible fundamental misalignments within the economy and second it can be used to analyse the process of nominal convergence.

The paper is divided into five parts. The first part presents some basic facts on nominal convergence in the Czech Republic. The second part gives the standard derivation of the BEER approach and presents some examples of its usage. The third part describes the data used for the estimation and the fourth part presents both the estimates of the behavioral exchange rate and also their analysis. The final part concludes.

1 Some stylized facts

The Czech Republic entered the EU in 2004 and ranks among the most prosperous countries in the region of the Middle and Eastern Europe. This is also reflected by real convergence of the economy. In 2001 GDP per capita in Purchasing Power Standards (EU-27 = 100) reached 70.2 while in 2008 it amounted to 80.4.

Figure 1 presents quarterly changes in real exchange rate (RER) of the Czech Koruna to euro (amount of domestic currency per unit of euro) together with the contributions of nominal exchange rate, domestic price level and foreign (eurozone) price level.

The average quarterly change from 2001 to the fourth quarter of 2009 in RER is negative: - 0.87 %, that is, it appreciated on average. The key contribution comes from the change in nominal exchange rate: - 0.84 per-
percentage point. The contribution of domestic price level is also negative while the contribution of eurozone price level is positive. One can readily observe that the contributions of price levels almost cancel themselves out. There was a strong real appreciation between the third quarter of 2007 and the third quarter of 2008 followed by a sharp real depreciation in the last quarter of 2008 and the first quarter of 2009. The strong appreciation was boosted by nominal appreciation and rapidly increasing domestic price level while the sharp real depreciation was driven by the behavior of nominal exchange rate.

2 The BEER

Following MacDonald (1997), the BEER approach may be derived as follows. The model built on the uncovered interest rate parity condition, which states that the return on domestic assets must equal the expected return on foreign assets measured in the units of domestic currency:

$$1 + IR_t^D = E_t \left[ \frac{S_{t+1}}{S_t} \left( 1 + IR_t^F \right) \right],$$

(1)

where IR denotes interest rate, S denotes spot exchange rate, E denotes the expectation operator and superscripts D and F refer to domestic and foreign, respectively. Taking logarithms of both sides of (1), the condition may be expressed in more familiar terms:

$$E_t (S_{t+1} - S_t) = IR_t^D - IR_t^F,$$

(2)

which means that the expected change in the spot exchange rate equals the interest rate differential. Subtracting inflation differential from both sides of (2):

$$E_t (S_{t+1} - S_t) - E_t (\pi_{t+1}^D - \pi_t^D) = IR_t^D - IR_t^F - E_t (\pi_{t+1}^D - \pi_t^F),$$

(3)

where $\pi$ refers to inflation rate, one obtains:

$$RER_t = E_t (RER_{t+1}) - (R_t^D - R_t^F),$$

(4)

where RER denotes real exchange rate and R refers to real interest rate.

Within the BEER approach the expectation of real exchange rate is considered to be the real equilibrium exchange rate which depends upon the fundamentals. Thus it can be
concluded that real exchange rate depends on real equilibrium exchange rate (REER) and real interest rate differential:

$$RER_t = REER_t - \left( R_t^D - R_t^F \right).$$ \hspace{1cm} (5)

The key question is: which fundamentals should be considered as determinants of the real equilibrium exchange rate? There is no single way to approach this task and the actual models differ in this respect.

Clark (2000) considers just two determinants of the real equilibrium exchange rate: the relative price of traded and nontraded goods which should, according to Balassa-Samuelson hypothesis, be an important factor especially in the cases of converging economies and net foreign assets which are considered as a general measure of the economy’s risk – higher net foreign debt means that investors require higher returns which can be achieved, given the interest rate differential, through depreciation.

However, the role of net foreign assets is ambiguous. First, as Egert and Halpern (2005) point out the effect may be quite the opposite in transitive economies. The reason is that, especially transitive economies, which suffer from low stock of domestic savings, may have a negative targeted stock of net foreign assets. While they converge toward that level, the additional inflows of foreign direct investment, which typically finances the negative foreign assets, tend to appreciate the real exchange rate. When the stock gets beyond the optimal (negative) level of net foreign assets, additional current account deficits will depreciate the real exchange rate. Second, Rogoff (1996) states that the supposed correlation between net foreign assets and real exchange rate tends to be detected at the lags between 5 – 10 years. However, the time series for the transitive economies may be too short to reveal such relationship when the method used does not estimate time-varying coefficients. As a result, an appreciating affect of larger negative stock of net foreign assets should not come as a surprise.

According to Balassa-Samuelson hypothesis the increase in domestic productivity should appreciate the real exchange rate as well as the rise of net foreign assets. The increase in real interest rate differential should depreciate the real exchange rate.

MacDonald (1997) uses a broader set of determinants of the real equilibrium exchange rate. Beside the factors just mentioned, many other determinants are included. He introduces the role of government spending in the sense that a larger share of government spending in total spending may induce faster increase in nontradable prices as government spending is often channelled into this sector. This may amplify the role of private sector in a growing economy when the income elasticity of demand for nontradable goods exceeds one.

Besides net foreign assets he also includes other measures of balances, namely: fiscal balance and personal savings. Higher fiscal surplus and personal savings should appreciate the real exchange rate.

He also adds an important determinant of terms of trade for small open economies, which is the real price of oil. Considering an importing economy, an increase in real price of oil should depreciate the real exchange rate.

Maeso et al (2001) consider a broader interpretation of government spending as they realize that in longer term higher government spending may be accompanied by various distorting effects pushing down the economic growth and leading to depreciation.

In this paper I consider a basic model which consists of only productivity, net foreign assets and the real interest rate differential. This model is further extended by including the effect of real price of oil, government spending and government debt.

The estimation procedure rests on using vector error correction models. Short introduction is presented below.

The principle of cointegration lies in the fact that a stationary linear combination of nonstationary series may exist, which is perceived as a long-run equilibrium towards which the dependent vector of series tends to return. A necessary condition for using cointegration technique is that the analyzed series have to be integrated of the same order. It is therefore necessary to test for the unit root in the levels of the series and check whether the same order of differencing stationarizes them.

Several tests to check for unit root behavior have been developed. It is important to note that testing the unit root behavior rests even in the simplest approaches on several choices: it has to be decided whether a constant
or linear trend will be included or both and the lag of the regressor must be chosen. Also, the problem of low power of unit root tests is reported in the “exchange rate” literature, for example in Rogoff (1996). In this paper augmented Dickey Fuller (ADF) test is used. The following regression is run to check for unit root behavior:

\[ \Delta y_t = \alpha + \beta_1 \Delta y_{t-1} + \cdots + \beta_p \Delta y_{t-p} + \epsilon_t, \quad (6) \]

where \( y \) is the examined series, \( x \) represents the exogenous factors (constant, linear trend or nothing).

The null hypothesis of unit root and the alternative one are stated as follows:

\[ H_0 : \alpha = 0 \]
\[ H_1 : \alpha < 0 \]

If the series meet the condition of being integrated of the same order, the cointegration test may be run to check for long-run relationships among the series. In this paper Johansen approach toward cointegration analysis is used.

First VAR model is set up for each problem:

\[ y_t = A_1 y_{t-1} + \cdots + A_p y_{t-p} + c + \varepsilon_t, \quad (7) \]

where \( y \) is the vector of endogenous variables, \( A \) is a matrix of coefficients, \( c \) is a vector of constants and \( \varepsilon \) is a vector of innovations.

An important task is how to set the lag order of VAR model. A number of information criteria may be used, however, as noted in Lutkepohl (2007) when the out-of-sample forecasting is not the prime goal of the model, a direct attention should be turned to the behavior of residuals. The lag should be chosen so that residuals do not display serial correlation or remaining heteroskedasticity at an acceptable statistical level of significance and follow normal distribution. This is a common practice when using cointegration in economic papers focused on exchange rates, for example Hwang (2001) or Bjorland (2006) and will be adopted in this paper as well. The lag chosen amounts up to 13 in case of monthly data in these papers.

Two widespread tests are used: portmanteau autocorrelation test which is based on Ljung-Box Q-statistic and Breusch-Godfrey (LM) test. When the number of lags at which the residuals are tested is relatively small, Breusch-Godfrey test is preferrable, see Lutkepohl (2007).

After a VAR model is set up, the vector of endogenous variables may be checked for presence of cointegration. Two statistic criteria were used to test the presence and number of cointegrating vectors:

\[ LR_{n var} = -T \sum_{i=1}^{k} \log(1 - \lambda_i) \]
\[ LR_{\lambda_{\text{max}}} = -T \log(1 - \lambda_{i_{\text{max}}}) \]

where \( T \) is the number of observations, \( \lambda_i \) is the i-th largest eigenvalue of matrix \( \Pi \) (further below) and \( k \) is the number of endogenous variables.

The first statistic tests the null of \( r \) cointegrating vectors against an alternative of the number of cointegrating vectors being equal the number of endogenous variables. The second statistic tests the null of \( r \) cointegrating vectors against an alternative of \( r+1 \) cointegrating vectors.

With the known number of cointegrating vectors, a vector error correction model is set up:

\[ \Delta y_t = \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \Pi y_{t-1} + c + \varepsilon_t, \quad (8) \]

where

\[ \Gamma_i = -\sum_{j=1}^{p} A_j \]
\[ \Pi = \sum_{i=1}^{p} A_j - I \]

The \( \Pi \) matrix may be divided into two matrices: \( \alpha \) and \( \beta \) so that \( \Pi = \alpha + \beta^\prime \). Each column of matrix \( \beta \) represents one cointegration vector while the coefficients of matrix \( \alpha \) represent the adjustment coefficients (how fast the system moves toward equilibrium with respect to the particular variables).

3 Data

Data were drawn from eurostat database. The variables mentioned above are defined as follows.

The real exchange rate is based on nominal exchange rate expressed as number of units of the Koruna per one unit of foreign currency (euro). The nominal exchange rate is then multiplied by a ratio of foreign (eurozone) and domestic price level.
The real interest rate differential (DIFF) was constructed by deducting the inflation differential based on average inflation rates from nominal interest rate differentials between the Czech economy and eurozone based on bond yields which are used as convergence criterion.

The Balassa-Samuelson effect (PROD) was expressed as the ratio of the price level measured by CPI to the price level measured by PPI and this ratio was further related to the corresponding ratio for eurozone. This approach was applied in most of the papers mentioned above.

The real price of oil (OIL) was obtained by converting the price of Brent oil measured in euro into the Koruna and deflating it by PPI.

Data on current account deficits and nominal GDP were retrieved from the database. Net foreign assets are computed cumulating current account deficits. The share of net foreign assets in nominal GDP was computed and the ratio of the share of the Czech economy to the share of eurozone was calculated. This serves as a measure of net foreign assets (NFA). For the upcoming part of the paper it should be noted that net foreign assets are negative throughout the sample for the Czech economy and eurozone. This means that increase in the ratio represents a relatively higher foreign indebtedness of the Czech economy.

The same approach was taken in the case of the variable of government spending (GOV). The share of government spending in nominal GDP was computed and a ratio of the share for the Czech economy to the one of eurozone was expressed.

The shares of total debt in GDP (DEBT) were directly retrieved from the database. Ratio of the Czech economy and eurozone shares was computed.

**Table 1** Co-integrating vectors (*,**,*** denotes the rejection of the null on 10%, 5% and 1% level of significance, respectively). Standard errors are given in parentheses.

<table>
<thead>
<tr>
<th>Lag of VAR</th>
<th>4</th>
<th>4</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cointegrating vector</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trace statistics</td>
<td>37,49812***</td>
<td>66,13929***</td>
<td>53,7342***</td>
</tr>
<tr>
<td>Max-Eigen statistics</td>
<td>20,53580*</td>
<td>27,80841*</td>
<td>55,61857***</td>
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<tr>
<td>RER</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>PROD</td>
<td>-1,736359</td>
<td>-1,968522</td>
<td>-1,803279</td>
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<tr>
<td>(0,173994)</td>
<td>(0,09967)</td>
<td>(0,11254)</td>
<td></td>
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<tr>
<td>OIL</td>
<td>-0,018631</td>
<td>-0,037169</td>
<td>-0,042675</td>
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<tr>
<td>(0,00347)</td>
<td>(0,00404)</td>
<td>(0,00520)</td>
<td></td>
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<tr>
<td>NFA</td>
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<td>0,037871</td>
<td>0,034212</td>
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<tr>
<td>(0,00522)</td>
<td>(0,00521)</td>
<td>(0,00466)</td>
<td></td>
</tr>
<tr>
<td>GOV</td>
<td>0,031926</td>
<td>0,037871</td>
<td>0,034212</td>
</tr>
<tr>
<td>DEBT</td>
<td>-0,775403</td>
<td>-0,704109</td>
<td>-0,559248</td>
</tr>
<tr>
<td>(0,22228)</td>
<td>(0,19502)</td>
<td>(0,13987)</td>
<td></td>
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<tr>
<td><strong>Error correction</strong></td>
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<td></td>
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<tr>
<td>Adjustment coefficient</td>
<td>-0,775403</td>
<td>-0,704109</td>
<td>-0,559248</td>
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<td><strong>Diagnostics</strong></td>
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<td>Adj. R squared</td>
<td>0,578854</td>
<td>0,759066</td>
<td>0,760899</td>
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<td>F-statistics</td>
<td>5,123421</td>
<td>9,101319</td>
<td>7,739053</td>
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<tr>
<td>AIC</td>
<td>-4,87712</td>
<td>-5,414479</td>
<td>-5,406557</td>
</tr>
</tbody>
</table>

Source: Eurostat; Own research
Even though some of the series have a longer history, the beginning of 2000 was set as a starting point for the estimation. Quarterly data were used.

For the use of VAR and VEC modeling, it is important to check the unit root behavior of the series. Except the real interest rate differentials, the series are in logs. The analysis of the unit root behavior is presented in the Appendix.

4 Estimates of REER

I present three estimated models in Table 1. The estimated coefficients of the respective cointegrating vectors are given as well as the key characteristics of the estimated vector error correction models.

In all three cases VARs with 4 lags were chosen so that the residuals passed standard autocorrelation, heteroskedasticity and normality tests. The first model is the base model which includes only productivity and net foreign assets variables in the cointegrating vector. Real interest rate differential is also used but is outside the cointegrating vector. All the estimated coefficients have the expected or possible signs: increase in productivity factor appreciates the real exchange rate, increase in real interest rate differential depreciates the real exchange rate and increase in net foreign assets (increase in relative foreign indebtedness) appreciates the real exchange rate. The second model includes the real price of oil in the cointegrating vector and the estimated coefficient is positive, which, as expected, means that increase in real price of oil depreciates the real exchange rate. The third model includes government spending in the cointegrating vector. The estimated coefficient is negative, which, as outlined above, means that increase in government spending leads to real appreciation.

Next I will focus on the estimated error correction models. The first model (model 1) explains approximately 58% of the variability of real exchange rate and the estimated adjust-

![Figure 2 Estimates of REER](image)

Source: Eurostat; Own research

Figure 2 Estimates of REER
error correction models.

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Next I will focus on the estimated error correction models. The first model (model 1) explains approximately 58% of the variability of real exchange rate and the estimated adjust-
real exchange rate and the half-life of a shock is little over one quarter. However, as one can see from the table the significance of government spending is not high (accepted at 10% level of significance) and the significance of real price of oil is lower than in the case of model 2. The variable of government debt did not enter significantly any of the models.

The estimates of real equilibrium exchange (REER) rates together with real exchange rate (RER) are captured in Figure 2. The estimates run from the first quarter of 2001 up to the fourth quarter of 2009.

The estimates show the real exchange rate was a little overappreciated on average. The extent of overappreciation depends on the model. The estimated equilibrium exchange rate appreciated from 2004 to app. 2008 (second quarter). Most of this period productivity variable contributed to REER appreciation as well as net foreign assets. Net foreign assets dropped in 2008 which contributed to REER depreciation. Also real price of oil contributed to temporary and mild REER depreciation in the second half of 2006 and then to rapid depreciation in 2008 (second half). Real interest rate differential contributed to REER appreciation in 2007 and in the beginning of 2008 (it had appreciating effects in the second half of 2008 too). It can be seen that the period of turbulences in the economy lead real exchange rate toward its equilibrium. This is even more apparent in Figure 3 where the misalignments (differences between RER and REERs) are given.

**Conclusions**

The estimated models of real equilibrium exchange rate show that real exchange rate of the Koruna was more or less overappreciated in the sample, the extent of which depends on the estimated model.

The estimated real equilibrium exchange rate shows appreciating tendency from 2004 to the second quarter of 2008. Main drivers of the real equilibrium appreciation are relative productivity, net foreign assets and real interest rate differential.

While the sharp appreciation of nominal and real exchange rate toward the end of 2007 pushed real exchange rate further from the equilibrium, the following adjustments of nominal and real exchange rate narrowed the gap between the estimated equilibrium given by the fundamentals and actual real exchange rate. All in all the turbulences in the financial markets (and real economy) brought real exchange rate closer to the equilibrium.

**References**


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Appendix

<table>
<thead>
<tr>
<th>Variable</th>
<th>t-statistic</th>
<th>Variable</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>real exchange rates</td>
<td></td>
<td>real exchange rates</td>
<td></td>
</tr>
<tr>
<td>BEER</td>
<td>-1.96625**</td>
<td>BEER</td>
<td>-2.29332**</td>
</tr>
<tr>
<td>interest rate differential</td>
<td>-1.840.21**</td>
<td>interest rate differential</td>
<td>-2.654.21**</td>
</tr>
<tr>
<td>PROD</td>
<td>-0.468.688</td>
<td>PROD</td>
<td>-4.387.48**</td>
</tr>
<tr>
<td>oil</td>
<td>-1.729.133</td>
<td>oil</td>
<td>-5.436.50**</td>
</tr>
<tr>
<td>net foreign asset</td>
<td>-1.804.50**</td>
<td>net foreign asset</td>
<td>-4.278.50**</td>
</tr>
<tr>
<td>government spending</td>
<td>-2.790.81**</td>
<td>government spending</td>
<td>-6.11.02**</td>
</tr>
<tr>
<td>GOV</td>
<td>-0.840.40**</td>
<td>GOV</td>
<td>-3.829.44**</td>
</tr>
</tbody>
</table>

Unit root tests: left column in levels, right column in first differences (***, ** denotes the rejection of the respective null on 10%, 5% and 1% level of significance, respectively)

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