INTERPRETING CONSTRAINTS IN FINITE CONTROL SET MODEL PREDICTIVE CONTROL

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Abstract: Aim of this paper is to show the possibilities of interpreting the constraints while using finite control set model predictive control. Paper explains the problem of the barrier functions and shows typical example. New possible candidates to barrier function are introduced in this article. All candidates are tested on the problem of the finite control set model predictive control of the PMSM drive control.

Keywords: barrier functions, model predictive control, finite control set, permanent magnet synchronous motor

1 INTRODUCTION

One of the features of the model predictive control is direct work with constraints on the states and inputs. Continuous forms of predictive control work with continuous control value therefore its value can be changed. Also, solvers of the optimization problem often provide built-in dealing with constraints.

On the other hand, in the finite control set this is not possible. Finite set of control vectors is given and cannot be changed during control. One of the possible ways to choose the best control vector is to compute cost function for every given vector. After that values of cost functions are compared and the vector with the lowest one is chosen. This approach offers an opportunity to compute whether limits were met or not. Barrier functions often used also in continuous optimization, which approximate inequality constraints.

In the case of the control of the drive with permanent magnet synchronous motor, finite control set model predictive control is used to find the optimal switching state of the voltage source inverter. [4] [3] Number of possible switching states is based on its architecture. Keeping current in the limits of its rated value is one of the most important constraints put on the control law.

2 ANALYSIS

Topology of the control problem contains of tje finite control set model predictive controller, voltage source inverter and PMSM. Controller chooses optimal switching states $s(k) \in S$ based on the measurement $x_m(k) = [i_d(k) \quad i_q(k) \quad \omega_m(k) \quad \vartheta_e(k)]^T$. Prediction of the future states is based on the discrete state space model of PMSM in dq reference axis with Clarke and Park transformation included

$$
\begin{aligned}
i_d(k+1) &= \left(1 - T_s\frac{R_s}{L_d}\right) i_d(k) + T_s P_p \frac{L_q}{L_d}\omega_m(k) i_q(k) + \frac{T_s}{L_d}\left(\sqrt{\frac{2}{3}}\cos\vartheta_e(k)\right) u_A(s(k)) + \\
&\quad + \frac{T_s}{L_d}\left(-\frac{1}{\sqrt{6}}\cos\vartheta_e(k) + \frac{1}{2}\sin\vartheta_e(k)\right) u_B(s(k)) + \frac{T_s}{L_d}\left(-\frac{1}{\sqrt{6}}\cos\vartheta_e(k) - \frac{1}{2}\sin\vartheta_e(k)\right) u_C(s(k)) \\
i_q(k+1) &= \left(1 - T_s\frac{R_s}{L_q}\right) i_q(k) - T_s P_p \frac{1}{L_q}\left(L_d i_d(k) - \Psi_{PM}\right)\omega_m(k) + \frac{T_s}{L_q}\left(-\sqrt{\frac{2}{3}}\cos\vartheta_e(k)\right) u_A(s(k)) + \\
&\quad + \frac{T_s}{L_d}\left(\frac{1}{\sqrt{6}}\sin\vartheta_e(k) + \frac{1}{2}\cos\vartheta_e(k)\right) u_B(s(k)) + \frac{T_s}{L_d}\left(\frac{1}{\sqrt{6}}\sin\vartheta_e(k) - \frac{1}{2}\sin\vartheta_e(k)\right) u_C(s(k)) \\
\omega_m(k+1) &= \omega_m(k) + T_s\frac{3}{2}\frac{P_p}{J}\left(\Psi_{PM} i_q(k) + (L_d - L_q)\, i_d(k) i_q(k)\right) \\
\vartheta_e(k+1) &= \vartheta_e(k) + T_s P_p \omega(k+1),
\end{aligned}
\tag{1}
$$

where

i_d, i_q are stator current components in dq frame,	L_d, L_q are rotor inductance components,
ω_m is rotor mechanical angular speed,	P_p is number of pole pairs,
ϑ_e is rotor electrical angle,	Ψ_{PM} us permanent magnet flux,
u_A, u_B, u_C are phase voltages,	J is moment of inertia.
R_s is stator winding resistance,	T_s is sampling period.

Phase voltages are generated by voltage source inverter (VSI). In this case 2-level VSI is used. Thus, there are 8 available combinations which can be used. Based on the length of the prediction horizon N, there are totally 8^N combinations available for the prediction.

3 BARRIER FUNCTIONS

Barrier functions are typically used for the approximation of inequality constraints. [2] They are usually defined for the inequalities of form $x(t) \leq 0$, which can be any other inequality converted to by simple algebra: $x(t) \leq a \rightarrow x(t) - a < 0$.

Mathematical form of the ideal barrier function is

$$
c(x) = \begin{cases} 0 & x \leq 0 \\ \infty & x > 0. \end{cases} \tag{2}
$$

This barrier function brings a few problems in practical realization. First of all, function is piecewise-defined, which brings problem during code execution. On some platforms, dealing with `if` statement means going through both branches and spending computational time there. Another problem can be ∞ in the definition of the function. Therefore, it is necessary to find sufficient approximation of the ideal barrier function.

Requirements for the barrier approximation are to keep cost function low, ideally zero, for the acceptable values and high otherwise. Typical example is logarithmic barrier [1]

$$
c(x) = -\frac{1}{k}\log_{10}(-x(t)), \tag{3}
$$

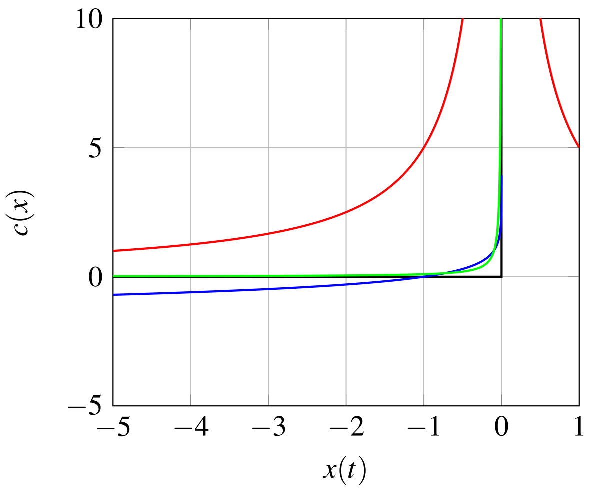


Figure 1: Comparison of used barrier functions; black - ideal, blue - $-\log_{10}(-x(t))$, red - $\frac{5}{|x(t)|}$, green - $100\frac{x}{|x|+0.001} + 100$.

where $k > 0$ is parameter setting the accuracy of approximation. Logarithmic barrier has one disadvantage. In its original form, for the values of $x(t) < -1$ it generates negative value of the cost function, which can lead to wrong decision by the control algorithm.

Similar results are obtainable by usage of the function

$$c(x) = \frac{a}{|x(t)|} \tag{4}$$

as a barrier function. This barrier function eliminates the problem of negative value of cost function, but brings another one. Division by zero can occur and for large exceeding of the limit the cost function starts to descend due to symmetry of the used function.

Problem of the function (4) can be eliminated by using approximation of the signum function

$$c(x) = \frac{a}{2}\frac{x(t)}{|x(t)|+p} + \frac{a}{2}, \tag{5}$$

where p is approximation coefficient and a is maximal value of the function. Limitation of this function is requirement of more computation operations, which can slow down execution of whole algorithm.

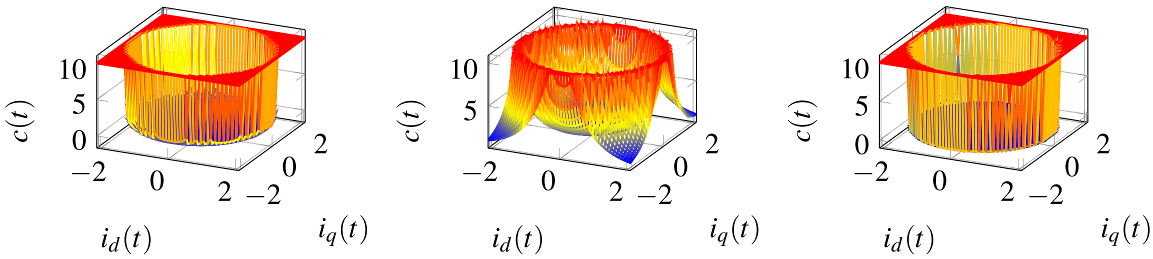
All introduced barrier functions are shown in the Figure 1. For the application of introduced barrier functions in the PMSM drive control it is necessary to state the constraint which is covered by the barrier. As was mentioned before, the most important constraint is the limit of the current. The limit can be in dq reference axis described as

$$i_q^2(t) + i_d^2(t) <= I_R^2, \tag{6}$$

where I_R is rated current. Values of the cost function generated by the introduced barrier functions are shown in the Figure 2. In graphs $I_R = 2$ is assumed.

4 SIMULATION RESULTS

Candidates for the barrier function were tested in PIL simulation using Jetson Nano for the execution of the control algorithm and Simscape model of the voltage source inverter and PMSM. Parameters



(a) logarithmic barrier **(b)** reciprocal of absolute value **(c)** signum approximation

Figure 2: Comparison of cost functions generated by the barrier function candidates; **a)** and **b)** limited to value 10.

Table 1: Parameters of the PMSM

R_s	$0.822\,\Omega$
L_d	$0.016\,\mathrm{H}$
L_q	$0.024\,\mathrm{H}$
Ψ_{PM}	$0.097 \times 10^{-3}\,\mathrm{Wb}$
Pp	5
J	$0.870 \times 10^{-3}\,\mathrm{kg\,m^2}$
I_R	$10\,\mathrm{A}$

of the PMSM are in the Table 1. Other weights in the computation of the cost function were kept the same during all simulations. Ability to behave like a barrier function was tested on the step change of the reference angular speed. All responses are shown in the Figure 3.

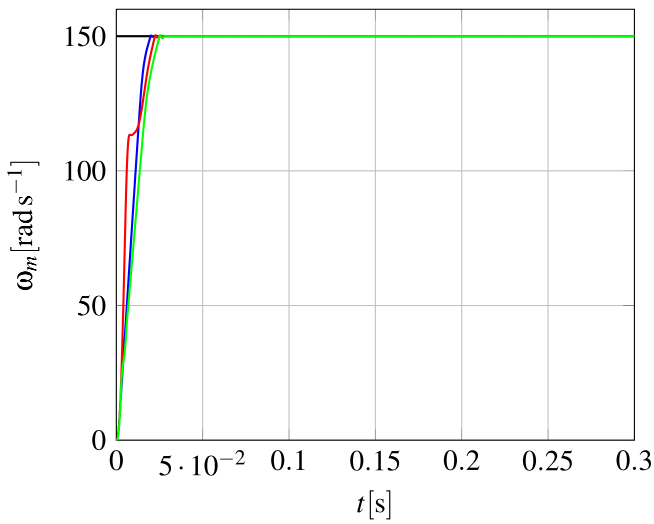


Figure 3: Step responses of the system controlled by controller with different barrier functions; black - reference, blue - logarithmic, red - reciprocal of the absolute value, green - signum approximation.

Behavior of the system controlled by controller with the barrier function given by reciprocal of the absolute value is not standard compared to other two. Reason to that can be seen in the current in dq reference frame in the Figure 4.

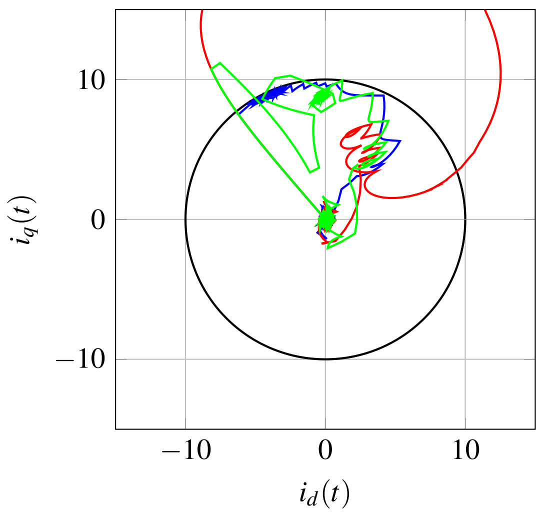


Figure 4: Current in *dq* frame; blue - logarithmic, red - reciprocal of the absolute value, green - signum approximation

Currents in the *dq* frame have shown that reciprocal of the absolute value is not eligible candidate for the barrier function. Controller was not able to keep the current within its limits. On the other hand, controller with the signum approximation was able to return the current back into its limits after a short time. There fore, it can be promising candidate for the barrier function.

5 CONCLUSION

Interpretation of the constraints is one of the main parts in optimization and model predictive control. Aim of this paper was to show the possibility of interpreting them by the barrier functions. Except standard approach of the logarithmic barrier, two more possible candidates were introduced: reciprocal of the absolute value and the approximation of the signum function. All three were tested on the PIL simulation of PMSM drive. Results have shown reciprocal of the absolute value is not able to perform functional blocking. Other candidate have shown promising results and its performance is highly dependent on the parameters of the function.

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