FIBER-COUPLED FABRY-PÉROT INTERFEROMETRIC SENSOR: ANALYSIS AND MODELLING

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Abstract: This paper presents a theoretical analysis of the Fabry-Pérot interferometric sensor utilizing an extrinsic fiber-optic cavity. Its behavior is modelled and simulated for various conditions, showing the key properties of the different sensor’s setups. The simple single-mode sensing tip and the GRIN collimator are compared. Further, new kind of sensing head is proposed exhibiting an inverse reflectivity and enabling a theoretical resolution less than 1 pm.

Keywords: Fiber-optic sensors, Fabry-Pérot, interferometry, displacement sensor, Faraday-rotator.

1 INTRODUCTION

Among various optical sensing principles, the Fabry-Pérot interferometer (FPI) formed by an optical fiber tip and a reflective surface represents an ultimate solution in terms of sensitivity, resolution, and also compactness. Therefore, it has been exploited in many applications. The best-known is the atomic force microscopy [1], where a noise density of several units of fm/√Hz was achieved [2]. Nevertheless, the FPI can be used to sense basically any physical quantity which can be transduced to displacement, refractive index, or wavelength. It is thus being utilized as vibration sensors, accelerometers, microphones, material properties sensors, etc [3]. Also, its manufacturing process may be very easy since the sensitive part consists only of cleaved fiber tip in the simplest case.

On the other hand, the development of such sensor itself is much less straightforward if particular parameters, such as range and sensitivity, are targeted since the actual behavior of the FPI is strongly dependent on material properties and cavity scales. Therefore, it is very difficult to develop and optimize such sensor unless we perform relevant calculations. From this point of view, simulations play a significant role for basic design and optimizing tasks. There has been proposed several analyses and models involving different physical aspects; one of the most comprehensive descriptions provide Wilkinson [4] and Kilic [5].

Compared to the ideal FPI with parallel plane mirrors, the proper characterization of the extrinsic fiber FPI is more complex since the beam divergency and fiber coupling must be taken into account. In this paper, the behavior of fiber FPI is simulated under different conditions, revealing some notable properties of the sensing capabilities. Further, there are also show some modifications of the fiber FPI, simulated by extending the model so their key advantages can be demonstrated.

2 MODELING OF FPI PROPERTIES

Likewise in any FPI, the fiber-optic arrangements is formed by resonance cavity passed by light beam and allowing its reflections with partial coupling outside the cavity (Fig. 1). In such a way, it can act as a periodic wavelength filter whose shape may be changed by the cavity size or other parameters. The filter selectivity increases with the number of reflections inside a cavity and is referred as finesse
of FPI. Each surface has some reflection and transmission; in case of fiber-air boundary they can differ for each direction. They basically determine the portion of light coupled to the cavity or reflected back inside the fiber. With only one passing inside the cavity ($p = 1$), we have two-beams’ interference yielding an output intensity response with a sine shape (low finesse). Conversely, by allowing multiple light passing through the cavity ($p > 1$) we achieve sharpening the response peaks in the output intensity and increasing the sensitivity (Fig. 2). The fiber FPI has typically the reflective response since the zero-order reflection ($p = 0$) interferes with the beams passing the cavity ($p > 0$).

![Figure 1: Fiber-optic FPI cavity.](image1)

![Figure 2: Typical FPI intensity response.](image2)

The reflected intensity $I_{out}$ is a linear function of the input intensity $I_{in}$ so we can express it in terms of reflectivity as $R_{FPI} = I_{out}/I_{in}$. However, since the light is a wave, rather than intensity we assume it in the form electric field $E$, having a magnitude and phase, and lowercase typing of reflection and transmission coefficients, as shown in Fig. 1. The relationship is then $R_{FPI} = |r_{FPI}|^2$ where

$$r_{FPI} = r_{f} + t_{f} \sum_{p=1}^{\infty} r_{m}^{p} e^{-j2kdp},$$

(1)

where $k$ is the wave number. The coefficients $r$ and $t$ are complex in general, reflecting phase shift caused by transmission and reflection. This approximation is valid for the optical fiber assuming much shorter cavity compared to beam mode field diameter (MFD). In case of common single-mode fibers, having MFD of several µm, thus cannot provide sufficient accuracy and more complex approach must be used.

The major problem is the beam divergence causing decreasing the coupling efficiency with every cavity pass, which affects both amplitude and phase of light portion coupled back to the fiber. Further, the model should also incorporate a possible misalignment $\alpha$ between reflective surface plane and fiber cleave plane to determine manufacturing tolerances. Based on the mathematical descriptions in [4, 5], the Matlab model of fiber FPI was created. Compared to [4] where the cavity passing is limited to $p = 5$, here the number of passing beams is only thresholded by carried energy (which can be arbitrarily small) and especially high-finesse FPI behavior can be thus approximated with higher accuracy.

In the following sections, the different structures of fiber FPI sensing head will be shown and compared to the simple single-mode fiber tip. The overview of the structures can be found in Fig. 3. Note that all simulations are performed for air-filled cavity, SMF-28 fiber and wavelength $\lambda = 1550$ nm.

2.1 Single-mode fiber (SMF) cavity

The simplest case of fiber FPI formed by cleaved single-mode fiber (SMF) is shown in Fig. 3a. The single-mode optical beam can be well approximated by Gaussian beam whose divergence angle $\theta$ outside the fiber is inversely proportional to the beam input waist, determined by the MFD. Assuming
Figure 3: The overview of analyzed fiber FPI sensing structures: a) SMF, b) GRIN collimator, c) Focused GRIN collimator, d) Faraday-rotator GRIN collimator.

a standard telecom SMF-28 and wavelength 1550 nm, the angle corresponds to 11°. The interesting fact is that the beam diverges as late as it reaches certain distance behind the fiber, until which is approximately collimated. This distance is referred as Rayleigh range and equals to 55 µm in the studied case. The Fig. 4 compares two modeled responses of SMF FPI: with no coating and a dielectric mirror having $R_m = 0.99$, and dielectric fiber coating $R_f = 0.5$ and a metallic mirror with $R_m = 0.95$, both with perfect fiber alignment $\alpha = 0°$. When using FPI as a distance sensor, we usually exploit a small portion of $R_{\text{FPI}}(d)$ between positive and negative peaks, which has positive or negative slope. Therefore, more than total reflected power we need to care about an efficient utilization of the detector dynamic range, i.e. minimizing the offset and maximizing the signal component, which is expressed by the visibility. In other cases, rather than SNR and measurement range, the peak sensitivity may be of higher importance, i.e. the slope of $R_{\text{FPI}}$ change. The Fig. 4 also shows normalized slopes (scaled on right vertical axis) reflecting the optimal usage of the detector range. The second case exhibits much higher sensitivity due to multiple reflections within the Rayleigh range, which is in accordance with the expectations. Note the positive and negative slopes difference due to Guoy phase shift caused by the beam divergency. Further, the resonant distances are shifted in Fig. 4b with respect to Fig. 4a thanks to metallic mirror absorbing a part of energy (see detailed view).

Figure 4: Modelled response of SMF FPI: a) no fiber coating ($R_f = 0.04$), dielectric mirror ($R_m = 0.99$); b) dielectric fiber coating ($R_f = 0.5$), metallic mirror ($R_m = 0.95$).

2.2 Graded-index (GRIN) Collimator

The drawback of the SMF FPI is decreasing coupling efficiency due to beam divergence and thus impossibility to reach high finesse since the intensity of the beam inside the cavity decreases rapidly with $p$. This can be overcome with a graded-index (GRIN) collimator at the fiber tip, as shown in
Fig. 3b. The gradient index fiber works as a lens which substantially extends the Rayleigh range of the output beam. The comparison of the simple SMF and GRIN collimator FPI response is shown in Fig. 5. The modified model was obtained by simple re-calculation of the Gaussian beam according to GRIN collimator performances.

The output reflectivity is here modeled in dependence on cavity size $d$ and also misalignment angle $\alpha$. Note that the GRIN collimator causes the FPI operates equally within much broader distance range. Conversely, SMF is much less sensitive to angle tilt due to broader beam cone.

In some cases, usage of the GRIN collimator may be limiting due to its bigger spot, which can even exceed 1 mm. This issue can be solved by focused GRIN collimator, shown in Fig. 3c. The simplest implementation may be performed by extending the collimator to form a convergence beam. However, to perform well for $p > 1$, the sensing had needs to have concave shape so the reflected beam did not diverge outside the cavity.

### 2.3 Faraday-rotator sensing head

The fiber FPIs inherently exhibit a reflection response (Fig. 2) which leads to small signal to DC bias contrast (see Fig. 5) if the reflectivities are not balanced. The transmission response does not contain the zero-order reflection and is thus unbiased. In fiber FPI we cannot couple the light from the opposite side of cavity; though, the problem may be overcome by special setup with Faraday polarization rotator (FR), whose structure is in Fig. 3d. Note the FPI cavity is not formed solely by the air gap, but comprises also the GRIN collimator with anti-reflection coating and the fiber portion behind the semitransparent mirror. The nonreciprocal rotation by 45° combined with polarizer at

![Figure 5: FPI reflectivity ($R_f = 0.04$, $R_m = 0.99$): a) simple SMF, b) GRIN collimator.](image)

![Figure 6: FR FPI head with different rotation $\beta$ ($R_f = 0.95$, $R_m = 1$): a) Back-coupled $|E|$ for separate pass order $p$, b) Normalized reflectivity peaks (detail).](image)

![Figure 7: FR FPI reflectivity with $\beta$ ($R_f = 0.5$, $R_m = 0.99$, $h = 10$ mm).](image)
the input eliminates the zero-order reflection. A small rotation of angle $\beta$ also exist in the cavity causing filtration of $p$-passed beam by factor $\sin(2\pi\beta)$. The advantage of this filtration also consists in suppressing the low-$p$ reflections and balancing the intensity of high-$p$ beams coupled back to the guiding fiber and thus increasing the sensor's finesse. Fig. 6a shows the theoretical portions of $|E|$ coupled back to the guiding fiber displaying the suppression of the zero-order reflection and balancing the higher-$p$ ones by different angle $\beta$. Due to eliminating the low-$p$ beams coupling, we can see the resonance peaks are sharper and with better linearity within the full dynamic range (Fig. 6b). The response of FR FPI with different rotation angle $\beta$ is shown in Fig. 7. The distance $d$ is the air cavity size, the sensing head length behind the mirror is assumed $h = 10$ mm.

3 CONCLUSION

Based on the mathematical description of fiber FPI, the Matlab model was created to simulate various behavior of different structures of fiber-coupled cavities. Firstly, it was shown at simple SMF FPI that the sensitivity can be greatly increased by fiber coating, albeit the distance $d$ must be maintained small. With GRIN collimator, we can extend it notably, however, the alignment is much more crucial. Nevertheless, we can reach extreme sensitivity with additional coating in that case or modify the GRIN lens to be used as very precise point sensor. Lastly, a novel solution of sensing head was proposed, using a Faraday rotator to achieve an inverse response, yielding maximizing the FPI performance yet more. With the moderate parameters ($\lambda = 1550$ nm, $R_f = 0.95$, $\beta = 1^\circ$), a relative intensity can reach a peak change of $0.377$ nm$^{-1}$. When 12-bit resolution is available, this yields $0.65$ pm is the smallest distinguishable distance change. Therefore, such kind of sensor has a high potential for the most accurate measurement applications.

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