VERIFICATION OF MODELING APPROACH BY USING MODEL OF CALCULABLE CAPACITOR AND DETERMINATION OF ITS ACCURACY

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Abstract: During the creation of any sort of model the inevitable question about the model correctness and accuracy, should always be answered. This paper proposes a simple method for modelers to prove their approach to modeling and to determine the accuracy of a simulation. The method consists of creating a model of calculable capacitor based on Thompson-Lampard theorem in Ansys Electronics desktop using Maxwell 3D electronics desktop. The model and its parts are described. The expected results are discussed and compared to data obtained from simulation. Problematic of rounding edges and its influence on simulation accuracy and time is also discussed and shown on two models. A part of the paper is a description of how to interpret the Ansys percentage error.

Keywords: Ansys Electronics Desktop, Calculable capacitor, Fringe effect, Maxwell 2D and 3D, Model accuracy, Simulation, Thompson-Lampard Theorem

1 INTRODUCTION

There are several methods to prove the correctness of a simulation, but most of them are based on a fact, that a creator has a great knowledge about the simulated model and the simulation itself and thus, has no doubt about the fundamentals. The method proposed by this paper aims mainly to beginners or intermediate modelers, who are learning Ansys Maxwell 2D or 3D environment with electrostatics solver. The main advantage of this method is the fact, that it does not require deep knowledge about the electrostatics. So even a learning modeler can, through this method, prove his modeling approach.

The proposed method is to find a model, which has precise analytic solution, therefore the modeler knows the results beforehand, which makes evaluation more comprehensive and understandable. This paper shows, how to evaluate simulation and how to interpret Ansys errors values and convergence data.

The main motivation for this paper was, for the sake of future research, to prove author’s own simulations and approaches.

2 CHOOSING THE MODEL

There are not many calculable capacitors. Most of the analytic solutions are only an approximation. For example, the best known equation (equation 1) for parallel plate capacitor, represents only a simplified solution, that does not include the fringing effect. The greater the distance between the plates, the greater the deviation from reality.

\[ C = \varepsilon_0 \cdot \varepsilon_r \cdot \frac{s}{d} \cdot [F] \]  

(1)
There is currently no precise analytic solution for a parallel plate capacitor, so it is not suitable for
the confirmation of the simulation results. There are a few configurations which have precise analytic
solution such as two spheres[1], but such a model is extremely simple and does not test the ability of
a model designer to create a model of intermediate level at least. The solution is to choose a capacitor
with electrode configuration, which has precise analytic solution and which is not overly simplified.
The calculable capacitor based on Thompson Lampard theorem meets those conditions.

![Diagram of capacitor configurations](image)

**Figure 1:** Principle of theoretical cross capacitor and N.S.L version of cross capacitor.

### 3 THOMPSON-LAMPARD THEOREM

In 1956 the Thompson, A. M., and Lampard, D. G. published their theorem based on the cross ca-
pacitance and its usage in calculable standards. [2] The theorem was an attempt to reduce variables
which influenced countable standards. Until then, the countable capacity standards were scarcely
used due to their inconsistency. As mentioned before, the problem was that the former standards
provided accurate results only if almost ideal conditions were met. Even a slight deviation from those
conditions made the results unusable. [3] One of such standard was based on capacitor with parallel
plates, whose problems were described in section 2.

This problem was eliminated with a capacitor based on cross capacitance design. Its design consists
of four rods/electrodes of arbitrary cross section and with infinite length. The configuration can be
seen in figure 1a. If the \( C_1 \) equals \( C_2 \) then the mean of those capacitances \( \bar{C} \) is the same. Lampard
has determined that an arbitrary cross capacitor is described by the following equation 2.[4]

\[
\exp(-4\pi C_1) + \exp(-4\pi C_2) = 1
\]

So if the \( C_1 \) equals \( C_2 \) then:

\[
C_1 = C_2 = \bar{C} = \frac{\ln 2}{4\pi^2} \text{[e.s.u. \cdot cm}^{-1}] = 1.953548 \text{[pF \cdot m}^{-1}] \tag{3}
\]

But for that to be true, two criteria have to be met. Firstly, the electrodes have to be placed in such a
manner, that the neighboring electrodes are almost in contact and create a gap in the middle. Secondly,
the cross section of the gap has to have one axis of symmetry, which connects two opposite spaces
between electrodes. The examples of such cross configurations are shown in figure 1c. Consequence
of such theorem is that the shape of electrodes does not matter as long as the cross section of a gap
is symmetrical. Or that size of cross section, either of rod or gap, does not matter as long as the
insulating space between neighboring rods is considerably small. The most important advantage is
that with this configuration only one dimension, the length, needs to be observed to determine the
capacitance.
4 REAL CALCULABLE CAPACITOR

Thompson discussed the physical realization of a countable capacitor[5], which resulted into a design of a real calculable capacitor. The theoretical capacitor has infinite length, and is therefore unachievable. With finite length, the fringing effect takes place at each end of the electrodes. Thompson came up with a few designs using shielding technics, which solved the fringe issue. [4]

That design was adopted and modified by many institutions. For purposes of this paper, the model from The National Standards Laboratory (N.S.L), which was renamed to National Measurement Laboratory in 1974, was chosen. The simplified model was used as a template. The real calculable capacitor is much more complicated, because unlike the simulation, the rods have to be somehow attached to the construction. The principle and design of this model can be seen in the figure 1b.

The design consists of one hollow cylinder with two holes on its ends for driving rods. The outer cylinder has shielding function. Inside the cylinder, 6 rods are placed. Two of them (A,C) have shielding function. The other two (B,D) serve as positive and negative electrodes. The last two rods (F,G) are the shielding driving electrodes. They serve to determine the effective length used for determining the capacity. For that purpose, the driving rods are movable. The precision of such calculable standard is dependent on an accuracy of the position measurement. So if a precise measurement principle, such us Fabry-Perot interferometer, is used, then the results can have precision as much as 0.1 ppm. [6]

5 SIMULATION

The main model was made according to previous description. The dimensions are unimportant, because of the mentioned fact, that the shape is more crucial than dimensions, but the length of the outer cylinder should be sufficiently larger than the maximal effective length. Gaps could be tighter, but as of now, they are sufficient. The resulting design can be seen in figure 2a. The materials of electrodes and shield were defined as a perfect conductor and the area was defined as a vacuum. The solution settings were as follows: Maximum number of passes 20, percent error 0.05, refinement per pass 30%. Parametric simulation was performed with parameter $l$ in range $<10;200>$ mm with a step of 10 mm and one simulation with $l = 5$ mm, for acquiring detail at short effective length. This parameter represents the effective length. The parameter was also bound to driving electrodes, thus creating a movement in an opposite direction of each other, so the effective area enlarges. The model was labeled as NR model (NR - Not Rounded).

Another model (R model (Rounded)) was made, but all the sharp corners were rounded. This was done because the sharp corners can cause convergence problem.[7] The second simulation was performed to find out if this models would cause mentioned problem. The settings remained the same.

(a) Model of a calculable capacitor in Ansys  
(b) Capacity vs effective length $l$

Figure 2: A model created in Ansys and dependence of capacity on effective length $l$
The driving electrodes, the outer shell and the blue electrodes were excited with 0 V for simulating the ground shielding. The orange electrodes were excited with -10 V and 10 V respectively.

6 RESULTS

In theory, the dependence of capacity on effective length should be linear. In reality, the dependence is exponential at the short lengths, due to the fringe effect. The fringe effect makes the electric field non-uniform around the end of the driving rods, but only to certain distance. Beyond said distance, a uniform electric field is generated. The uniformity ensures the stable increment of capacity. This phenomenon is in accordance with W. K. Clothier’s research [5]. In the figure 2b, the exponential growth at the short lengths can be seen. Beyond 40 cm the growth is linear and is roughly 19.5 fF per cm. In the same figure, it seems like, that the model with rounded edges gives exactly the same results as the model without the rounded edges.

![Progress of increment in dependence on l with deviation.](image1.png)  
(b) Convergence data for l=100

**Figure 3:** Data obtained by simulation.

7 ACCURACY AND CONVERGENCE

The advantage of used models is the known analytical solution, which is precise even in real environment. That fact presents the possibility to determine the accuracy of a model and to help comprehend simulation settings or errors. Both models converged with total error under the 0.05 criteria. To a direct comparison of both models, the figure 3a has to be observed. There, the increment of capacity for both models and absolute deviations $\Delta$ from the theoretical increment, can be seen. The deviations for $l \in (0;40)$ were not counted due to aforementioned phenomenon. The deviation with NR model was mostly larger, hence the NR model had overall inferior accuracy. But even in the worst case, the relative deviation is under 1%. The average deviation is 0.0234 pF for R model and 0.0362 pF for NR model. The 1% simulation accuracy could indicate that the 0.05 % error criteria was not met. However those numbers are of different meaning.

The Ansys software uses two error criteria to evaluate convergence. The energy error and the delta error. The first is calculated by sophisticated algorithm and means, that the solution is in the x% vicinity of the simulated value. The former is the difference between last two energy error values in percentage. Be aware, that the simulation can converge into false convergence due to low quality mesh or wrong model. The convergence does not mean that the simulation reached real value.

The percentage error criteria means, that the solver aims to get both of previously mentioned values under that criteria value. The 0.05 % error value may seem too strict, but we have to take in notion,
that with increasing capacity the error increases accordingly. Also the increment is calculated from two values so in a worse scenario, the deviation could be as much as: $\Delta = ((1.0005 \cdot 361.76) - (0.9995 \cdot 342.25)) - 19.53 = 0.33\,pF$ which is almost 1.7%.

Graph concerning convergence, is presented in figure 3b. There we can observe, that the R model converged faster and with fewer tetrahedrons (the mesh is less dense). Simultaneously, it was found out, that one parametric solution for R model was 30% faster. The slightly rounded edges caused finer mesh at details (edges), so the refinement of the mesh was focused around those details. On the other hand, the mesh of NR model was refined mostly in the whole volume, which caused the need of finer mesh. In this case, the R and NR model are almost equal. The NR model could perform better, by slightly changing the simulation settings. But we could run into the risk of converging into a false solution. Rounding corners makes converging into right solution more certain, but for models with more corners it could also mean an exponential increase in the number of tetrahedrons.

8 CONCLUSION

The calculable capacitor based on Thompson-Lampard theorem was described and model of its N.S.L version was created for simulation. Two simulations were performed. Results show that the model with rounded edges converges faster and provides more precise results. The model with sharp edges is faster to create, but convergence is slower and needs more tetrahedrons (as seen in figure 3b), so more memory is needed. Model designer should be cautious when using sharp edges, because there could be a chance of a false convergence. The verification of a correct convergence was possible due to knowledge of analytic results beforehand. Without that knowledge it would be necessary to make more simulations with different settings. However, the results from both models were approximately the same. The increment deviates only slightly. The knowledge obtained during the work on this paper will shall lay a great foundations for future research about planar capacitive sensing. Simultaneously, reader can make use of this paper as a guide for proofing the reader’s modeling approaches by making the same simulation.

ACKNOWLEDGEMENT

The completion of this paper was made possible by the grant No. FEKT-S-20-6205 - “Research in Automation, Cybernetics and Artificial Intelligence within Industry 4.0” financially supported by the Internal science fund of Brno University of Technology.

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