COMPARISON OF MEASUREMENT METHODS FOR CHARACTERIZATION OF 3D PRINTED LOW-LOSS ARTIFICIAL DIELECTRIC SUBSTRATE BASED ON CROSS UNIT CELL

Petr Kaděra
Doctoral Degree Programme (3), FEEC BUT
E-mail: kadera@ftec.vutbr.cz

Supervised by: Jaroslav Láčík
E-mail: lacik@ftec.vutbr.cz

Abstract: This paper deals with the effective complex permittivity determination of low-loss 3D printed artificial dielectric substrate based on a cross unit cell. The two resonators working with TE$_{011}$ and TM$_{010}$ modes suitable for a uniaxial anisotropy determination are compared with the transmission / reflection waveguide method. To evaluate the methods’ performance, the eigenmode analysis is carried out to provide precise reference values.

Keywords: Complex permittivity, artificial dielectric substrate, 3D printing, dielectric characterization, uniaxial anisotropy, resonators, waveguide

1 INTRODUCTION

A precise determination of an effective complex permittivity of low-loss 3D printed dielectric substrates plays an important role in the antenna and microwave circuit design [1]-[2]. For its determination, two measurement methods [3], [4] can be used. The first one is based on the exploitation two resonators [4] with known electric and magnetic field distributions allowing to calculate the material properties from change of the resonant frequency and cavity quality factor due to inserted sample of the material in the cavity. The second measurement method is based on the measurement of the transmission and reflection coefficient of a sample located in a waveguide and exploiting the non-iterative Nicolson-Ross-Weir (NRW) algorithm [5]. Both these measurement methods are able to measure anisotropy of a measured sample which is a typical feature of 3D printed structures.

In this paper, both the measurement methods are compared for the characterization of a 3D printed artificial low loss dielectric substrate based on a cross unit cell which is one of the basic elements exploited in the field of 3D printing [3]. For the evaluation of the suitability of the methods, the data of the eigenmode analysis of the cross unit cell is used [6]-[7]. Note that in this phase of our study, the data for the comparison was not obtained by the measurement of samples in the laboratory, but it was obtained by the modeling of the measurement methods in CST Studio Suite.

2 EFFECTIVE COMPLEX PERMITTIVITY DETERMINATION

2.1 EIGENMODE ANALYSIS METHOD

The method of eigenmode analysis is based on the modal analysis of an enclosed structure (defined by boundary conditions) when the electric and magnetic field distribution of each mode (representing a standing wave) are calculated. As a result, the eigenfrequencies and eigenvectors are determined and can be further used for calculation of the effective relative permittivity and tangent loss according to (1)-(3) [6]-[7].
\[
\varepsilon_{\text{eff}} = \frac{\beta^2 - \left(\frac{c}{f}\right)^2}{4\pi^2},
\]

(1)

\[
\tan\delta = \frac{P_D}{\pi \cdot f + 2 \cdot W},
\]

(2)

\[
W = \frac{1}{2} \varepsilon_0 \varepsilon_{\text{eff}} \int \left| \vec{E} \right|^2 \partial V + \frac{1}{2} \mu_0 \mu_i \int \left| \vec{H} \right|^2 \partial V,
\]

(3)

where \(\varepsilon_{\text{eff}}\) is the effective relative permittivity, \(\varepsilon_0\) is the permittivity of vacuum, \(\mu_i\) is the relative permeability, \(\mu_0\) is the permeability of vacuum, \(\beta\) is the propagation constant, \(c\) is the speed of light, \(f\) represents the eigenfrequencies, \(\tan\delta\) is the tangent loss, \(P_D\) denotes the dielectric power loss, \(W\) represents the total energy stored in the calculated structure, \(\vec{E}, \vec{H}\) are eigenvectors corresponding to the electric and magnetic field, respectively and \(V\) is the volume of calculation domain.

The eigenmode analysis is applied in CST Studio Suite with the eigenmode JD (Jacobi-Davidson Method) solver to determine the exact values of the effective relative permittivity and tangent loss of the investigated loss-less dielectric substrate. The dielectric substrate composition and boundary conditions settings are shown in Fig. 1. To evaluate this method, we assume the host permittivity \(\varepsilon_h\) of 1 (air) with the \(\tan\delta_h\) of 0 and the inclusion permittivity \(\varepsilon_i\) of 2.5 with the \(\tan\delta_i\) of 7.5.10^{-3} at a frequency of 7 GHz, corresponding to the 3D printing filament (Prusament PLA Jet Black).

**Fig. 1:** The cross unit cell based dielectric substrate (a) and boundary conditions settings for the eigenmode analysis (b).

### 2.2 TE_{011} AND TM_{010} RESONATOR METHOD

Two resonant cavities with diameters of 56 mm and 32 mm and heights of 55 mm and 12 mm are exploited to find the effective complex permittivity of a cross unit cell dielectric substrate in the longitudinal (TE_{011} mode) and the transversal (TM_{010} mode) polarization direction, respectively [4]. The material of the cavity is gold with the electric conductivity \(\sigma\) of 5.7.10^{7} S/m. The calculation is processed for a dielectric substrate of thickness 7.2 mm and the inclusion volume fraction (defined as the ratio of the volume of the inclusion material to the volume of the whole cross unit cell) between 0.2 and 1. The sample is located in the middle and at the bottom of TE_{011} and TM_{010} resonant cavities, respectively (Fig. 2). The feeding coupling loops are not depicted in this figure. The resonant frequency of the TE_{011} mode in the empty cavity is 7.06637 GHz with the quality factor of 35263. Similarly, the resonant frequency of the TM_{010} mode in the empty cavity is 7.17177 GHz with the quality factor of 8780.
Fig. 2: The electric field distribution of TE\textsubscript{011} resonator mode (a), electric field distribution of TM\textsubscript{010} resonator mode (b). The cross unit cell based dielectric substrate with inclusion volume fraction 0.2 is located inside the resonators.

2.3 TRANSMISSION / REFLECTION NRW WAVEGUIDE METHOD

The WR137 waveguide with inner dimensions of 34.8488 mm x 15.7988 mm and propagation mode TE\textsubscript{10} is assumed. The NRW algorithm with input transmission and reflection coefficients is utilized for the 3.6 mm thick sample. In the default sample arrangement (Fig. 3a), the extracted values of the effective complex permittivity correspond to the longitudinal direction. To obtain the values in the transversal direction, the elementary dielectric substrate unit cells have to be rotated by 90-degree with respect to the x axis. The waveguide model with inserted sample and electric field distribution at a frequency of 7 GHz is shown in Fig. 3.

Fig. 3: The WR137 waveguide with cross unit cell based dielectric substrate with inclusion volume fraction 0.2 (cut in the middle of structure) (a), electric field distribution of TE\textsubscript{10} mode at a frequency of 7 GHz (b).

3 RESULTS AND DISCUSSION

The achieved effective complex permittivity of the artificial dielectric substrate based on the cross unit cell in the longitudinal and transversal direction are presented in Fig. 4. The relative error (4-5) of the measurement methods (waveguide and resonators) related to the eigenmode analysis in Fig. 5 is calculated as follows:

\[ \Delta_{\varepsilon_{\text{eff}}} = \left| \frac{\varepsilon_{\text{eff, waveguide/resonator}} - \varepsilon_{\text{eff, eigenmode}}}{\varepsilon_{\text{eff, eigenmode}}} \right| \times 100, \]  

(4)

\[ \Delta_{\tan\delta} = \left| \frac{\tan\delta_{\text{waveguide/resonator}} - \tan\delta_{\text{eigenmode}}}{\tan\delta_{\text{eigenmode}}} \right| \times 100, \]  

(5)

where \( \Delta_{\varepsilon_{\text{eff}}} \) denotes the relative error of the effective relative permittivity and \( \Delta_{\tan\delta} \) represent the relative error of tangent loss.

By comparison of the achieved results, the suitability of the measurement methods to accurately determine the effective complex permittivity of the artificial dielectric substrate based on the cross
unit cell can be confirmed. The relative error of the effective relative permittivity is under 2.2% for both NRW waveguide and resonator methods, respectively, in both longitudinal and transversal directions for a PLA 3D printing material. The relative error of the tangent loss lies below 3.1% and 7.1% for NRW waveguide and resonator methods, respectively, in the longitudinal direction, and below 16% and 5.2% for NRW waveguide and resonator methods, respectively in the transversal direction.

Our study shows that both the NRW waveguide and resonator measurement methods are precise enough to determine the effective complex permittivity of the selected structure. However, in practice for low-loss materials, the resonator method could be more convenient because of its lower sensitivity to parasitic air gaps between sample and resonator’s walls or sample defects compared to the transmission/reflection waveguide method. Such advantage of resonator method is pointed out in the recent study [8], where a multimode resonator cavity based on TM_{01n} modes was employed with the tangent loss sensitivity in order of 10^{-4}-10^{-5}.

![Graphs](https://via.placeholder.com/150)

**Fig. 4:** The effective relative permittivity (a, b) and tangent loss (c, d) of the artificial dielectric substrate in longitudinal (a, c) and transversal (b, d) direction determined by the eigenmode analysis, NRW waveguide and resonator methods.

### 4 CONCLUSION

In this paper, we presented a comparison of the transmission/reflection waveguide method based on Nicolson-Ross-Weis algorithm and two resonator method based on TE_{011} and TM_{010} modes enabling to effectively describe effective complex permittivity of low-loss 3D printed artificial dielectric substrate based on the cross unit cell in the longitudinal and transversal directions. Both methods showed good ability in the anisotropy characterization. To evaluate the accuracy of the methods, the data of the eigenmode analysis of the cross unit cell was used. Future work will focus on resonators fabrication and methods’ verification on various artificial dielectric substrate samples.
Fig. 5: The relative error of effective relative permittivity and tangent loss of the artificial dielectric substrate in longitudinal (a) and transversal (b) direction.

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REFERENCES


