Comparison of two loss minimization algorithms for induction motors

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Abstract—This paper discusses two loss minimization algorithms for induction motor drives. Firstly, equivalent models in the form of Gamma and inverse Gamma are shown, from which the loss models were obtained. Then, the procedure for determining the two algorithms is presented. The results show that it is easier to determine the loss minimization algorithm for rotor flux. In terms of accuracy, both algorithms are similarly accurate.

Keywords—calculation comparison, induction motor drives, loss minimization algorithm, power losses

1. INTRODUCTION

The largest consumers of electricity are electric induction motor drives. These motors are simple in design, robust, reliable, and have high efficiency, but this can drop if the speed and torque are varied over a wide range.

The efficiency of an induction motor can be increased by reducing losses through the appropriate linkage flux. One of the commonly used strategies is model-based method determining the optimal linkage flux. [1].

The aim of this paper is to present the procedure of determining the algorithm of optimal linkage flux from the motor parameters for stator and rotor fiel-oriented control. The calculations are different for both types of control and this paper shows a description of the differences and an evaluation of the results.

2. INDUCTION MACHINE MODEL IN STEADY STATE

For determining the optimal linkage flux algorithm, it is useful to add a resistor represent the iron resistance to the model. This resistor is placed parallel to the magnetizing inductance on the stator side. Detailed development of the model can be found in [2].

The models most commonly used are in the form of a gamma network (Γ-network) and an inverse gamma network (Γ-network). In general, the Γ-network is used for stator field-oriented control and the Γ-network for rotor field-oriented control.

The steady-state model in the form Γ-network shown in Fig. 1 is defined in the rotating (d,q) stator flux frame, hence \( \Psi_{sd} = \Psi_s \) and \( \Psi_{sq} = 0 \).

\[
\begin{align*}
\frac{d \Psi_s}{dt} & = \omega_s L_s i_{rd} + \omega_m \Psi_s, \\
\end{align*}
\]

Figure 1: Steady-state induction machine equivalent circuit (Γ-network) in: a) d-axis and b) q-axis

In steady state, the coils can be considered as a short circuit. Then, for the stator voltage in q-axis

\[
u_{sq} = R_s i_{sq} + \omega_s \Psi_s = \omega_r L\sigma i_{rd} + R_r i_{rq} + \omega_m \Psi_s, \tag{1}
\]
where \( \omega_r \) is the rotor frequency, which is defined as the difference of synchronous frequency \( \omega_s \) and mechanical speed \( \omega_m \). \( \omega_s \Psi_s \) represents the voltage on the rotor side. Since resistance \( R_{Fe} \) is many times greater than rotor resistance \( R_r \), currents \( i_{Fe,d} \) and \( i_{Fe,q} \) can be considered zero. It is advantageous to express the current in the q-axis as a function of torque and stator linkage flux. For stator currents on the d- and q-axes, we can write

\[
i_{sd,T} = i_{Fe,d,T} + i_{\mu,T} + i_{rd,T} = \frac{\Psi_s}{L_s} + \frac{\omega_r L_{oR} i_{rq,T}}{R_r},
\]

\[
i_{sq,T} = i_{Fe,q,T} + i_{rq,T} = \frac{2M}{3p \Psi_s},
\]

where \( p \) is the number of pole-pairs. Furthermore, the Fig. 1 shows that the voltage across the resistor \( R_{Fe} \) is \( \omega_s \Psi_s \).

Fig. 2 shows the steady-state model in the form \( \Gamma \)-network which is defined in the rotating (d,q) rotor flux frame. Then \( \Psi_{rd} = \Psi_r \) and \( \Psi_{rq} = 0 \).

![Figure 2: Steady-state induction machine equivalent circuit (\( \Gamma \)-network) in: a) d-axis and b) q-axis](image)

Similarly to the previous model, the coils in steady-state can be considered as a short circuit. The rotor current in the d-axis can be considered as zero, due to the zero voltage on the \( L_R \) inductance. The induced voltage from the q-axis lies across the resistor \( R_{Fe} \). This voltage is very small due to the small leakage inductance \( L_{oS} \). Then for the current in d-axis

\[
i_{sd,T} = i_{Fe,d,T} + i_{rd,T} = \frac{\Psi_r}{R_R}.
\]

Since the resistance of \( R_{Fe} \) is many times greater than the resistance of \( R_R \), the current through \( R_{Fe} \) can be neglected. Then the current in the q-axis can be described as follows

\[
i_{sq,T} = i_{Fe,q,T} + i_{rq,T} = \frac{2M}{3p \Psi_r}.
\]

3. LOSS MODEL

In the rotating flux frame (d, q), electrical losses can be determined very easily. First, iron losses can be calculated by dividing the voltage \( u_{R,Fe}^2 \) by the resistance \( R_{Fe} \). However, the iron resistance must not be constant because the iron losses consist of eddy current losses and hysteresis losses. According to [2], these two values can be combined to create a simple linear dependence of the total iron losses on frequency. Then the iron losses can be calculated

\[
P_{Fe} = \frac{3}{2} \frac{u_{R,Fe}^2}{R_{Fe}} = \frac{3}{2} \frac{u_{R,Fe}^2}{R_{Fe} \left( \omega_s / \omega_0 \right)} = \frac{3}{2} \frac{u_{R,Fe}^2 \omega_0}{R_{Fe} \omega_s},
\]

where \( \omega_0 \) is the frequency at which the resistance \( R_{Fe0} \) is determined.

Stator currents flow through the stator winding in both axes. Then for joule losses in the stator winding

\[
P_{js} = \frac{3}{2} R_s (i_{sd}^2 + i_{sq}^2).
\]
The same situation occurs when determining the joule losses in the rotor. Currents flow through the rotor winding in both axes then
\[ P_{jr} = \frac{3}{2} R_r (i_{rd}^2 + i_{rq}^2). \] (8)

To determine the analytical expression of the optimum linkage flux, it is useful to express all quantities as functions of torque, linkage flux, and speed. Therefore, the synchronous frequency can be described by neglecting the rotor frequency as \( \omega_s = \omega_m. \)

The expression of the iron losses in the \( \Gamma \)-network is given by equation (6). As mentioned above, the voltage across the resistor \( R_{Fe} \) is determined by \( \omega_s \Psi_s. \) Then for iron losses is true
\[ P_{Fe,\Gamma} = \frac{3}{2} \omega_m p \omega_0 \Psi_s^2 \frac{R_s}{R_{Fe0}}. \] (9)

In the case of Joule losses, it is easy to determine both currents in the q-axis using the equation (3). More complicated is the expression of currents in the d-axis. \( \omega_r \) appears in equation (2), which can be expressed from equation (1). In [3] is shown that \( \omega_r \) can be calculated as follows
\[ \omega_r,\Gamma = \frac{2 R_s M}{3 p \Psi_s^2}. \] (10)

Substituting equation (10) into (2) gives an expression for the Joule losses in the rotor
\[ P_{jr,\Gamma} = \frac{3}{2} \left( \frac{R_s M^2}{p^2 \Psi_s^2} \right) + \frac{8}{27} \left( \frac{R_s M^4 L_s^2}{p^4 \Psi_s^6} \right). \] (11)

Equation (2) is substituted into the equation for the stator current (2). Then for the Joule losses in the stator
\[ P_{js,\Gamma} = \frac{3}{2} \left( \frac{R_s \Psi_s^2}{L_s^2} \right) + \frac{4}{3} \left( \frac{R_s M^2 L_s^2}{p^2 \Psi_s^2 L_m} \right) + \frac{8}{27} \left( \frac{R_s \cdot M^4 \cdot L_s^2}{p^4 \Psi_s^6} \right) + \frac{3}{2} \left( \frac{R_s M^2}{p^2 \Psi_s^2} \right). \] (12)

The term in equations (11) and (12) that contains \( \Psi_s^{-6} \) has very little effect on the accuracy of the loss calculation, as demonstrated in [3]. So, this term can be neglected.

For the \( \Gamma \)-network, iron losses can be expressed similarly according to equation (6), with the voltage on \( R_{Fe} \) defined as \( \Psi_r \omega_s + L_s i_{sd,\Gamma} \omega_s \) (from Fig. 2). Then the iron losses can be written
\[ P_{Fe,\Gamma} = \frac{3}{2} \frac{\omega_m p \omega_0 \Psi_s^2 (1 + L_s \omega / L R)^2}{R_{Fe0}}. \] (13)

Since the rotor current in the d-axis is zero and the rotor current in the q-axis is determined by equation (5), determining the rotor Joule losses is easy:
\[ P_{jr,\Gamma} = \frac{3}{2} R_r \frac{4 M^2}{9 p^2 \Psi_s^2}. \] (14)

Since the stator current in the d-axis is equal to the magnetizing current, then for Joule losses in the stator
\[ P_{js,\Gamma} = \frac{3}{2} \frac{R_s \Psi_s^2}{L_s^2} + \frac{4 M^2}{9 p^2 \Psi_s^2}. \] (15)

4. LOSS MINIMIZATION ALGORITHM
Finding the optimal steady-state linkage flux can be done by solving the following equation
\[ \frac{\partial}{\partial \Psi} (P_{Fe} + P_{js} + P_{jr}) = 0. \] (16)

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Analytical expression for the optimal stator flux for \( \Gamma \)-network

\[
\Psi_{s,\text{opt}} = \sqrt{M} \sqrt[3]{\frac{2L_s}{3p} \sqrt{\frac{R_s(2k_l + 1) + R_r}{\omega_m L^2_s F + R_s}}},
\]

where \( k_l = L_{\sigma R}/L_s \) and \( F = \omega_0 p/R_{Fe} \). Finally, the analytical expression for the optimal rotor flux for \( \Gamma \)-network

\[
\Psi_{r,\text{opt}} = \sqrt{M} \sqrt[3]{\frac{2L_R}{3p} \sqrt{\frac{R_{Fe} L^2_R (R_s + R_R)}{\omega_m \omega_0 p L^2_F (1 + L_{\sigma S}/L_R)^2 + R_{Fe} R_s}}},
\]

5. RESULTS

The algorithm was verified on a real induction machine, whose parameters can be found in [4]. The dependence of the total losses on the stator flux, respectively, rotor flux for three different torques and two mechanical speed (3000 min\(^{-1}\) and 1500 min\(^{-1}\)) can be seen in Fig. 3 respectively Fig. 4. Furthermore, the points of optimum flux calculated according to the equation (17) respectively (18) for the corresponding torque and speed are marked with a cross in the graph.

![Graph showing the dependence of total power losses on stator flux \( \Psi_s \) at different torques and speeds.](image)

**Figure 3:** Dependence of total power losses on stator flux \( \Psi_s \) at different torques and speeds

Table I shows the calculated values of the optimum linkage flux according equation (17) respectively (18) and the values of the ideal linkage flux at which the motor has the lowest losses, deviation and percentage deviation for a given torque, and speed.

<table>
<thead>
<tr>
<th>( n ) [min(^{-1})]</th>
<th>( M ) [Nm]</th>
<th>( \Gamma )-network</th>
<th>( \Gamma )-network</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Psi_{s,\text{id}} ) [Vs]</td>
<td>( \Psi_{s,\text{opt}} ) [Vs]</td>
<td>( \delta ) [Vs]</td>
</tr>
<tr>
<td>3000</td>
<td>0.8</td>
<td>0.74</td>
<td>0.67</td>
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<td></td>
<td>1.4</td>
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<td>0.85</td>
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<td>2.0</td>
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<tr>
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<td>0.87</td>
<td>0.73</td>
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<tr>
<td></td>
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<td>1.36</td>
<td>1.13</td>
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Table I: Comparison of ideal linkage flux \( \Psi_{id} \) and calculated optimal linkage flux \( \Psi_{opt} \)
Furthermore, the Table I shows that the deviation of the calculated optimal linkage flux differs from the ideal value by tens of percent. However, the graphs show that the total loss curve is very flat in the region of the minimum. Therefore, the resulting power loss when applying the calculated optimum linkage flux will not be as large compared to the power loss at the ideal flux value.

6. CONCLUSION

The aim of this paper was to present a comparison of the computation loss minimization algorithm for the equivalent circuit in form Γ-network and Γ-network of the induction motor. The minimization algorithms are derived from both equivalent circuits. The derivation of the algorithm for rotor flux orientation is simpler. The algorithm for stator flux orientation leads to a complex expression for the losses. In general, the rotor flux oriented algorithm performs better. However, both algorithms can be considered as accurate because, as shown in the Fig. 3 and Fig. 4, the flux-dependent loss curve is very flat. Hence, the deviations in the algorithm will not cause a large loss increase.

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