Maximum Available Accuracy of FM-CW Radars

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Abstract. *This article deals with the principles and above all with the maximum available measuring accuracy analyse of FM-CW (Frequency Modulated Continuous Wave) radars, which are usually employed for distance and velocity measurements of moving objects in road traffic, as well as air traffic and in other applications. These radars often form an important part of the active safety equipment of high-end cars – the so-called anti-collision systems. They usually work in the frequency bands of mm waves (24, 35, 77 GHz). Function principles and analyses of factors, that dominantly influence the distance measurement accuracy of these equipments especially in the modulation and demodulation part, are shown in the paper.*

Keywords

FM-CW radar, continual wave, frequency modulation, measurement, distance, velocity.

1. Principle and Mathematical Expression of the FM-CW Radar Function

Doppler's radar with frequency modulated continuous wave is often used the as most suitable system for anti-collision equipment of the motor cars or other moving objects (airplanes, boats etc.). Such radar makes possible not only the relative velocity v measurement, but also the measurement of the distance d between the considered objects.

A simplified functional diagram is shown in Fig. 1. The frequency modulated signal $s_t(t)$ (with the central frequency f_{to}) is transmitted by means of a directive antenna $\mathbf{A_D}$ situated in front of the moving object (e.g. car). A reflected and time delayed frequency modulated signal $s_r(t)$ is received and mixed (by the mixer **MX2**) with the transmitted and frequency converted signal $s_o(t)$ (its central frequency is $f_{to} - f_o$). Mixer **MX1** with low frequency oscillator $\mathbf{O_F}$ executes this frequency shift. The time delay t_d is proportional to the distance d from the front object.

It holds

$$t_d = \frac{2d}{c} \tag{1}$$

where c is the velocity of electromagnetic wave propagation (c = 3.10^8 m/s). It may vary slightly in the diverse conditions of propagation.

The mathematical expression of the signal instantaneous frequency $f_{MX2}(t)$ at the frequency demodulator **FMD** input can be described by an equation, which holds only for sinusoidal frequency modulation of the transmitter oscillator

$$f_{MX2}(t) = \frac{1}{2\pi}\cdot\frac{d}{dt}\left[\Phi_{MX2}(t)\right] = f_o \pm f_D - 2\Delta f_{to}\cdot\sin\frac{\Omega\cdot t_d}{2}\cdot\sin\Omega\left(t - \frac{t_d}{2}\right) \tag{2}$$

where Ω is the modulation angular frequency [rad/s]

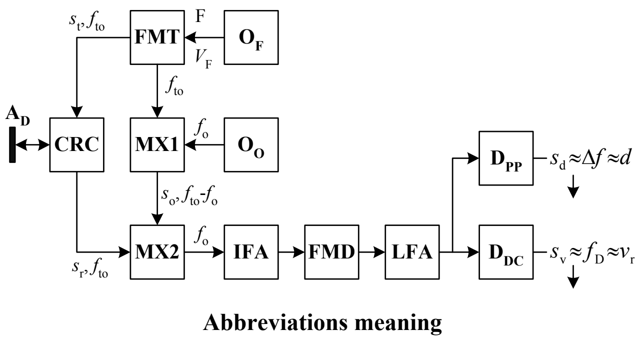
$$\Omega = 2\pi F\,,$$

F is the frequency of the low frequency oscillator $\mathbf{O_F}$ [Hz], f_D is the Doppler's frequency [Hz]. It holds

$$f_D = f_{to}\cdot\frac{2v_r}{c}$$

and subsequently

$$v_r = \frac{c}{2}\cdot\frac{f_D}{f_{to}}\ . \tag{3}$$



Abbreviations meaning

$\mathbf{A_D}$	directive antenna	**IFA**	IF selective amplifier
CRC	circulator	**FMD**	frequency demodulator
FMT	frequency modulated transmitter	**LFA**	LF amplifier
MX1, MX2	frequency mixers	$\mathbf{D_{PP}}$	peak-to-peak detector
$\mathbf{O_F}$	LF oscillator	$\mathbf{D_{DC}}$	d.c. component detector
$\mathbf{O_o}$	intermediate heterodyne oscillator		

Fig.1: Simplified functional diagram of the analog part of the Doppler's FM-CW radar without circuits for the output signal processing and display (in digital form).

Δf_{to} is the frequency deviation of the transmitted frequency modulated signal $s_{to}(t)$. It depends on the hetero-dyne oscillator $\mathbf{O_F}$ voltage V_F and the modulation characteristic steepness S_{FM} of the frequency modulated transmitter **FMT**: $\Delta f_{to} = S_{FM} \cdot V_h$. If $2\pi F \cdot t_d/2 \rightarrow 0$, the formula (2) can be simplified into the form

$$\begin{aligned} f_{MX2} &\cong f_o \pm f_D - \Delta f_{to} \cdot t_d \cdot 2\pi F \cdot \sin 2\pi F\left(t - \frac{t_d}{2}\right) = \\ &= f_o \pm f_D - \Delta f \cdot \sin 2\pi F\left(t - \frac{t_d}{2}\right) \end{aligned} \quad (4)$$

where Δf is the frequency deviation at the frequency demo dulator input which is equal to

$$\Delta f = \Delta f_{to} \cdot t_d \cdot 2\pi F = \frac{4\pi \cdot \Delta f_{to} \cdot F}{c} \cdot d \,. \quad (5)$$

The amplitude of the a.c. component of the frequency demodulated signal is proportional to the frequency deviation Δf and therefore it is proportional to the measured distance d.

$$d = \frac{\Delta f \cdot c}{4\pi \cdot F \cdot \Delta f_{to}} \,. \quad (6)$$

The central frequency f_o of the IF signal $s_o(t)$ before the frequency demodulator **FMD** has to be reduced to achieve sufficient accuracy of the frequency demodulation.

The d.c. component of the frequency demodulated signal on the frequency demodulator **FMD** output varies linearly with the Doppler's frequency f_d and therefore it is proportional to the relative velocity v_r (see (3)).

Output signals s_d and s_v are subsequently digitized and processed by a control microprocessor, which operates the anti-collision system.

Another configuration of the CW-FM radar is shown in Fig.2. A frequency multiplier **FMP** with multiplying factor n (n > 1 is a natural number) and oscillator **VCO** (Voltage Controlled Oscillator) are used in this configuration.

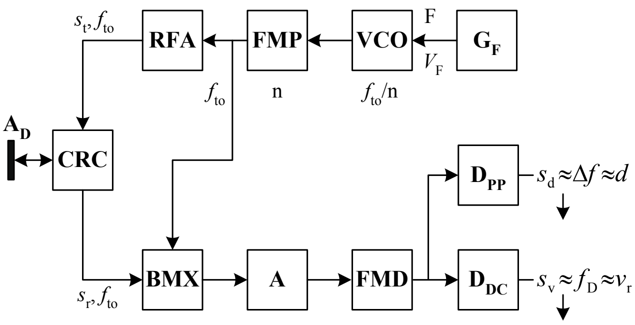


Fig.2: Other configuration of the analog part of the Doppler's FM-CW radar block diagram without the circuits for the output signal processing and display (in digital form).

There are many other sophisticated configurations of the FM-CW radar. Their description is beyond the scope of this paper.

2. Factors Influencing the Maximum Available Accuracy of the FM-CW Radar

The main factors influencing the maximum available accuracy of the distance measurement of time uncompensated FM-CW radar system are:

- inaccuracy of the used approximation $\sin\alpha \cong \alpha$ (see formula (4)),
- time delay Δt_{CR} in the cross arm (**MX1**) of the CW-FM radar structure,
- frequency instability($\Delta F/F$) of the oscillator $\mathbf{O_F}$,
- voltage variation ($\Delta V_F/V_F$)of oscillator $\mathbf{O_F}$,
- non-linearity of the frequency modulation characteristic steepness ($S_{FM} \neq$ const.) of the frequency modulated transmitter **FMT**,
- non-linearity of the frequency demodulation characteristic steepness ($S_{FD} \neq$ const.) of the frequency demodulator **FMD**,
- variations ($\Delta G_A/G_A$) of the amplifier **LFA** gain,
- variations ($\Delta K_{DPP}/K_{DPP}$) of the detectors' $\mathbf{D_{PP}}$ transfer factor,
- changes ($\Delta c/c$) of the electromagnetic wave propagation velocity in different environments.

The first and the second errors are the so called **systematic errors**. They can be compensated by means of a consequent digital processing of the signal s_d. The remaining errors are **random errors**, which decrease the attainable measurement accuracy. Their influence on the measured distance d is **uncorrelated**. This presumption does not always have to be accurate. For instance, the mentioned variations caused by the temperature changes should be mutually dependent.

2.1 Error Caused by the Simplifying Premise $\sin\alpha \cong \alpha$

This simplification imports a systematic error into the radar function. The function $\sin\alpha$, where

$$\alpha = \frac{\Omega \cdot t_d}{2} = \frac{2\pi \cdot F \cdot d}{c},$$

can be approximated by the Mac Laurin series (α is expressed in circular measure)

$$\sin\alpha = \alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \cdots\cdots \mathrm{R_n}(\alpha) \cdot \quad (7)$$

Assuming validity of the condition $0 < \alpha < \alpha_{max}$, the absolute value of the series remainder can be expressed as

$$\left|R_n(\alpha)\right| = \frac{\left|\alpha_{max}\right|^{2n+1}}{(2n+1)!}. \qquad (8)$$

When using only the first series member (n = 1) , the series remainder is

$$\left|R_1(\alpha_{max})\right| = \frac{\left|\alpha_{max}\right|^{3}}{6}.$$

The maximum value of the relative error $\delta_{\alpha rm}$ is expressed by the relation

$$\delta_{\alpha rm} = \frac{\left|\alpha_{max}\right|^{3}}{6\sin\alpha_{max}}. \qquad (9)$$

Numerical example

For d_{max}= 100 m, c = $3\cdot10^8$ m, $F = 10^5$ Hz,

$$\alpha_{max} = \frac{2\pi F\cdot d_{max}}{c} = \frac{2\pi\cdot10^5\cdot10^2}{3\cdot10^8} = 0{,}209 \text{ rad/s},$$

and $\sin\alpha_{max} = 0{,}208$,

the maximum value of the relative error is

$$\delta_{\alpha rm} = \frac{\left|\alpha_{max}\right|^{3}}{6\sin\alpha_{max}} = \frac{0{,}209^3}{6\cdot0{,}208} = 0{,}069 = 0{,}69\ \%.$$

2.2 Error Caused by a Time Delay Δt_{CR} in the Cross Arm of the FM-CW Radar Structure

Selective circuits containing accumulative elements realized by an arbitrary technology (e.g. resonant LC circuits in the frequency mixer **MX1,** connected in the cross arm CW-FM radar according to the Fig.1), most often evoke the time delay Δt_{CR}. It holds for a simple resonant LC circuit quality factor Q

$$\Delta t_{CR} \cong \frac{2Q}{2\pi f_{to}} \qquad (10)$$

This equation is valid for $\Delta f_{to}/f_o \rightarrow 0$ [7].

For coupled resonant (oscillating) circuits it holds

$$\Delta t_{CR} = \frac{4Q}{2\pi f_{to}\left(1+k^2\cdot Q^2\right)}, \qquad (11)$$

where k is a coupling coefficient [-].

For the critical coupling (k = 1), time delay according to (11) is identical as in (10).

Partial time delays are added for number of resonant circuits subsequently connected in the cross arm:

$$\Delta t_{CRsum} = \Delta t_{CR1} + \Delta t_{CR2} + \dots\dots + \Delta t_{CRi}. \qquad (12)$$

This total time delay decreases the measured distance *d,* see formula (13). Such error is able to influence the essentially attainable measurement accuracy and must be compensated.

3. Estimation of the Greatest Value of Total Relative Error of FM-CW Radar

It holds for the peak-to-peak value of the output signal s_d, which is proportional to the measured distance d

$$\begin{aligned} s_d &= V_{dem}\cdot G_A\cdot K_{DPP} = S_{FD}\cdot\Delta f\cdot G_A\cdot K_{DPP} = \\ &= S_{FD}\cdot G_A\cdot K_{DPP}\cdot\Delta f_{to}\cdot\Omega\cdot\left(t_d-\Delta t_{CR}\right) = K_{sum}\cdot d \end{aligned} \qquad (13)$$

where S_{FD} is the steepness of the frequency demodulator **FMD** characteristic [V/Hz] – $V_{dem} = S_{FD}\cdot\Delta f$, G_A is the amplification (gain) of the low frequency amplifier **LFA** [-], K_{DPP} is the transfer factor of the peak-to-peak detector $\mathbf{D_{PP}}$ [-],K_{sum} is the equivalent transfer factor of the FM-CW radar system [m/V].

Time compensated FM-CW radar output signal s_d can be expressed (after inserting the equation (5) in (13) and exploiting the relation $\Delta f_{to} = S_{FM}\cdot V_h$) in the final form (for $\Delta t_{CRsum}\rightarrow 0$)

$$\begin{aligned} s_d &= 4\pi\frac{S_{FD}\cdot G_A\cdot KD_{DPP}\cdot F\cdot\Delta f_{to}\cdot d}{c} = \\ &= 4\pi\frac{S_{FD}\cdot G_A\cdot K_{DPP}\cdot F\cdot S_{FM}\cdot V_h\cdot d}{c} \end{aligned}. \qquad (14)$$

This equation is valid for the configuration according to Fig. 1 and in the case of the time delay Δt_{CR} compensation in the cross arm ($\Delta t_{CRsum}\rightarrow 0$). The individual components in the formula (14) aren't constant. They may be influenced by accidental variations of various factors, which may cause measurement errors. In this paper these errors are considered to be uncorrelated. However, distribution functions of particular errors are unknown. Therefore, only the maximum value of the total relative error $|\delta_{tr}|$ of the measured distance d may be expressed as a sum of anticipated maxima of all errors

$$\left|\delta_{tr}\right| = \left|\delta_\alpha\right| + \left|\delta_{FD}\right| + \left|\delta_A\right| + \left|\delta_{DPP}\right| + \left|\delta_F\right| + \left|\delta_{FM}\right| + \left|\delta_{Vh}\right| + \left|\delta_c\right| \qquad (15)$$

The partial maximal values of errors in (15) can be described by the following expressions:

$$\delta_\alpha = \frac{\left|\alpha_{max}\right|^3}{6\sin\alpha_{max}}, \qquad \delta_{FD} = \frac{\Delta S_{FD}}{S_{FD}},$$

$$\delta_A = \frac{\Delta G_A}{G_A}, \qquad \delta_{P\text{-}P} = \frac{\Delta K_{DPP}}{K_{DPP}},$$

$$\delta_F = \frac{\Delta F}{F}, \qquad \delta_{FM} = \frac{\Delta S_{FM}}{S_{FM}},$$

$$\delta_{Vh} = \frac{\Delta V_F}{V_F}, \qquad \delta_c = \frac{\Delta c}{c}. \qquad (16\text{ a-h})$$

Numerical example

Calculation of the maximal value of the total relative error $|\delta_{tr}|$ of the measured distance d can be realized by means of a preliminary estimation of the maximal values of partial functional block errors. Assume these greatest error values, which can be achieved in the practical technical realization of the FM-CW radar:

$|\delta_\alpha| = 69\cdot10^{-3}$ (see paragraph 2.1)

$|\delta_F| \le 10^{-4}$, $|\delta_{Vh}| \le 10^{-2}$, $|\delta_A| \le 2.10^{-2}$, $|\delta_{DPP}| \le 2.10^{-2}$,

$|\delta_c| \le 10^{-4}$ (see e.g. [7]) .

Values of the errors δ_{FM} and δ_{DM} depend on the technical design of the FM modulator and demodulator. An LC oscillator with capacitive diode (varicap, varactor) controlled by LF oscillator voltage v_F which is connected in a parallel resonant circuit is used often as frequency modulator. Differential capacitance $C_d(v_F)$ of the capacitive diode can be approximated in this elementary case by the relation

$$C_d(V_F) = \frac{C_d(0)}{\left(1-\dfrac{v_F}{V_C}\right)^p} = \frac{C_d(0)\cdot V_c^p}{(V_c - v_F)^p} \tag{17}$$

where V_c is the contact potential [V] (for a silicon diode $V_c \approx 0{,}6$ V), p = 1/3 for a diode with diffuse PN junction.

Instantaneous frequency of this oscillator is expressed by the Thompson relation

$$f(v_F) = \frac{1}{2\pi\sqrt{L\left[C + C_d(0)\left(\dfrac{V_C}{V_C - v_F}\right)^{\frac{1}{3}}\right]}} \tag{18}$$

The resulting relation of frequency deviation $\Delta f(v_F)$ is obtained using equation (18) in the form

$$\pm\Delta f(v_F) = \left(\frac{1}{2\pi}\right)\frac{1}{\sqrt{L\left[C + C_d(0)\left(\dfrac{V_c}{V_c + |V_0|}\right)^{\frac{1}{3}}\right]}} - \left(\frac{1}{2\pi}\right)\frac{1}{\sqrt{L\left[C + C_d(0)\left(\dfrac{V_c}{V_c + |V_0 \pm v_F|}\right)^{\frac{1}{3}}\right]}} \tag{19}$$

where V_0 is capacitive diode bias voltage for the requested value of the central frequency f_0.

This relation shows that the dependence of the oscillator frequency $\Delta f(v_F)$ on the tuning voltage is nonlinear, introducing correlated nonlinear modulation and demodulation errors. A frequency swing from the nonlinear dependence of the frequency modulation characteristic (and corresponding error δ_{FM}) can be obtained after a substitution of concrete numbers. This error may be estimated in this numerical example only as

$$|\delta_{FM}| = |\delta_{FD}| = 1\cdot10^{-2} .$$

After substitution into Eq. (15) it holds for the maximum value of the total relative error $|\delta_{tr}|$ of the measured distance d (sum of anticipated maxima of all errors)

$$|\delta_{tr}| = \left(0{,}69\cdot10^{-3}\right) + \left(10^{-2}\right) + \left(2\cdot10^{-2}\right) + \left(2\cdot10^{-2}\right) + \left(10^{-4}\right) + \left(10^{-2}\right) + \left(10^{-2}\right) + \left(10^{-4}\right) = 0{,}079 = 7{,}9\ \% .$$

This calculated value presents a only very approximate estimation of the maximum value of the total distance measurement error.

4. Conclusion

The basic function description and simplified analyses of a maximum available measurement accuracy especially of the time uncompensated Doppler's FM-CW radar are shown in this paper. Anti-collision systems support safe traffic of various vehicles (especially cars, but no only them) and they make use of this principle. It results from the introduced analyses, that the total relative error isn't insignificant. Therefore it is necessary to eliminate the errors using a suitable circuit configuration option of the partial function blocks parameters. Description of these sophisticated and time compensated configurations of CW-FM radar is beyond the scope of this paper. More detailed information may be found for example in [1], [2], [7].

FM-CW radar makes possible to obtain an information on the relative velocity v_r, too (by means of a second signal s_v processing), if it is necessary for the anti-collision system's function.

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