# The Time Difference of Arrival Estimation of Wi-Fi Signals

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**Abstract.** *The papers deals with a modeling of a Time-Difference of Arrival system for a subscriber station localization, based on the 802.11 standard wireless network. In the case of severe multipath effects the standard TDOA estimation methods, based on correlation of signals, received by conveniently displaced receiving stations show large errors. Thus, a new algorithm is proposed using received signals decomposition to a set of delayed replicas. This represents a linear estimation of reflected signals amplitudes. The described method leads to a better estimation of time differences of the signals, propagating on the direct paths between the emitter and the receiving stations.*

## Keywords

Positioning, multipath propagation, correlated signals, wall reflections, signal parameters estimation, signal processing.

## 1. Introduction

A considerable number of radio positioning systems are based on measurement of the Time Difference of Arrival (TDOA) of two or more signals. The primary and secondary radars, passive radars and satellite navigation systems belong to this group. While the satellite navigation systems are designed to find the receiver (user) position, the radars detect external objects and determine their positions. Although the TDOA measurement problems are similar in both groups of systems, in this paper we will discuss the TDOA measurement only in the radar system. The active (primary and secondary) and passive radars are largely used for localization of outdoor objects in the line-of-sight positions. This paper is devoted to the problem of the indoor object position measurement in the system, using Wi-Fi communication signals. Indoor objects localization brings a challenging problem of TDOA measurement in presence of multiple correlated received signals environment.

Wireless networks are more and more used not only for communication but also for localization of wireless device user. The mobile phone networks have the leading position at this market due to a perfect coverage of large areas. A combination of mobile phone and GPS satellite systems is also wide spread mainly in the field of transport but in this case mobile wireless networks do not perform localization function.

Unfortunately no localization system was designed for indoor or urban area application until recently either due to a low signal level (GPS) or for a poor accuracy (GSM) available in such environment. Only for a short time the wireless data communication networks at 2.4 GHz, based on 802.11 standard, are increasingly used for this application [1], [2]. This communication technology is available now in many types of electronic equipments as in notebooks, PDAs, mobile phones etc., is called Wi-Fi.

The localization systems based on this technology use cell identification principle or signal level principle in areas covered by many access points of wireless networks. Some suppliers declare that in the case of three or more overlapping access points the theoretical available accuracy of such system is about 1 m. These systems require cooperation with tracked devices with the localization server and optimization of access points distribution. To achieve a higher position measurement accuracy calibration procedure in the whole building is necessary. Unfortunately the calibration data are extra sensitive to instantaneous positions of various objects in the building including persons, chairs etc. That is why in real cases the accuracy of such systems is very poor being in the range of 10 to 100 m.

## 2. The Positioning System Description

The described system locates positions of subscriber station transmitters, using wireless communication technology (e.g. notebooks, PDAs, mobile phones etc.). All the stations are situated in a building or in outdoor areas with barriers such as building walls etc. A system locating the positions of these transmitters consists of a few (three or more) receiver stations and of a central processing station. Each receiver station with a known position receives signals in the full frequency band of the 802.11b standard.

After demodulation the received signals are transmitted to the central station. Time-differences of arrivals of signals emitted by individual transmitters received by the receiving stations are evaluated at the central station. The positions of all transmitters are then estimated.

The receiving stations of our system receive not only a direct signal from each transmitter present in the station range, but also reflected signals from walls, ceilings and floors in the building. We suppose that these reflected signals differ from the direct signal only in the amplitude and the delay. All direct and reflected signals give the total signal $s_{\mathrm{Am}}(t)$ at the $m$-th receiving station, which can be written in the following way:

$$s_{Am}(t) = \sum_{n=1}^{N} \sum_{k=0}^{K} A_{nmk} s_n \left(t - t_{nm} - \tau_{nmk}\right) \tag{1}$$

where $A_{nmk}$ is the amplitude of the $k$-th reflected signal of the $n$-th transmitter at the $m$-th receiving station, $t_{nm}$ is the delay of the direct signal of the $n$-th emitter coming to the $m$-th receiving station, and $\tau_{nmk}$ is the delay of the $k$-th reflected signal of the $n$-th transmitter to the $m$-th receiving station in respect to the direct signal.

It is necessary to find an estimation of the delay $t_{nm}$ of the direct signal of the $n$-th transmitter to the $m$-th receiving station, respectively to find time-differences of signal arrivals $t_{nm} - t_{n1}$ to the individual receiving stations. In the case of severe multipath effects the standard TDOA estimation methods, based on correlation of signals, received by conveniently displaced receiving stations show large errors.

## 3. Decomposition of RF Data Signals

At first a full data recovery is necessary which provides us with one replica of the original transmitted signal. Signal of each receiving station is then decomposed to a linear combination of the delayed replicas. This represents linear estimation of amplitudes of delayed signals. Thus we are able to estimate time-differences of arrival $t_{nm}$ of direct signals of the $n$-th transmitter by the $m$-th receiving station to the central station. We suppose signal is sampled in times $t_p$, where $p$ = 1, 2, …, $P$. Thus we can rewrite equation for demodulated signal of the $n$-th channel at the $m$-th station by the following way:

$$s_{nm}\left(t_p\right) = \sum_{k=0}^{K} A_{nmk} Q_n \left(t_p - t_{nmk}\right) + e_{Rnm}\left(t_p\right) \tag{2}$$

where $Q_n(t)$ is a known chip sequence transmitted by the $n$-th transmitter after passing through demodulator filter, $e_{Rnm}(t)$ is a total clutter in the $n$-th channel at the $m$-th station after demodulator with zero mean value, and $A_{nmk}$is the amplitude of the $k$-th delayed signal at $n$-th transmitter received at the $m$-th station after demodulation.

To solve this nonlinear multidimensional problem is a very difficult task. A significant simplification is obtained if we accept a hypothesis that delays $t_{nmk}$ are restricted only to the multiples of some time step $\Delta t$. Now the values of $t'_{nmk} = k.\Delta t$ and quantities $Q_n(t_p\text{-}t'_{nmk})$ are known constants.

The minimized equation is transformed to a system of linear equations for unknown $A_{nmk}$ which can be written in matrix form:

$$\mathbf{S}_{nm} = \mathbf{Q}_{nm}\mathbf{A}_{nm} + \mathbf{E}_{nm} \tag{3}$$

where

$$\mathbf{S}_{nm} \equiv \begin{vmatrix} s_{nm}(t_1) \\ s_{nm}(t_2) \\ . \\ . \\ s_{nm}(t_P) \end{vmatrix}; \quad \mathbf{A}_{nm} \equiv \begin{vmatrix} A_{nm0} \\ A_{nm1} \\ . \\ . \\ A_{nmK} \end{vmatrix}; \quad \mathbf{E}_{nm} \equiv \begin{vmatrix} e_{Rnm}(t_1) \\ e_{Rnm}(t_2) \\ . \\ . \\ e_{Rnm}(t_P) \end{vmatrix},$$

$$\mathbf{Q}_{nm} \equiv \begin{vmatrix} Q_n\left(t_1 - t'_{nm0}\right) & . & .. & Q_n\left(t_1 - t'_{nmK}\right) \\ Q_n\left(t_2 - t'_{nm0}\right) & . & .. & . \\ . & . & .. & . \\ Q_n\left(t_P - t'_{nm0}\right) & . & .. & Q_n\left(t_P - t'_{nmK}\right) \end{vmatrix}.$$

The estimation of delays $t'_{nm}$ is processed by a localization data processor. Here the delays are corrected by a signal transmission time between the receiving station and the central station. For the $n$-th transmitter position estimation the least squares method is used.

## 4. Estimation of Delay Measuring Error

A computer model of signal and data processing was created to verify functionality of the proposed localization system and to evaluate its position accuracy. Simulations run in Simulink/Matlab SW. The Simulink is used for the signal processor model and a pure Matlab software is used for mathematical evaluations of data processor model including position estimation of a localized device.

The following errors affect the described system:

- Clutter and data communication in adjacent channels $\sigma_{tN,}$
- Reflected signals with very small differences in propagation distance $\sigma_{tOd}$
- Time quantizing after demodulation $\sigma_{tQ}$
- Propagation of direct signal through walls $\sigma_{tZ.}$

We may suppose the errors are mutually independent and their variances could be summed considering different source of particular errors of delay measurement. Thus we can write equation for total error of delay estimation at the subscriber station by the following way:

$$\sigma_{tTotal} \cong \sqrt{\sigma_{tN}^2 + \sigma_{tOd}^2 + \sigma_{tZ}^2 + \sigma_{tQ}^2} \tag{4}$$

where $\sigma_{tTotal}$ is the root-mean-square error of signal delay at the subscriber station.

The total error is in the range of 1 to 5 ns by error analysis (see [4]). Thus we choose the mean value $\sigma_t$ = 3 ns for demonstrative calculations.

Although measurement errors of delay at individual stations take different values we may consider their variances being identical at all stations. In this case a covariance matrix of error estimation of position of transmitting station can be defined (see [3]). By this matrix a localization error of transmitting station in vertical and horizontal planes $\sigma_V$ and $\sigma_H$ can be estimated. In many cases we determine the total error in the horizontal plane $\sigma_H$ and in the vertical plane $\sigma_V$ separately.

The mentioned errors are functions of the receiving and subscriber stations positions. In [3] different situations including buildings dimensions and the receiving stations locations are analyzed. In this paper results for only one building of 100 m by 100 m (horizontal) and by 20 m (vertical) and one receiving stations distribution are presented. In Fig. 1 we may see the four receiving station P1 – P4 locations at the outer wall of the building. In Fig. 2 – Fig. 5, the computed vertical and horizontal position errors as functions of the emitter position inside the building are shown. Fig. 2 shows the vertical error of localization in meters.

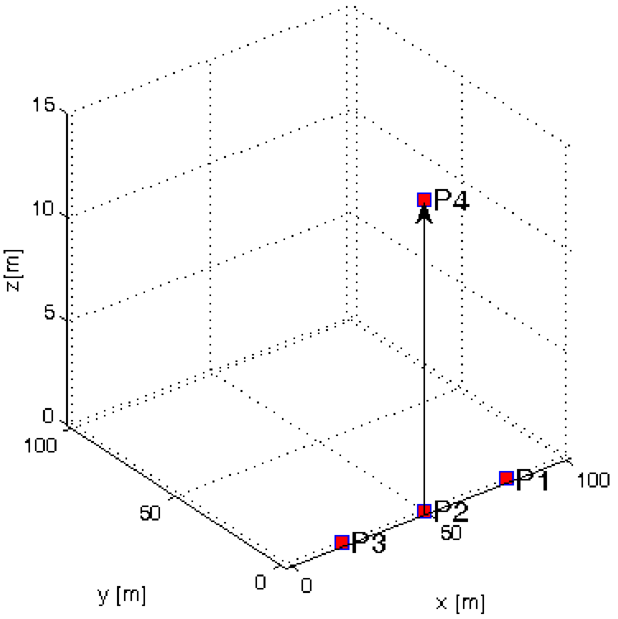

**Fig. 1.** Distribution of receiving stations.

The vertical error in this case is approximately in range from 2 to 9 m with maxima near building corners mainly at walls without receiving stations.

Fig. 2 and Fig. 3 show error calculations corresponding to the subscriber station position at the ground floor. For given distribution of receiving stations maximal error increases about 10 % of the values shown at the half of the building height and up to 30 % at the building top.

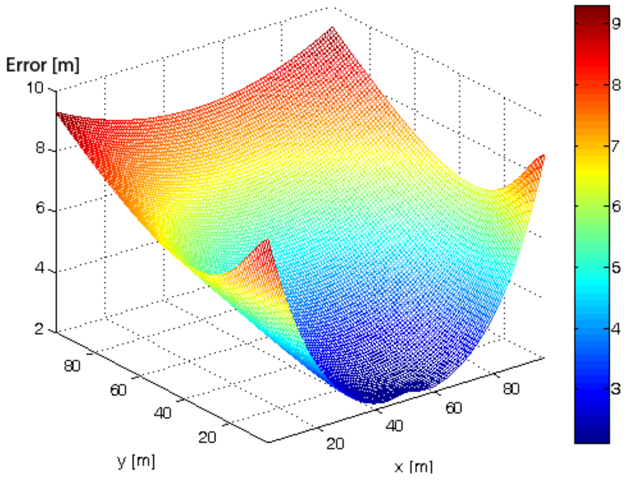

**Fig. 2.** Vertical error of localization.

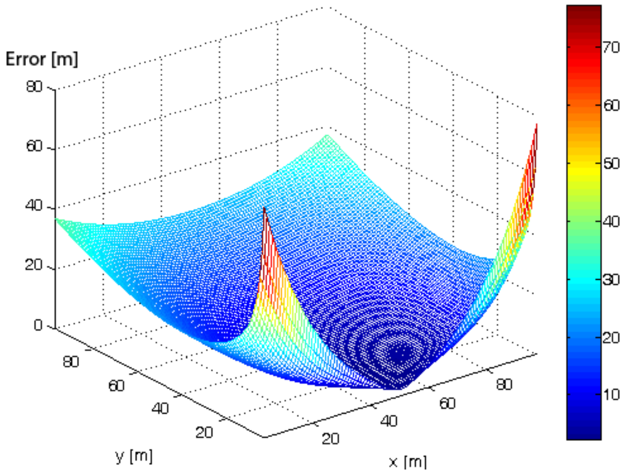

**Fig. 3.** Horizontal error of localization.

We can show the distribution of localization errors also in 2-D graph (Fig. 4 and Fig. 5) where axes define size of the building base. Individual curves determine error bands in meters. Ground coordinates of receiving stations are also displayed for illustration.

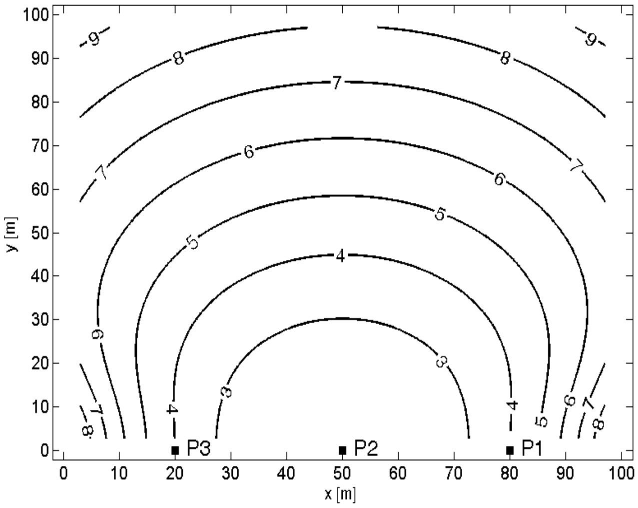

**Fig. 4.** Distribution of vertical error of localization in 2-D representation for four receiving stations.

Results of these computer analyses show that designed system is able to localize the subscriber stations with better accuracy than three or four meters in dependence on character of building and distribution of receiving stations.

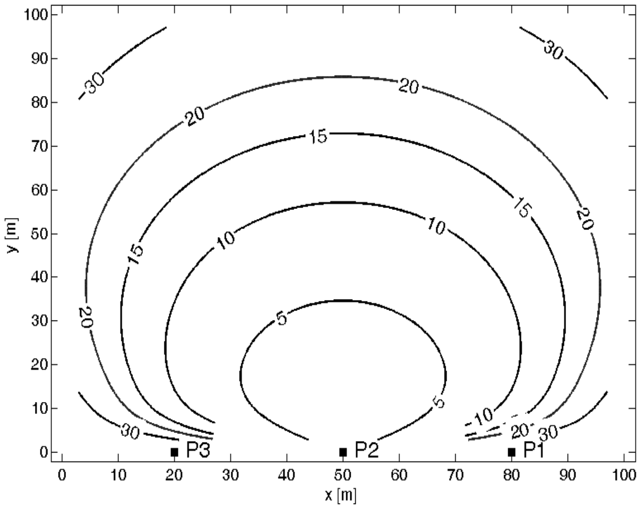

**Fig. 5.** Distribution of horizontal error of localization in 2-D representation for four receiving stations.

Optimization of stations distribution and their number in particular cases is the complicated task. Propagation of signal through walls has dominant effect on localization error of delay. On the other hand effect of clutter and noise as well as multipath effect were minimized to a considerable rate due to the samples selection method and the proposed method of received signals decomposition to a set of delayed replicas.

## 5. Conclusions

The described method leads to a better estimation of time-differences of the signals, propagating on the direct paths between the emitter and the receiving stations. The proposed TDOA system brings mainly a higher localization accuracy useful in different applications, because of the mentioned wireless technology at 2.4 GHz is implemented today in many electronic equipments as in notebooks, PDAs, mobile phones and so on. The TDOA system also eliminates any problem of outdoor - indoor transition.

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