Switched-Capacitor Filter Optimization with Respect to Switch On-State Resistance and Features of Real Operational Amplifiers

Lukáš DOLÍVKA, Jiří HOSPODKA

Dept. of Circuit Theory, Czech Technical University, Technická 2, 166 27 Prague, Czech Republic

dolivl1@fel.cvut.cz, hospodka@fel.cvut.cz

Abstract. *The optimization of a switched-capacitor filter, which implements a biquadratic section, is described in this paper. The aim of the optimization is to obtain a required magnitude frequency response of the filter. The optimization takes into account both one of the features of real switches – their on-state resistance, and the features of real operational amplifiers – finite voltage gain and finite unity-gain bandwidth. An optimal dynamic range is to be achieved as well. The differential evolution – a kind of evolutionary algorithms – is employed for the optimization. The filter is designed by the usual way with ideal switches and ideal operational amplifiers at first. The analysis of this filter with real switches and real operational amplifiers proves that there is a significant difference between its magnitude frequency response and the one with ideal components. Hence, the optimization is applied for finding component values so that the magnitude frequency response is as similar to the one with ideal components as possible. As for other main real features of operational amplifiers – input and output resistance – it is shown that their effect is small.*

Keywords

Switched-capacitor circuit, band-pass filter, optimization, evolutionary algorithm, differential evolution.

1. Introduction

One of common methods for circuit implementation, in particular integrated circuits, is the switched-capacitor (SC) technique. This technique is widespread because it has a few advantages in comparison with other techniques [1], for instance:

- The transfer of SC circuits depends not on capacitor values, but on the ratios of them. These ratios can be substantially more accurate than the capacitor values.
- A clock frequency signal, which is needed for SC circuit operation, can be used for their tuning.
- SC circuits do not require resistors, whose implementation is difficult in integrated form.

As the switches in SC circuits, field effect transistors are commonly used [1]. However, this switch implementation has several nonidealities: nonzero off-state conductance, nonzero on-state resistance, and parasitic capacitances. The transfer of SC circuits is affected negatively by these nonidealities.

From the mentioned switch nonidealities, one can say that nonzero on-state resistance R_{ON} shows itself mostly. Therefore, just this nonideality (linear resistance) was taken into account in this paper. On-state resistance causes that a capacitor C in a SC circuit is charged not in zero but in nonzero time. It is apparent that the higher on-state resistance is, the longer the time is, and the stronger effect of on-state resistance on the SC circuit behavior is.

Another nonideality that can occur in SC circuits is the effect of the features of real operational amplifiers. Their main nonidealities are these: finite input resistance, nonzero output resistance, finite slew rate, finite unity-gain bandwidth, and finite voltage gain. The effect of input resistance is usually insignificant, especially when field-effect transistors on the operational amplifier inputs are used. The output resistance effect is also less important. The slew rate is not considered in this paper. However, the remaining two features – unity-gain bandwidth and voltage gain – affect the transfer function of an SC circuit substantially.

In this paper, the effect of the three chosen nonidealities on the magnitude frequency response of an SC circuit was eliminated using optimization method based on one of evolutionary algorithms. Evolutionary algorithms are a group of optimization techniques, see, e.g., [2], [3], and [4], which are applicable for finding the global extreme of a mathematical function, called the objective function. Hence, they try to either maximize or minimize its value. This is accomplished by finding suitable values for its variables. A few methods belong in evolutionary algorithms, e.g., genetic algorithms [5] and the differential evolution (DE) [6], [7]. For the purpose of optimization in this paper, the DE was chosen. The applications of the DE are, e.g., [7], [8], and [9].

2. Design of Switched-Capacitor Filter with Ideal Switches

The SC circuit chosen for the optimization was an SC biquad (biquadratic section) with schematic diagram in Fig. 1 [10]. Four kinds of filters can be implemented by this biquad: low-pass, high-pass, band-pass, and notch filter. From these types, the band-pass filter was chosen.

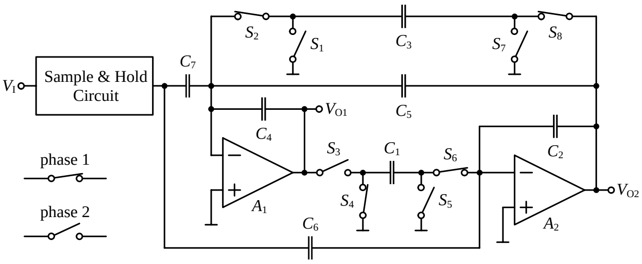

Fig. 1. SC biquad.

The Sample & Hold Circuit in Fig. 1 converts the input signal V_I, which is continuous-time, to a discrete-time one. In phase 2, the circuit samples the input signal and in phase 1, the circuit holds it.

The filter was required to have these parameters:

- center frequency: f_0 = 400 kHz,
- clock frequency: f_C = 6 MHz,
- gain at f_0: G = 20 dB,
- quality factor: Q = 10,
- transfer function implemented from the input V_I to the output V_{O2} of the biquad.

The transfer function from the input V_I to the output V_{O1} is denoted P_1 and the transfer function from the input V_I to the output V_{O2} is denoted P_2. This transfer function labeling is for the biquad with ideal components. Hence, the following equations are valid for P_1 and P_2

$$P_1 = \frac{V_{O1}}{V_I}, \quad P_2 = \frac{V_{O2}}{V_I}. \qquad (1), (2)$$

In this paper, the transfer functions of the biquad with both ideal and nonideal components are considered from phase 1 on the input to phase 1 on the output.

The way of obtaining component values of the filter was according to the common method [1], which is described briefly below.

1. Prewarping the frequency f_0 owing to the bilinear transformation that is used afterwards for creating a discrete-time transfer function. This prewarping yields a prewarped frequency f_{0P}.
2. Substituting the prewarped frequency f_{0P}, the gain G at frequency f_0, and quality factor Q into the general form of the transfer function of an analogue band-pass filter.
3. Using the bilinear transformation, which gives the required transfer function of the equivalent SC band-pass filter (biquad).
4. Calculating capacitor values by comparing the coefficients of the required and symbolic transfer function of the SC biquad.
5. Scaling capacitor values so that an optimal dynamic range is achieved (i.e., the magnitude maxima of the transfer functions P_1 and P_2 are at the same level).

The resulting capacitor values given by the previous design are listed in Tab. 1. These values were adjusted so that the lowest capacitor value is equal to 1. This could be done thanks to the fact that the transfer of SC circuits depends on capacity ratios (mentioned in Section 1). This fact is also the reason why the unit of capacity in Tab. 1 is not specified.

i	1	2	3	4	5	6	7
C_i	10.235	25.086	4.2511	10.235	1	5	10

Tab. 1. The resulting capacitor values for the transfer functions P_1 and P_2 with ideal components.

3. Analysis of Switched-Capacitor Biquad

Mathematical program Maple™ was used for both the analysis and the optimization of the biquad. The analysis was carried out by PraSCAn [10], which is a package for analyzing both ideal and real switched-capacitor and switched-current circuits.

All the frequency responses presented in this paper were verified by WinSpice – a general-purpose circuit simulation program [11]. This was done according to [12] and it confirmed the correctness of PraSCAn's results.

3.1 Biquad with Ideal Components

At first, the biquad with ideal switches and ideal operational amplifiers was analyzed and its magnitude frequency responses were obtained.

The magnitude of a transfer $P(z)$ is symbolized by $M(f)$. In case of the transfers $P_1(z)$ and $P_2(z)$, the magnitudes are calculated as follows

$$M_1(f) = \left| P_1\left(e^{j\frac{2\pi f}{f_C}} \right) \right|, \quad M_2(f) = \left| P_2\left(e^{j\frac{2\pi f}{f_C}} \right) \right|. \qquad (3), (4)$$

In Fig. 2, there are the magnitude frequency responses M_1 and M_2 of the biquad with the capacitors listed in Tab. 1.

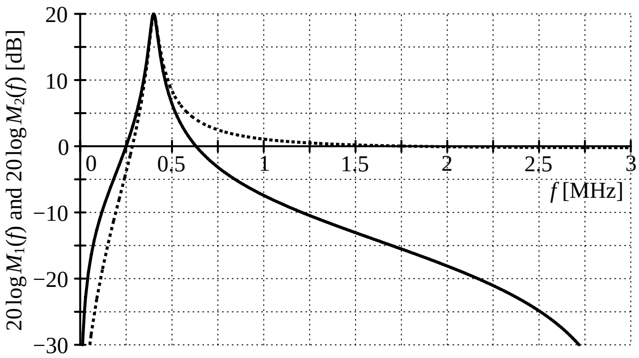

Fig. 2. The magnitude frequency responses of the SC biquad, dotted line: the magnitude M_1, solid line: the magnitude M_2.

3.2 Biquad with Real Components

In this case, switches and operational amplifiers with real features were used. The features that were considered for real switches and operational amplifiers have been already presented in Section 1.

For finding out the effect of switch on-state resistance R_{ON}, knowing the ratios of capacitors is not sufficient. It is necessary to known also concrete capacitor values. The reason is that the effect of the resistance corresponds to the time constant τ of charging the capacitors. The value of the constant is dependent not on the capacity ratios but on the capacity itself. Thus, the unit picofarad was chosen for the calculated capacitor values in Tab. 1, which were multiplied by two. Therefore, the capacitor values used for analysis with real components were, e.g., C_5 = 2 pF. The value of on-state resistance has to be determined as well. A suitable value for the SC circuit switches is 1 kΩ, which was used.

The model in Fig. 3 was used for the operational amplifiers in the biquad. R_{IN} means the input resistance and R_{OUT} means the output resistance. The value of the transadmittance g is 1 S and the value of the voltage gain a is 1.

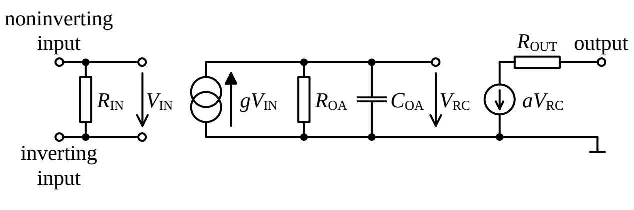

Fig. 3. Model of operational amplifier.

The value of the resistor R_{OA} and the capacitor C_{OA} depends on the unity-gain bandwidth B_1 and the voltage gain A_0 of the applied operational amplifier and it can be calculated according to the following formulae

$$R_{OA} = A_0, \quad C_{OA} = \frac{\sqrt{A_0^2 - 1}}{2\pi B_1 A_0}. \qquad (5), (6)$$

For the transfer functions of the biquad with nonideal components, the labeling P_{1N} and P_{2N} are used instead of P_1 and P_2, respectively. The magnitudes of the transfer functions $P_{1N}(z)$ and $P_{2N}(z)$ are denoted $M_{1N}(f)$ and $M_{2N}(f)$, respectively. They can be calculated in the same way as in case of P_1 and P_2 – according to (3) and (4).

Fig. 4 shows the magnitude frequency response of the biquad with both ideal and real components. The used value of the switch on-state resistance has been already mentioned (1 kΩ). The parameter values of the operational amplifiers were the following: R_{IN} = 1 TΩ, R_{OUT} = 50 Ω, B_1 = 20 MHz, A_0 = 2·10^5. From this figure, one can see that the magnitude frequency responses are different.

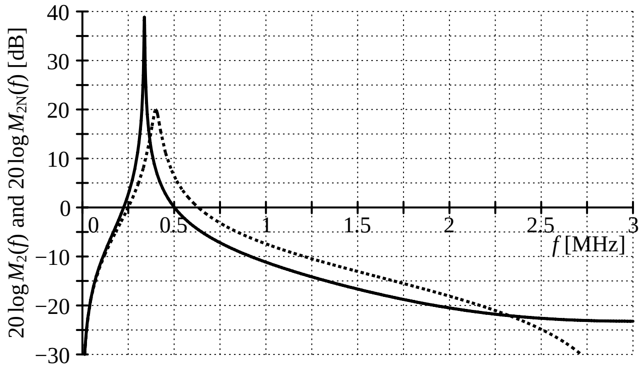

Fig. 4. The magnitude frequency responses of the SC biquad before the optimization, dotted line: the magnitude M_2, solid line: the magnitude M_{2N}.

4. Optimization of Switched-Capacitor Biquad Transfer Function

The optimization had the following aims, which shall have been satisfied by finding suitable capacitor values:

- The magnitude frequency response M_{2N} should fulfill a defined magnitude filter specification, which was derived from the magnitude frequency response M_2.
- The magnitude frequency responses M_{1N} and M_{2N} should have their maximum values as similar as possible (because of obtaining an optimal dynamic range).

Achieving the stability of the optimized biquad was the third aim. However, this aim is evident. (The condition of fulfilling it is well known – all the poles of the transfer function have to have the absolute value lower than 1.)

The spread of capacitor values was not considered in the optimization. Nevertheless, the obtained capacitor values (see Tab. 2) have a spread, which is acceptable.

The only publications dealing with the optimization of an SC circuit which the authors know are [8], [13], and [14]. In [8], switch on-state resistance is considered; parasitic capacitances are respected in [13]. Ref. [14] describes finding capacitor values of an SC filter so that a required transfer function is obtained and simultaneously the spread of the capacitor values and the dynamic range are optimized, but for all the components being ideal.

The magnitude filter specification mentioned in the first requirement consists of ranges $\langle B_L(f_i), B_U(f_i)\rangle$ for several frequencies f_i [7], [8]. After the optimization, the magnitude M_{2N} at a frequency f_i should be in the range that is assigned to this frequency, i.e., $\langle B_L(f_i), B_U(f_i)\rangle$. The following formulae express this fact

$$M_{2N}(f_i) \in \langle B_L(f_i), B_U(f_i)\rangle \quad \forall f_i. \tag{7}$$

If the unit of the magnitude M_{2N} is decibel, formulae (8), (9), and (10) are used instead of (7)

$$20\log M_{2N}(f_i) \in \langle B_{LdB}(f_i), B_{UdB}(f_i)\rangle \quad \forall f_i, \tag{8}$$

$$B_{LdB}(f_i) = 20\log B_L(f_i), \tag{9}$$

$$B_{UdB}(f_i) = 20\log B_U(f_i). \tag{10}$$

However, in general, both of the bounds need not be determined for all frequencies. The upper bound $B_U(f_i)$ for some frequencies f_i can be equal to ∞. Alternatively, for other frequencies f_i, the lower bound $B_L(f_i)$ can be equal to 0 (generally, $-\infty$, but magnitude cannot be negative). For these frequencies f_i, the formula (7) can be modified to one of inequalities

$$M_{2N}(f_i) \ge B_L(f_i) \quad \text{or} \quad M_{2N}(f_i) \le B_U(f_i). \tag{11), (12}$$

It is also possible that both of the bounds are the same at some frequencies f_i. Then (7) is changed into this equation

$$M_{2N}(f_i) = B_L(f_i) = B_U(f_i) \tag{13}$$

and the magnitude M_{2N} at these frequencies should be equal to one value.

The values of the lower and upper bounds were derived from the magnitude frequency response M_2 due to the first aim of the optimization. Addition of a small number to $M_2(f_i)$ was carried out to obtain the value of the upper bound $B_U(f_i)$. The value of the lower bound $B_L(f_i)$ was obtained by a similar way – by subtraction of a small number from $M_2(f_i)$.

Chosen frequencies f_i for the magnitude filter specification are apparent from Fig. 5. In Fig. 6, there is a detail of Fig. 5 around the frequency f_0. The number of the frequencies f_i is 23. Only the upper bounds are specified at fourteen of them. Both the upper and the lower bounds are defined at the others. In order to determine the peak of the magnitude frequency response M_{2N} exactly, the upper and lower bound at the frequency 400 kHz (f_0) are the same (20 dB). This means that the value of the optimized magnitude M_{2N} at this frequency should be just this one.

As mentioned in Section 1, the optimization employed the DE. For the description of it, refer to the presented references. In this paper, only the applied objective function is described. Its value was minimized in case of this optimization. Usually, several forms of the objective function can be used for a particular optimization task. The following form of the objective function F was found to be the most convenient for this optimization [7], [8]:

$$F(C_1,\dots,C_7) = \begin{cases} \left|\max M_{1N}(C_1,\dots,C_7) - M_{2N\max}\right| + \\ + \sum_{i=1}^{L} F_{Li}(C_1,\dots,C_7) + \sum_{i=1}^{U} F_{Ui}(C_1,\dots,C_7) \\ \qquad \text{if the biquad is stable,} \\ 1000 \quad \text{if the biquad is unstable,} \end{cases} \tag{14}$$

where

$$F_{Li}(C_1,\dots,C_7) = \begin{cases} \dfrac{B_L(f_i) - M_{2N}(f_i, C_1,\dots,C_7)}{B_L(f_i)} \\ \quad \text{if } M_{2N}(f_i, C_1,\dots,C_7) < B_L(f_i), \\ 0 \quad \text{else,} \end{cases} \tag{15}$$

with this symbol meaning:

$C_1, C_2,\dots, C_7$	capacitor values,
L	the number of the lower bounds, $L = 9$,
U	the number of the upper bounds, $U = 23$,
$\max M_{1N}$	the maximal value of the magnitude M_{1N},
$M_{2N\max}$	the required maximal value of the magnitude M_{2N}, $M_{2N\max} = 10$ (= 20 dB).

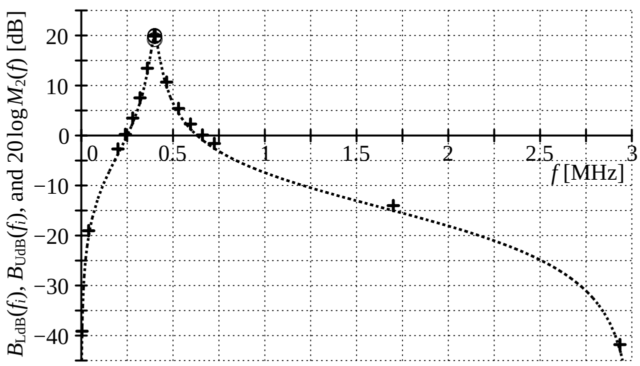

Fig. 5. The magnitude filter specification for the optimization, circles: the lower bounds B_{LdB} of the magnitude ranges, crosses: the upper bounds B_{UdB} of the magnitude ranges, dotted line: the magnitude frequency response M_2.

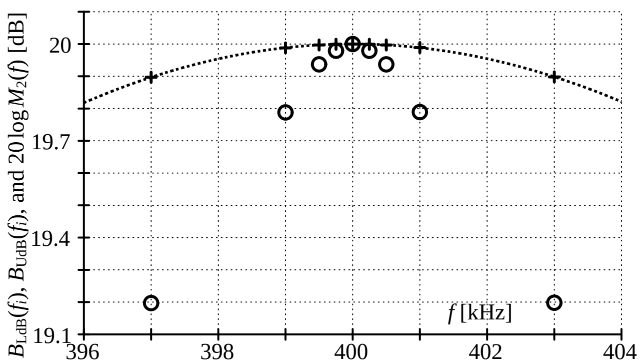

Fig. 6. Detail of Fig. 5 for a vicinity of the frequency f_0.

If the requirements for the optimization result are satisfied, the value of the objective function is 0. Capacitor values for the optimization could be within a range of ⟨1 pF, 100 pF⟩. One generation was composed of 70 members and the values of optimization parameters F and CR were 0.5 and 0.9, respectively.

The optimization was performed while using the parameter values of the real components listed in Section 3.2. However, the parameters R_{IN} and R_{OUT} in the model of the operational amplifiers were not used (so $R_{IN} = \infty\ \Omega$ and $R_{OUT} = 0\ \Omega$). The input and output resistances of the operational amplifiers were neglected because of simplifying the analysis of the biquad during the optimization and thereby speeding it up. This simplification was done since their effect was supposed to be not significant. After the optimization, the analyses of the biquad with the input and output resistances and without them were carried out and these analyses confirmed this assumption. The difference between these analyses can be seen in Fig. 10 and 11.

5. Results of Optimization

The optimization reached the value of the objective function of 0.000034 during 1523 generations. Using more generations did not improve the objective function value (the total number of generations was 4000). Tab. 2 shows the capacitor values arisen from the optimization.

i	1	2	3	4	5	6	7
C_i [pF]	65.319	43.877	69.006	58.628	13.450	4.1664	52.682

Tab. 2. The resulting capacitor values for the transfer functions P_{1N} and P_{2N} with real components.

The magnitude frequency responses M_{1N} and M_{2N} with using the resulting capacitor values are shown in Fig. 7. The magnitude frequency response M_2 is also shown in this figure for comparing. In Fig. 8, there is a detail of Fig. 7 around the frequency f_0. The magnitudes M_{1N} and M_{2N} have their maxima on almost the same level; the difference between them is only about 0.00025 dB. The difference between the frequencies of the maxima occurs even in case of the magnitudes M_1 and M_2.

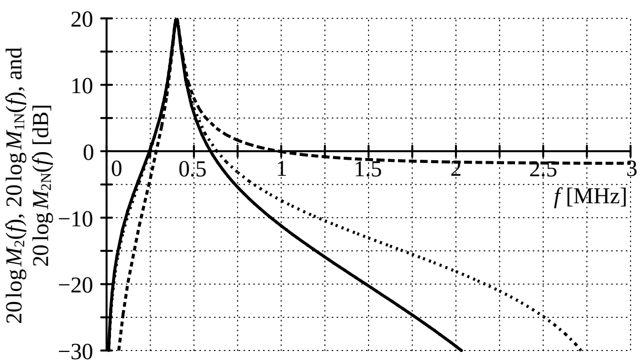

Fig. 7. The magnitude frequency responses of the SC biquad after the optimization, dotted line: the magnitude M_2, dashed line: the magnitude M_{1N}, solid line: the magnitude M_{2N}.

Fig. 9 depicts the magnitude frequency response M_{2N} computed by WinSpice (see Section 3). It is obvious that it is very similar to the magnitude M_{2N} in Fig. 7.

The magnitudes M_{1N} and M_{2N} which are plotted in Fig. 7 and 8 are with $R_{IN} = \infty\ \Omega$ and $R_{OUT} = 0\ \Omega$ – these parameters were not considered since the optimization was carried out without them (see Section 4). However, their effect on the magnitude frequency responses of the biquad is not significant. The magnitude frequency responses M_{1N} and M_{2N} with using the parameters R_{IN} and R_{OUT} in the model of the operational amplifiers are denoted M_{1NR} and M_{2NR}, respectively.

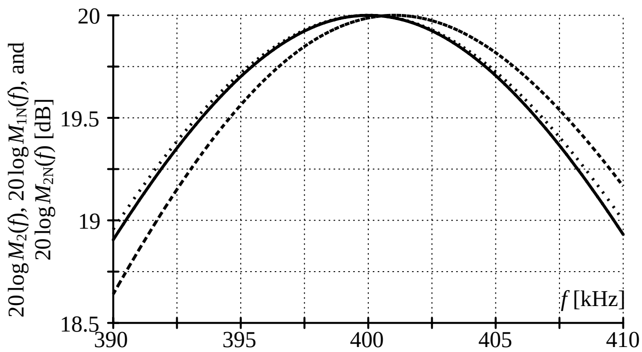

Fig. 8. Detail of Fig. 7 for a vicinity of the frequency f_0.

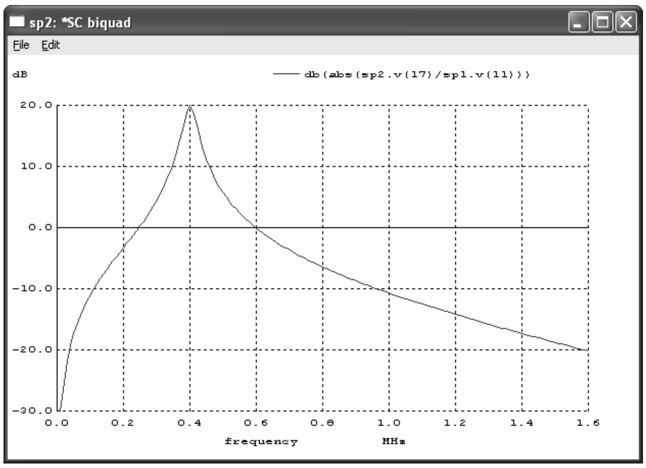

Fig. 9. The optimized magnitude frequency response M_{2N} of the SC biquad computed by WinSpice.

In Fig. 10, there is the difference between the magnitude frequency responses M_{1NR} and M_{1N}. Fig. 11 shows the difference between M_{2NR} and M_{2N}. It is apparent from these two figures that the differences are not high. Higher values (but not too high) are only in the stop-band of the difference between M_{2NR} and M_{2N}.

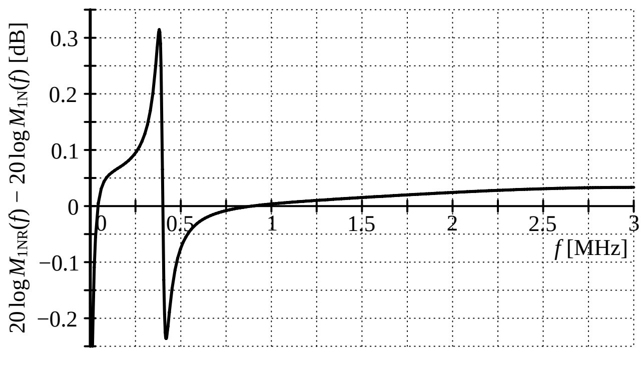

Fig. 10. The difference between the magnitude frequency responses of the SC biquad M_{1NR} and M_{1N}.

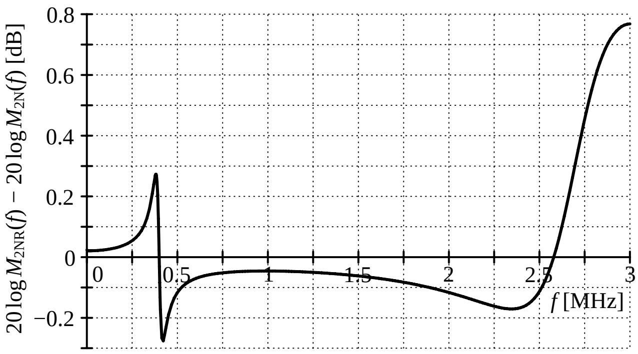

Fig. 11. The difference between the magnitude frequency responses of the SC biquad M_{2NR} and M_{2N}.

Of course, the optimization could be accomplished including the parameters R_{IN} and R_{OUT} but it would take a longer time than without them (about three times).

6. Conclusion

A possible method for elimination of nonidealities that affect SC biquad features was presented in this paper. These nonidealities were represented by one of the real features of switches – on-state resistance – and the real features of operational amplifiers. The elimination was achieved by utilization of the differential evolution, which proved to be sufficiently powerful. The method was successful in meeting the determined requirements. The applied way of optimization can be considered as suitable for this intention.

Acknowledgment

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About Authors...

Lukáš DOLÍVKA was born in 1980. He graduated from the Czech Technical University in Prague in 2005. Since then, he has been a student of a doctoral study program at the Department of Circuit Theory at the same university. His research interests are concerned with discrete-working circuits and the optimization of them.

Jiří HOSPODKA was born in 1967. He received the M.Sc. and Ph.D. degree 1991 and 1995 from the Czech Technical University in Prague. Research interests: circuit theory, analog electronics, filter design, switched-capacitor and switched-current circuits.