

# Simulation of “Tsunami Waves” Propagating along Non-Linear Transmission Lines

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**Abstract.** *The paper compares three methods for computer simulation of transients on transmission lines with losses and nonlinear behavior, namely distributed LC model, FDTD (Finite-Difference Time-Domain) method, and a new and very effective Method of Slices. The losses are responsible for attenuation and shape changes of the waves as function of time and distance from the source. Special behavior of the line due to voltage-dependent capacitance of the line is considered in detail. The non-linear nature of the line causes that the higher is the voltage the higher is the velocity of propagation. Then, the waves tend to tilt over so that their top moves faster than their base. As a result “tsunami waves” are created on the line. Fundamental algorithms are presented in Matlab language. Several typical situations are solved as an illustration of individual methods.*

## Keywords

Non-linear transmission line, wave propagation, shock waves, tsunami waves.

## 1. Introduction

A wave on a linear transmission line is described by partial differential equations for voltage  $u(t,x)$  and current  $i(t,x)$ . Both quantities are functions of time  $t$  and distance from near end of the line  $x$ . The equations are [2, 3]

$$\frac{\partial u(t,x)}{\partial x} = -R_0 i(t,x) - L_0 \frac{\partial i(t,x)}{\partial t} \quad (1a)$$

and

$$\frac{\partial i(t,x)}{\partial x} = -G_0 u(t,x) - C_0 \frac{\partial u(t,x)}{\partial t} \quad (1b)$$

with corresponding initial and boundary conditions.

The symbols  $L_0$ ,  $C_0$ ,  $R_0$  and  $G_0$  are primary line parameters, i.e. inductance, capacitance, resistance and conductance per unit length. In a general case, however, they may be functions of voltage or current.

Numerical solution of these equations can be best based on the Laplace transformation.

In the case of the one-dimensional transformation the voltages  $u(t,x)$  and currents  $i(t,x)$  are transformed from time domain to the domain of complex variable  $s$  to get  $U(s,x)$  and  $I(s,x)$ , respectively [3]. The resulting differential equations with independent variable  $x$  are not partial any more. The sought functions of time can be obtained by any numerical inversion method, for instance [7].

A very effective method is the two-dimensional transformation. Time  $t$  is transformed to  $s$  and in the second step distance  $x$  is transformed to another complex variable  $q$ . Again, very effective methods for numerical inversion are available [8, 9].

In case of a non-linear line another procedure has to be applied. In this paper we present 2 classical simulation procedures, namely the modeling of the line by a cascade connection of a finite number of LC sections and the so-called FDTD (Finite-Difference Time-Domain) method. We shall show their possibilities and limitations and then, based on results of some typical problems, we develop a new Method of Slices. This method is very effective and allows solving even situations where the two previously mentioned procedures fail.

Further on, we assume that the line may be nonlinear due to its voltage-dependent capacitance

$$C_d = \frac{C_0}{(1 + u/U_p)^2} \quad (2)$$

where  $C_d = dq/du$  is the dynamical capacitance and  $U_p$  is a parameter characterizing the non-linearity (it is the voltage for which the capacitance drops to  $C_0/4$ ).

## 2. Cascade Connection of LC Sections

The total length  $len$  of the line is divided into  $m$  partial sections. Each of them is simulated with the help of an LC network. The line is thus substituted by the ladder network shown in Fig. 1 [4, 5, 6].

Each section has an inductor  $L_0$  and two capacitors  $C_d/2$  so that the first and the last condensers are  $C_d/2$  while all the inner capacitors have the capacitances  $C_d$ . In parallel with each capacitor there is a conductance  $G_0$  and in series with each inductance there is a resistance  $R_0$ . The circuit

with  $m$  nodes is described by  $m$  independent node voltages and  $m-1$  currents of the inductors.

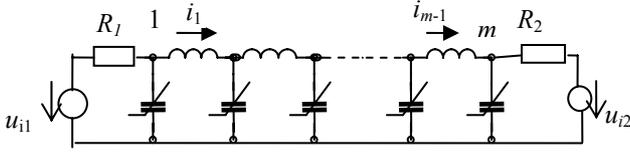


Fig.1. The LC equivalent scheme.

For the first and the last node there holds (consider only a signal source at the near end)

$$\frac{du_1}{dt} = 2 \frac{u_{i1}G_1 - u_1(G_1 + G_0/2) - i_1}{C_d}, \quad (3a)$$

and

$$\frac{du_m}{dt} = \frac{i_{m-1} - (G_2 + G_0/2)u_m}{C_d}, \quad (3b)$$

respectively.

For the  $k$ -th inner node voltage we have

$$\frac{du_k}{dt} = \frac{i_{k-1} - i_k - G_0u_k}{C_d}, \quad 2 \leq k \leq m-1 \quad (3c)$$

and for the current of the  $j$ -th inductor there is

$$\frac{di_j}{dt} = \frac{u_{j-1} - u_j - R_0i_j}{L_0} \quad j = 1, m-1. \quad (3d)$$

The equations take into consideration even the losses caused by resistances  $R_0$  and conductances  $G_0$  so that they hold for any general transmission line.

The simulation can be done by Matlab language of mathematical computations. In the main program we define the basic parameters of the problem. Then we call the differential equation solver

```
[t,u]=ode45('NLCfun',[0 tx],zeros(2*m-1,1));
```

The function NLCfun is used to define the system of ordinary differential equations (3a, b, c, d):

```
function xdot=NLCfun(t,x)
```

```
global m Co Lo Ro Go G1 G2 A to Ti Up
sig=A*G1*(sin(pi*t/Ti))^2*(t<=Ti);
xd(1)=(sig-(G1+Go/2)*x(1)-
      x(m+1))/(Co/(1+x(1)/Up)^2)*2;
      % node voltage u1
xd(m)=(x(2*m-1)-
      (G2+Go/2)*x(m))/(Co/(1+x(m)/Up)^2)*2;
      % node m
for k=2:m-1 % node k and current k-1
xd(k)=(x(m+k-1)-x(m+k)-
      Go*x(k))/(Co/(1+x(k)/Up)^2);
xd(m+k-1)=(x(k-1)-x(k)-Ro*x(m+
      k-1))/Lo;
```

end

```
xd(2*m-1)=(x(m-1)-x(m)-Ro*x(2*m-1))/Lo;
      % current of the last inductor
xdot=xd';
```

**global** transfers the parameters from the main program. The signal (current) at the near end is defined as

$$U_{i1}G_1 \sin^2\left(\pi \frac{t}{T_i}\right)$$

with amplitude  $U_{i1}G_1$  and duration (at the bottom level)  $T_i$ .

Since in practical situations the primary parameters  $L_0$ ,  $C_0$  have values that differ by many orders of magnitude the program uses normalized quantities. Each section represents a sector of 1 meter, the time step is 1 s. Fig.2 shows an example of the situation with parameters  $C_0=2$  F/m,  $L_0=1$  H/m,  $G_0=0$ ,  $R_0=0$ ,  $U_p=4$  V,  $G_1=1.414$  S,  $G_2=0.4714$  S. The line is loss-less and matched at the source (neglecting the non-linearity) and terminated at the far end by resistance  $R_2$  causing reflection with reflection coefficient +0.5.

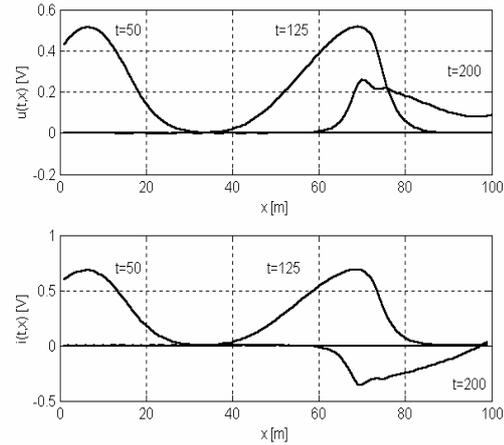


Fig.2a. Distribution of voltage and current along the line at  $t=50$ , 125 and 200 s; Gauss impulse  $U_{i1}=1$  V,  $t_0=40$  s,  $T_i=12$  s,  $U_p=4$  V.

### 3. FDTD Model of the Line

The basic equations can be rearranged to get

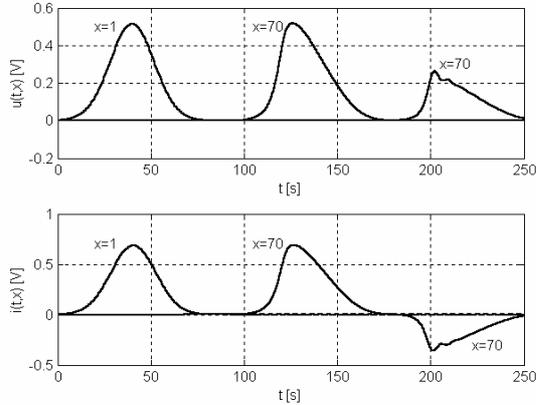
$$\frac{\partial u(t,x)}{\partial t} = -\frac{1}{C_d} \frac{\partial i(t,x)}{\partial x} - \frac{G_0}{C_d} u(t,x), \quad (4a)$$

$$\frac{\partial i(t,x)}{\partial t} = -\frac{1}{L_0} \frac{\partial u(t,x)}{\partial x} - \frac{R_0}{L_0} i(t,x). \quad (4b)$$

The line will be again divided into  $m$  sections with the length  $\Delta x = l_{en}/m$ . Also the total solution interval is divided into  $nt$  subintervals  $\Delta t = t_{max}/nt$ . This will be the base of the FDTD method. The derivatives in Eq. (4a,b) are replaced by approximate expressions containing finite differences to get equations (5a) and (5b)

$$\frac{i_{k-1/2}^{n-1} - i_{k+1/2}^{n-1}}{\Delta x} + G_0 \frac{u_k^{n-1} + u_k^n}{2} + C_0 \frac{u_k^n - u_k^{n-1}}{\Delta t} = 0 \quad (5a)$$

$$\frac{u_k^{n-1} - u_{k-1}^{n-1}}{\Delta x} + R_0 \frac{i_{k+1/2}^{n-1} + i_{k+1/2}^n}{2} + L_0 \frac{i_{k+1/2}^n - i_{k+1/2}^{n-1}}{\Delta t} = 0 \quad (5b)$$



**Fig.2b.** Voltage and current waveforms as functions of time at the near end ( $x=1$  m) and at  $x=70$  m.

Thus the voltage

$$u_k^n = \frac{1 - \frac{G_0 \Delta t}{2C_0}}{1 + \frac{G_0 \Delta t}{2C_0}} u_k^{n-1} + \frac{2 \frac{G_0 \Delta t}{G_0 \Delta x} \frac{2C_0}{2C_0}}{1 + \frac{G_0 \Delta t}{2C_0}} (i_{k+1/2}^{n-1} - i_{k-1/2}^{n-1}) \quad (6a)$$

and the current

$$i_{k+1/2}^n = \frac{1 - \frac{R_0 \Delta t}{2L_0}}{1 + \frac{R_0 \Delta t}{2L_0}} i_{k+1/2}^{n-1} - \frac{2 \frac{R_0 \Delta t}{R_0 \Delta x} \frac{2L_0}{2L_0}}{1 + \frac{R_0 \Delta t}{2L_0}} (u_k^{n-1} - u_{k-1}^{n-1}) \quad (6b)$$

The values of  $u(t,x)$  and  $i(t,x)$  have been replaced by the average value from the two neighboring sections as follows from the fact that the table of currents is shifted by  $\Delta x/2$  with respect to the table of voltages.

The numerical calculations are realized in loops for  $m-1$  voltages and  $m-1$  currents according to the following scheme:

$$\begin{matrix} t_{n-1} & u_1^{n-1} & u_2^{n-1} & \dots & u_{k-1}^{n-1} & u_k^{n-1} & u_{k+1}^{n-1} & \dots & u_{m-1}^{n-1} & u_m^{n-1} \\ & i_{3/2}^{n-1} & i_{5/2}^{n-1} & \dots & i_{k-1/2}^{n-1} & i_{k+1/2}^{n-1} & \dots & i_{m-1/2}^{n-1} & i_{m+1/2}^{n-1} & \\ t_n & u_1^n & u_2^n & \dots & u_{k-1}^n & u_k^n & u_{k+1}^n & \dots & u_{m-1}^n & u_m^n \\ & i_{3/2}^n & i_{5/2}^n & \dots & i_{k-1/2}^n & i_{k+1/2}^n & \dots & i_{m-1/2}^n & i_{m+1/2}^n & \end{matrix} \quad (7)$$

The calculation proceeds in steps of  $\Delta t$ . In each step the actual voltage is calculated from the value of voltage and 2 currents from the previous step. The calculation of current proceeds in a similar manner. The process is straightforward with no need for iterations. Therefore it is very fast and efficient.

Due to a certain lack of symmetry of both tables (the table of currents is shifted by  $\Delta x/2$  with respect to the table of voltages) the voltage at the near end as well as the current at the far end are zero. Therefore, in the original version described in [1] the near end is considered short-circuited and the far end open. To enable simulation of arbitrary sources and terminating resistors at both ends it is necessary to complete the tables by the relations  $u(1) = u_{i1}(t) - R_{i1}i(1)$  and  $i(m) = (u(m) - u_{i2}(t))/R_{i2}$ .

In Matlab the decisive part of the algorithm may have the following form:

```
% ftd3 - lossy and non-linear line
% signals at both ends
clear
m=100; % number of sections
nt=250; % number of time steps
dtx=1; % delta t/delta x
```

```
Co=2; Lo=1; Ro=0; Go=0; % primary
% parameters
Up=4; % parameter of non-linearity
dRL=Ro/(2*Lo); %auxiliary time constant
%(dGC is in general voltage dependent)
Ri1=1/sqrt(2);
Ri2=3/sqrt(2); %terminations
u=zeros(m,1);
i=zeros(m,1); % prepare u and i
tsave=[50 125 200];
% what times for the results?
xsave=[1 70];
% what distances for the results?
to=40; % center of the Gauss impulse
Ti=12; % length of the Gauss impulse
Ui1=1; % amplitude of the impulse
Ui2=0; % amplitude of the far end
% sin^2 impulse
Ti2=40; % length of the far end
% sin^2 impulse
nn=0; % counter of ux, ix curves
for t=1:nt % main loop for time tt
    ui1=0; ui2=0;
    kk=1; % counter of ut curves
    if xsave(1)==1 kk=2;
end;
for ku=2:m % loop for u(x)
```

```

dGC=Go/(2*Co/(1+u(ku)/Up));%auxiliary
if (kk<=length(xsave))
    ksu=xsave(kk);
end;
u(ku)=u(ku)*(1-dGC)/(1+dGC)...
+dtx/(1+dGC)/(Co/(1+(u(ku)/Up)^2))*
(i(ku-1)-i(ku));
if ku==ksu
    ut(t,kk)=u(ksu);
    kk=kk+1;
end;
end;
ui1=Ui1*exp(-0.5*((to-t)/Ti)^2);
                                %Gauss impulse
u(1)=ui1-Ri1*i(1); % near end signal
                                % source
u1(t)=u(1);
if xsave(1)==1
    ut(t,1)=u(1);
end;
kk=1; % counter of it curves
for ki=1:m-1 % loop for i(x)
    if (kk<=length(xsave))
        ksi=xsave(kk);
    end;
    i(ki)=i(ki)*(1-dRL)/(1+dRL)...
    +dtx/(1+dRL)/Lo*(u(ki)-u(ki+1));
    if ksi==ki
        it(t,kk)=i(ksi);
        kk=kk+1;
    end;
end;
ui2=Ui2*(sin(pi*t/Ti2))^2*(t<=Ti2);
i(m)=(u(m)-ui2)/Ri2; % far end signal
                                source
if xsave(length(xsave))==m
    it(t,kk)=i(m);
end;
if any((t-tsave)==0) % save according
                                to tsave
    nn=nn+1; ux(:,nn)=u;
    ix(:,nn)=i;
end;
end; % end of main loop for t

```

The program allows choosing the ratio  $\Delta t/\Delta x$ . If it equals to 1 (as given in the program) it is considered that  $\Delta t=1$  s,  $\Delta x=1$  m.

The results obtained with this program are virtually equivalent to those from the LC model with the equal number of sections  $m$ . The computation, however, is much faster.

### 3.1 Stability Limits of FDTD Method

Practical experience with the program shows that the explicit numerical method may be unstable under some circumstances. Instability (voltage and current rise very fast and unlimitedly) is demonstrated mainly with loss-less lines. The stability limits depend on  $\Delta t/\Delta x$  and on

parameters  $L_0$  and  $C_0$ . For  $L_0 < 1$  or  $C_0 < 1$  the solution is always unstable. Maximal allowable values of the internal source resistance for lower values of  $L_0$  and  $C_0$  are given in Tab.1.

	Co=1	1.25	1.5	1.75	2	2.25	2.5	2.75	3
Lo=1	1.10	1.45	1.58	1.65	1.70	1.74	1.77	1.79	1.81
1.25	1.81	2.00	2.10	2.16	2.21	2.25	2.27	2.29	2.31
1.50	2.36	2.52	5.61	2.67	2.71	2.75	2.77	2.79	2.81
1.75	2.89	3.03	3.11	3.17	3.21	3.25	3.27	3.29	3.31
2.00	3.40	3.58	3.62	3.67	3.71	3.74	3.77	3.79	3.80
2.25	3.91	4.04	4.12	4.17	4.21	4.24	4.26	4.28	4.30
2.50	4.42	4.54	4.62	4.67	4.71	4.74	4.76	4.78	4.80
2.75	4.92	5.04	5.12	5.17	5.21	5.23	5.26	5.28	5.29
3.00	5.42	5.54	5.61	5.66	5.70	5.73	5.75	5.77	5.79

Tab.1. Maximal allowable values of source resistance as function of parameters

For  $C_0 > 3$  F the value of  $R_{imax}$  depends only a little on  $C_0$  but with higher  $L_0$  it grows linearly.

## 4. Discussion of the Two Methods

Fig.3 shows four examples of wave propagation solved with the program *fdtd3*. The solution parameters were  $C_0=1$  F/m,  $L_0=2$  H/m,  $m=100$  m,  $nt=100$  s  $R_{i1}=1$   $\Omega$ , input signal was a Gauss impulse. Individual curves correspond to  $U_p=1000$  V (virtually linear line),  $U_p=4$  V, 2.5 V and 1.5 V. Lower values of  $U_p$  result in larger delay of top of the impulse. The waveforms for  $U_p=1.5$  V oscillate since they are not unique functions of time and distance.

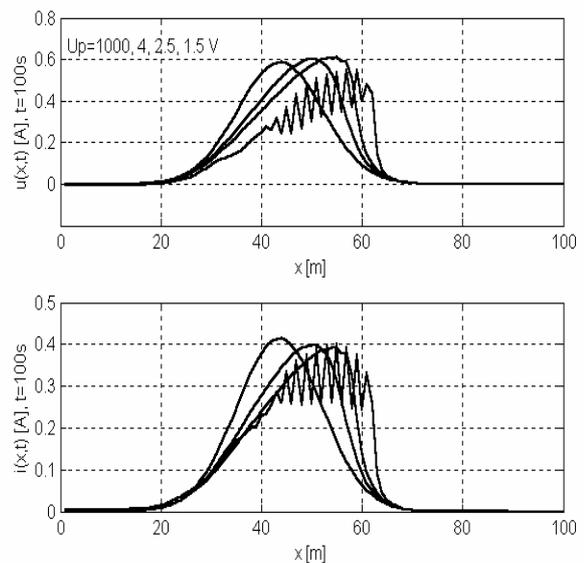


Fig.3. Simulation of voltage and current waves for different values of parameter  $U_p$

It is interesting that the voltage and current waveforms have a very similar shape in spite of the non-linear character of the line. Ratio of voltage to current at any point for higher  $U_p$  (lower effect of non-linearity) is virtually constant and equal to the wave resistance  $R_{vs}=u(t)/i(t)$ . However, this resistance is not

$$R_v = \sqrt{\frac{L_0}{C_d}} = \sqrt{\frac{L_0(1+u/U_p)^2}{C_0}} \quad (8)$$

as expected. The velocity of propagation of the impulse is

$$v = \frac{1}{\sqrt{L_0 C_d}} = \sqrt{\frac{(1+u/U_p)^2}{L_0 C_0}} \quad (9)$$

The results are collected in Tab.2. In the table  $u_x$  and  $i_x$  are the maximal values of voltage and current for the corresponding  $U_p$ ,  $u_x/i_x$  is their ratio,  $d_{max}$  is the distance of the top from the signal source. Values of the specific delay  $\tau=l/v$  [s/m], the wave resistance  $R_v$  and the actual distance  $d'$  are calculated from parameters  $L_0$  and  $C_d(u_x)$ . It is evident that the ratio  $u_x/i_x=R_{vs}$  and also the voltage at the near end (and also voltage the at any point along the line) depends on the transfer factor

$$K(u) = \frac{R_{vs}}{R_{i1} + R_{vs}}$$

$U_p$	$u_x$	$i_x$	$d_{max}$	$u_x/i_x$	$\tau$	$R_v$	$R_{vs}$	$u'_x$	$i'_x$	$u'_x/i'_x$	$d'$
1000	0.586	0.415	44	1.414	1.414	1.414	1.414	0.586	0.415	1.414	42.4
4	0.6027	0.399	50	1.51	1.155	1.224	1.520	0.603	0.397	1.519	51.9
2.5	0.6105	0.392	55	1.56	1.137	1.760	1.577	0.611	0.387	1.579	52.7

Tab.2. The important values taken from Fig.3.

### 5. Method of Slices

The suggested method is based on the facts revealed in the previous section. It considers a loss-less line. The input signal is composed of individual thin voltage slices and these slices propagate along the line quite independently. Due to the assumed loss-less character of the line the voltage level of each slice is keeping its value constant during the propagation. Its velocity depends on the corresponding dynamical capacitance  $C_d$  (9). The wave resistance however is dependent on the static capacitance  $C_s$  (11).

The near end voltage equals to

$$u_1(t) = u_{i1}(t) \frac{R_{vs}(u_1(t))}{R_{i1} + R_{vs}(u_1(t))} \quad (14)$$

with

$$R_{vs} = \sqrt{\frac{L_0}{C_s}} = \sqrt{\frac{L_0(1+u/U_p)}{C_0}} \quad (10)$$

where

$$C_s = \frac{q(u)}{u} = \frac{1}{u} \int_0^u C_d du = \frac{C_0}{1+u/U_p} \quad (11)$$

is static capacitance of the line.

In case of  $U_p \leq 2$  V the nonlinear character of the line causes that the wave top is ahead of the base. The wave tilts and converts into a shock wave. Thus the solution of differential equations is not a unique function of time any more. The solution is not representative. Therefore it is necessary to find another simulation algorithm.

Note: similar results were obtained even with the non-linearity given by

$$C_d = \frac{dq(u)}{du} = \frac{C_0}{1+u/U_p}, \quad (12)$$

$$C_s = \frac{q(u)}{u} = \frac{1}{u} \int_0^u C_d du = \frac{C_0 U_p \log(1+u/U_p)}{u}. \quad (13)$$

Since the voltage  $u_1(t)$  appears at both sides of the equation it is necessary to calculate it for each time instant by solving the non-linear equation

$$u_1(R_{i1} + \sqrt{\frac{L_0(1+u_1/U_p)}{C_0}}) - u_{i1}(t) \sqrt{\frac{L_0(1+u_1/U_p)}{C_0}} = 0 \quad (15)$$

In Matlab language this task can be solved simply as

```
u1=fzero('ful',1); % near end voltage
```

The corresponding function is then

```
% ful near end voltage, called by fzero
function u1=fuu(u)
```

```
global m Co Lo Up Ril R2 A Ti uil Rvs1
Rvs1=sqrt(Lo*(1+u/Up)/Co);
u1=u-uil*Rvs1/(Ril+Rvs1);
```

The point with voltage level  $u_1(t)$  covers the distance  $d$  in  $tt1(t)=d/v(u_1(t))+t$ . As soon as the wave reaches the far end of the line it is reflected with the coefficient of reflection

$$\rho_2(u_f, u_r) = \frac{R_2 - R_{vs2}(u_r)}{R_2 + R_{vs1}(u_f)} \quad (16)$$

dependent on both the voltages of forward and reflected waves. The reflected voltage  $u_r$  will be again obtained by iteration just like in case of the near end voltage.

The main loop of the program may be for instance

```

for t=1:tx % main loop for t
    uil=A*sin(pi*t/Ti)^2*(t<=Ti); % int.source voltage
    u1=fzero('fu1',1); % near end voltage
    ut1(t)=u1;
    it1(t)=u1/Rvs1;
    temp=1+u1/Up; % auxiliary constant
    tau1=sqrt(Lo*Co/temp^2);
    tt1(t)=tau1*d+t; % time delay of uf
    if tx>=m*sqrt(Lo*Co)
        uf=u1; ur=fzero('fu2',0.5);
        tau2=sqrt(Lo*Co/(1+ur/Up));
        tt2(t)=tau1*m+(m-d)*tau2+t; % time delay of ur
        ut2(t)=ur; it2(t)=-ur/Rvs2;
    end;
end;

```

end;

end;

with function

```

% fu2, far end voltage after reflection
function ur=fu2(x)

```

```

global m Co Lo Up R1 R2 A Ti uil uf
Rvs1 Rvs2
Rvs2=sqrt(Lo*(1+x/Up)/Co);
ur=x-uf*(R2-Rvs1)/(R2+Rvs2);

```

It is still necessary to solve the problem of setting up the vectors of time, distance, voltage and current. The tables of distances for the forward and reflected waves do not contain equal points. The two tables have to be merged in a single table and values of voltages and currents in the new points obtained by interpolation. The part of the program that secures this operation looks like this

```

% merge tables of u and i
for jtt=1:length(tt1)
    if tt1(jtt)>tt2(1)
        jttmn=jtt-1;
        break;
    end;
end;
for itt=1:length(tt2)
    t1=tt2(itt);
    if t1>tx
        ittmx=itt;
        break;
    end;
    % max. index in tt2

```

```

end;
for itt=1:ittmx-1
    for jtt=jttmn:length(tt1)-1
        % interval tt1
        t1=tt1(jtt);
        t2=tt1(jtt+1);
        t3=tt2(itt);
        % instant from tt2
        if (t3>t1)&(t3<=t2)
            % interpolate
            p=polyfit([t1 t2],[ut1(jtt)
                ut1(jtt+1)],1);
            ut2(itt)=ut2(itt)+polyval(p,t3);
            p=polyfit([t1 t2],[it1(jtt)
                it1(jtt+1)],1);
            it2(itt)=it2(itt)+polyval(p,t3);
            break;
        end;
    end;
end;

```

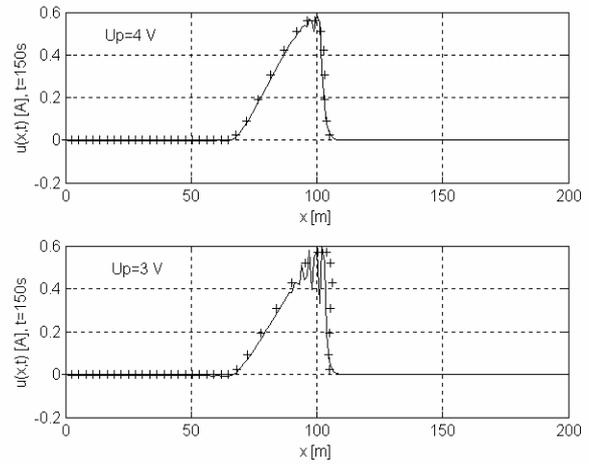


Fig.4. Comparison of the FDTD (solid line) with the method of slices (denoted by crosses)

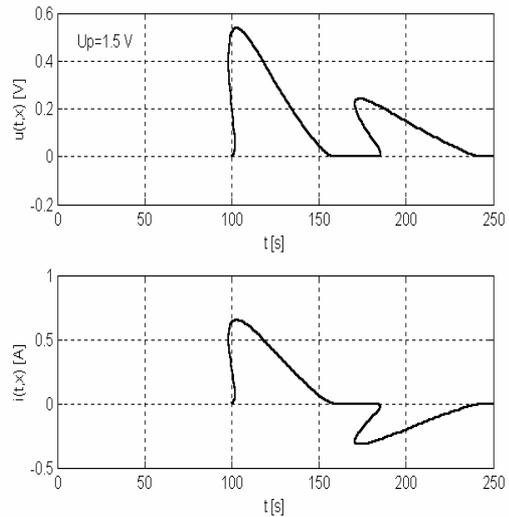


Fig.5. Shock waves simulated by the method of slices

Fig. 4 compares the voltage waves with  $U_p=4$  V and  $U_p=3$  V simulated by two methods, namely the FDTD method and the method of slices. The other solution parameters are  $m=200$ ,  $nt=150$ ,  $C_0=2$  F/m,  $L_0=1$  H/m,  $R_{11}=0.6$   $\Omega$  with  $\sin^2$  input signal of 1 V amplitude and 60 s duration. It is evident that if  $U_p>=4$  V, the agreement of the results is very good. For larger non-linearity the method FDTD (and obviously even the LC ladder network) do not give usable waveforms while the results of the method of slices look quite satisfactory.

Fig. 5 shows the voltage and current waves obtained by the method of slices for  $U_p=1.5$  V at the distance of 70 m with the total line length of 100 m and the terminating resistance  $R_2=2.121$   $\Omega$ . Both forward and backward waves are plotted. The shape of all the shock waves suggests that the classical simulation methods would not offer reasonable results.

## 6. Conclusion

The paper deals with possible ways of simulating wave propagation along a non-linear transmission line. It shows that the classical LC and FDTD methods give valuable results only if the non-linear character of the line is not very distinct and no shock waves are formed. Shock ("tsunami") waves are not unique functions of time or distance and classical mathematical methods fail to produce usable results (the solution oscillates). That is why a new Method of Slices was developed. This method does not suffer from such limitations and is more effective. A certain drawback of the method is that in its present form it cannot solve the problems with lossy lines. On the other hand, when the wave is attenuated its amplitude is low and the non-linear character of the line does not demonstrate itself so markedly.

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## About Author

**Juraj VALSA** was born in 1933. He graduated as MSc. in radio engineering in 1956 and in 1965 obtained the PhD at the Military Academy in Brno, Czechoslovakia. From 1960 to 1971 for total 6 years he taught Circuit Theory at MTC in Cairo, Egypt. Then he worked for 15 years as leader of a research group at the Research Institute for Rotating Electrical Machines. Since 1988 he was assistant professor and since 1991 professor of theoretical electrical engineering at the Brno Technical University. During academic year 1993/94 he acted as visiting professor at the University of Waterloo, Ontario, Canada. He retired in 2003.