DC ANALYSIS OF AN IDEAL DIODE NETWORK USING ITS DECOMPOSED PIECEVISE-LINEAR MODEL

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Abstract

A new method of finding the operating points in circuits containing ideal diodes which utilizes the decomposed form of the state model of an one-dimensional piecewise-linear (PWL) system is developed. The universal procedure shown gives all the existing solutions quite automatically.

Keywords:

DC analysis, ideal diode network, piecewise-linear network

1. Introduction

The process of finding the operating points in networks containing ideal diodes, having a simple rectangular *u-j* characteristic (Fig. 1a), represents a relatively difficult analytical problem. The methods known usually utilize different algorithms for the unique solution and for the multiple solution [1] - [12]. For the existence of the unique solution certain necessary and sufficient conditions [13] must be fulfilled.

In this paper a new method based on the decomposed form of the state model of an one-dimensional piecewise-linear (PWL) system [15], [16] is developed. The main advantage of the corresponding procedure is a relatively simple and universal algorithm generating automatically all the existing solutions without fulfilling any conditions. The network analysed may also include transistors determined by their Ebers-Moll models containing ideal diodes [12].

2. Description of the network

Decomposing the whole network by separating all the *n* diodes as a PWL loading *n*-port and any of the internal dc voltage sources as a power supply E_0 we can form a linear nonreciprocal transformation (*n*+1)-port (Fig. 1b). Its description can be considered for example in the conductance submatrix form

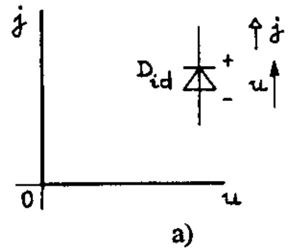
$$i = G_{11}v + G_{12}u + i_0 \qquad (1)$$

$$j = G_{21}v + G_{22}u + j_0 \qquad (2)$$

where the vectors of *internal* variables

$$u = [u_1, \dots, u_n]^T \text{ and } j = [j_1, \dots, j_n]^T$$

represent the ideal diode voltages and currents, respectively.



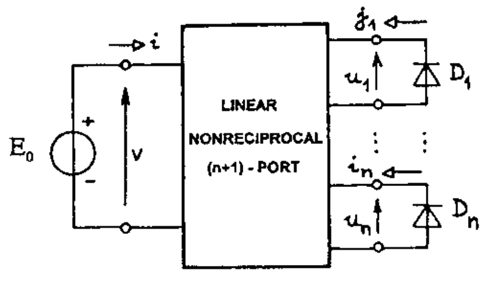


Fig. 1
Characteristic of the ideal diode (a) and the whole network decomposition (b)

The DC supply current *i* and voltage *v* are the *external* variables of the whole system. The PWL block can be described by the so called *linear complementary condition* [14]

$$u \geq 0\ ,\ j \geq 0\ ,\ u^T j = 0 \qquad (3)$$

representing the simple rectangular *u-j* characteristic of the loading ideal diodes. Then eqns (1),(2),(3) correspond to the so called *state model* [14] of an one-dimensional PWL system ($y = i$, $x = v$; u , j - the state vectors) and represent the PWL mapping:

$$R^1 \to R^1 \ , \ x \mapsto f(x) \ ,$$

where

$$G_{11} \in R^{1\times 1}, \quad G_{12} \in R^{1\times n}, \quad i_0 \in R^1 ,$$

$$G_{21} \in R^{n\times 1}, \quad G_{22} \in R^{n\times n}, \quad j_0 = R^n .$$

To obtain all existing sets of u , j for certain $x = v = E_0$ the so called *linear complementary problem* [14] must be solved. As only the one-dimensional PWL system is considered it is convenient to rewrite the basic form (1), (2) of the state model to more suitable *decomposed parametric form* [15] to [19]

$$i = A'u + B'j + f' , \tag{4}$$

$$v = C' + D'j + g' , \tag{5}$$

$$0 = Mu + Nj + q \tag{6}$$

where

$$A' \in R^{1\times n}, \quad B' \in R^{1\times n}, \quad f' \in R^1 ,$$

$$C' \in R^{1\times n}, \quad D' \in R^{1\times n}, \quad g' \in R^1 ,$$

$$M \in R^{(n-1)\times n}, \quad N \in R^{(n-1)\times n}, \quad q \in R^{(n-1)} ,$$

while the linear complementary condition (3) remains the same.

The main difference between the basic and decomposed forms consists in the choice of the independent variables. In the basic conductance form the numbers of the dependent and independent variables (all the currents and voltages, respectively) are equal while in the decomposed form they are different, i.e. the external ones *(i,v)* are considered dependent and the internal ones *(u,j)* independent. This is the reason why eqns (4), (5) express the resultant *i-v* characteristic in the PWL *parametric* form where for each of its segments the corresponding nonzero state variables act as parameters. As the total number of equations (n+1) must remain the same formula (6) represent (n-1) equations containing only state variables u and j . Such a decomposed description provides some new possibilities for the universal approach to the given problem.

To rewrite the basic conductance form (1),(2) into the decomposed one (4),(5),(6) the following simple procedure (easily implementable in PC programme) is recommended:

1. Select from n eqns (2) one eqn having nonzero coefficient by variable v and a minimum number of nonzero coefficients by other variables.
2. Express from this selected equation v as the function of u , j and rewrite it to the form (5).
3. Substitute v into eqn (1) and rewrite it to the form (4).
4. Substitute v into (n-1) equations remaining from n eqns (2) and rewrite them to the form (6).
5. Using the current linear operations simplify these eqns to the form having a minimum number of nonzero coefficients by u and j.

3. Principle of the new analysis procedure

This procedure starts directly from (n-1) eqns (6) which determine the existence of the individual diode-state combinations (the particular states) and their sequence, i.e. *the existence of the individual breakpoints* of the resultant *i-v* characteristic [15], [16]. This step is simplified by the fact that each existing segment of the one-dimensional PWL characteristic can contain maximum two breakpoints so that it is not necessary to verify all the transient states between two adjoining diode-state combinations. The algorithm utilizing the negative delimitation has been described in [17].

The corresponding values of diode voltages u and currents j in the considered points can also easily be calculated from eqns (6). Substituting them into eqns (4),(5) the co-ordinates of these points are obtained so that for the external port the shape of its *i-v* characteristic is determined. The intersections of the vertical line $v = E_0$ and the corresponding segments represent *the dc operating points* of the analysed network (Fig. 2). Then the uniqueness or the multiplicity of dc solution is directly given by the shape of the resultant PWL characteristic.

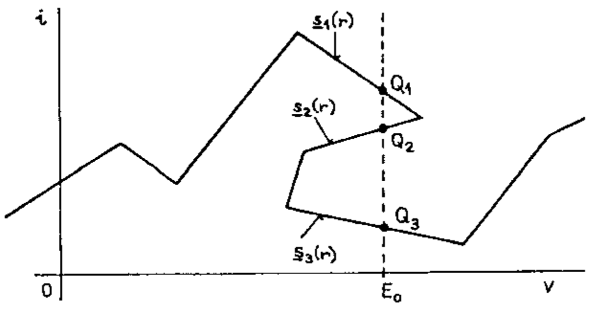


Fig. 2
Resultant PWL characteristic of the network and its DC operating points

In the last step the values of diode voltages and currents for all these operating points are calculated. Each of the related segments corresponds to a certain particular diode-state combination represented by state vector [14]

$$s_p(r) = [s(r)] \ , \quad r = 1, \dots , n$$

where

$$s(r)=0 \;\rightarrow\; u_r=0\;,\; j_r\geq 0\;,$$

$$s(r)=1 \;\rightarrow\; u_r\geq 0\;,\; j_r=0\;.$$

Then the corresponding *nonzero* diode voltages and currents can be expressed as:

$$s_p 1 u \quad \text{and} \quad (1-s_p 1)j \tag{7}$$

and the remaining state variables for this particular state *equal zero*, i.e.

$$(1-s_p 1)u=0 \quad \text{and} \quad s_p 1 j=0 \tag{8}$$

where 1 is the unity matrix. In order to cancel additional zero rows and columns in eqns (7),(8) diagonal matrices $s_p 1$ and $(1-s_p 1)$ can be decomposed to the product of two oblong matrices

$$s_p 1 = P_p\, {}^T P_p \quad \text{and} \quad (1-s_p 1)=R_p\, {}^T R_p\;, \tag{9}$$

respectively. Denoting p the number of nonzero diode voltages for this particular state the individual oblong matrices then are

$$P_p \in R^{n\times p}\;,\; {}^T P_p \in R^{p\times n}\;, R_p \in R^{n\times(n-p)}\;,\; {}^T R_p \in R^{(n-p)\times n}\;.$$

The corresponding nonzero and zero state variables are

$$u_p = {}^T P_p u > 0\;,\; j_p = {}^T R_p j > 0\;, \tag{10}$$

and

$$u_{p0} = {}^T R_p u = 0\;,\; j_{p0} = {}^T P_p j = 0\;, \tag{11}$$

respectively. Substituting $v=E_0$ into eqn (2), and expressions (7),(8) into eqns. (2),(3), we obtain a set of n linear equations for n nonzero state variables u_p, j_p containing only the related columns of the individual matrices, i.e.

$$\begin{bmatrix} C'P_p & D'R_p \\ MP_p & NR_p \end{bmatrix} \begin{bmatrix} {}^T P_p u \\ {}^T R_p j \end{bmatrix} = -\begin{bmatrix} g' \\ q \end{bmatrix}. \tag{12}$$

The procedure suggested must be repeated for each of the related particular state vectors (e.g. in Fig. 2 - $s_1(r)$, $s_2(r)$, $s_3(r)$).

The corresponding algorithm can be divided to the following steps:

1. Derive by the simplified systematic analysis of eqns (6) - [17], [18]:
   - the existence of all the individual diode-state combinations, i.e. all particular state vectors $s_p(r)$ of the given PWL system,
   - the sequence of these particular states, i.e. the existence of the individual breakpoints of the resultant PWL characteristic,
   - the corresponding values of diode voltages u and currents in each of these points.
2. Substitute them into eqn (5) and calculate the voltage coordinates of the PWL characteristic individual breakpoints.
3. Select only the related states (segments of the resultant PWL characteristic), i.e. where the value of supply voltage E_0 is between the voltage co-ordinates of their adjacent breakpoints.
4. Calculate for each selected particular state (determined by state vector s_p):
   - mutually transposed oblong matrices according eqn (9),
   - corresponding n zero state variables according eqn (11),
   - corresponding n nonzero state variables as the solution of n eqns (12).

4. Conclusion

The new method of DC analysis of networks containing ideal diodes evidently relates with the so called *linear complementary problem* of the one-dimensional PWL system [14] and represents one method of its solution which can easily be extended also for the multi-dimensional cases. The analytical part of the PC programme for the automatic design of the decomposed state model of an one-dimensional PWL system [17] has been modified for this purpose and the computing time is comparable with that in other methods. The main advantage of the algorithms described is their universality.

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