A Simple Generator of Forward Scattering Functions on Spherical Dielectrics

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Abstract

The described program generates the forward scattering functions of dielectrics of spherical shape, while the input parameters are: frequency, radius of the sphere and complex refractive index. The part enabling to evaluate the complex refractive index of water in the dependence on frequency and temperature is added.

1. Introduction

It is usefull to have a program able to generate forward scattering functions of dielectrical spheres, which allows us to describe, for example, the interaction of electromagnetic wave with the rain medium. Using the program described below to calculate the specific rain attenuation we have to take into account that it serves only as an approximation, due to the fact that rain drops are not exactly spherical.

The algorithm used is based on Mie's scattering theory [1]. This mathematical description requires to evaluate the Bessel's and Hankel's functions as well as Legendre's polynoms. In simplifying this problem it was found, that Deirmendjian in [2] simplified the generation of forward scattering functions using only much more simple complex sinus, cosinus and cotangent functions.

2. definition of the scattering function

The forward scattering function $\hat{S}$ is defined by Deirmendjian through the expression:

$$\hat{E}_s = \hat{E}_i \hat{S}(x, \hat{m}) \frac{e^{-jkr}}{jkr} \tag{1}$$

where:

$\hat{E}_s$, $\hat{E}_i$ is the scattered (incident) electrical intensity

$x = \frac{2\pi a}{\lambda}$ is a parameter

a is the drop radius

λ is the wave length

m is the complex refractive indices

k is the wave number

3. Algorithm

$$\hat{S} = \frac{1}{2}\sum_{n=1}^{\infty} (2n+1)\left(\hat{a}_n + \hat{b}_n\right) \tag{2}$$

$$\hat{a}_n = \frac{\left[\frac{\hat{A}_n}{\hat{m}} + \frac{n}{x}\right] \mathrm{Re}\hat{O}_n - \mathrm{Re}\hat{O}_{n-1}}{\left[\frac{\hat{A}_n}{\hat{m}} + \frac{n}{x}\right]\hat{O}_n - \hat{O}_{n-1}} \tag{3}$$

$$\hat{b}_n = \frac{\left[\hat{m}\hat{A}_n + \frac{n}{x}\right] \mathrm{Re}\hat{O}_n - \mathrm{Re}\hat{O}_{n-1}}{\left[\hat{m}\hat{A}_n + \frac{n}{x}\right]\hat{O}_n - \hat{O}_{n-1}} \tag{4}$$

$$\hat{A}_n = -\frac{n}{\hat{y}} + \left[\frac{n}{\hat{y}} - \hat{A}_{n-1}\right]^{-1} \tag{5}$$

$$\hat{A}_0 = cotg\,\hat{y} \tag{6}$$

$$\hat{y} = \hat{m}x \tag{7}$$

$$\hat{O}_n = \frac{2n-1}{x}\hat{O}_{n-1} - \hat{O}_{n-2} \tag{8}$$

$$\hat{O}_0 = \sin x + j\cos x \tag{9}$$

$$\hat{O}_{-1} = \cos x - j\sin x \tag{10}$$

In the described program formulas were used which were published in [3] to evaluate $\hat{m}$ (the complex refractive index), where the temperature and frequency occur as parameters. The author supposes that the formulas are valid for temperatures from −20 °C to 50 °C in the case of water and for frequencies between 1 MHz and 150 GHz.

We have found, that due to the strongly decreasing argument of the series (2) with n increasing it is sufficient to limit the evaluation of this sum for n from 1 to 5.

4. Example

The example shows (besides the scattering functions depending on the radius of the sphere) also the evaluation of specific rain attenuation in the case of propagation of radiowave on the frequency 12.0 GHz through the medium with rain of intensity 5 mm/h.

The formula to compute the specific rain attenuation s.a. may be expressed as follows:

$$s.a. = 8.6859 \, . \, 10^5 \frac{\lambda^2}{2\pi} \int_0^\infty \mathrm{Re}\, \hat{S} N(r) dr \qquad \left[\frac{dB}{km}\right] \tag{11}$$

where wave length is in cm and $N(r)$ is the drop spectrum in cm^{-4} depending on the rain intensity R [4].

SCATTERING FUNCTIONS of DIELECTRICAL SPHERE:

Frequency = 12.0 GHz
Rain intensity = 5.0 mm/h
Temperature = 20.0 Centigrade
Refractive Index = 7.743613 +j 2.302602

RADIUS	REAL	IMAGINARY
[cm]	-	-
0.025000	0.000007	-0.000241
0.050000	0.000095	-0.001987
0.075000	0.000615	-0.007053
0.100000	0.003011	-0.017778
0.125000	0.011921	-0.035324
0.150000	0.030522	-0.051873
0.175000	0.045694	-0.067331
0.200000	0.062697	-0.096187
0.225000	0.091565	-0.134367
0.250000	0.132723	-0.179261
0.275000	0.191339	-0.230132
0.300000	0.272025	-0.279185
0.325000	0.372247	-0.316508
0.350000	0.483353	-0.335899
0.375000	0.594887	-0.338691

Specific Attenuation = 0.13 dB/km

5. Conclusion

The method, enabling to evaluate the forward scattering functions of dielectrical spheres by very easy way, was shown. The results were successfully compared with published tables. The described program can be used, for instance, in order to evaluate the specific rain attenuation and to study it's dependence on the temperature as well as on the rain intensity or on the rain drop spectrum. It is not possible to study the depolarization by the described method.

REFERENCES:

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[2] Deirmendjian D.: Electromagnetic scattering on spherical polydispersions. Elsevier Publ.Comp.,INC.New York, 1966

[3] Ray S.P.: Broadband Complex Refractive Indices of Ice and Water. Applied Optics, 11, 8, p. 1836, 1972

[4] Marshall J.S., Palmer W: The distribution of raindrops with size. J. Meteorol., 5, pp 165-166, 1948

Coming Events

Czechoslovakia Section of IEEE is proud to invite you to a lecture given on May, 21. by

Ferdo Ivanek,

IEEE-MTT distinguished lecturer

The lecture title is

Progress and Change in Microwave Radio Communications.

The lecture is scheduled from 10.00 am.
(Czech Technical University, Faculty of Electrical Eng., Room 80).