CALIBRATING THE SIX-PORT REFLECTOMETER USING A MATCHED LOAD AND FOUR UNITY-REFLECTION STANDARDS

PART 2: SLIDING TERMINATION

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Abstract
A convenient and exact method of using a sliding load to the calibration of the single six-port reflectometer is described. Neither reflection coefficient nor positions of the sliding load settings need be known. Reflection coefficient of the load is not limited to small values. No numerical iterations are involved in the calibration.

Keywords:
Six-port reflectometer, calibration

1. Introduction

For accurate measurement of low reflection coefficients with six-port reflectometers, a matched termination should be used as one of the calibration standards. The measurement accuracy in the vicinity of Smith chart center is then limited mainly by the residual reflection of the termination. To reduce this error, a sliding matched load can be used. Existing methods employing the sliding load have certain limitations; they are briefly reviewed in the next section. Following this, a new procedure is introduced which is both more practical and free of limitations of the preceding methods.

The method is an extension to and is based on the calibration procedure described in [1], which is a prerequisite to studying the present theory. Most of the quantities used here are defined in [1]. The formulas of the paper [1] will be referred to by their respective numbers preceded by "1-", e.g. "Eq. (1-4)".

2. Existing approach

The sliding load technique is currently based on the fact that when a load is slid, the power readings $P_i$ ($i = 1, 2, 3, 4$) of the four detectors vary around the values $P_o$, corresponding to the perfectly matched load. The values $P_o$ can be determined e.g. by moving the load and observing for the minimum and maximum readings of the four detectors (Method I). A more practical procedure (Method II) has been suggested by Somlo [2] using only three sliding load settings, however with known relative positions. Assuming the reference port being $i = 4$, the normalized powers corresponding to the perfectly matched load are then

$$P_{oi} = \frac{P_{oi}}{P_{o4}} \quad i = 1, 2, 3$$ (1)

The methods based on the above techniques have certain limitations. Method I is tedious. An inconvenience of Method II from operator’s point of view is that the sliding load must be set to known positions. Method II is not theoretically exact: it is not suitable for loads with higher residual reflections. Generally, it is not possible to carry out the procedures of either of the methods with normalized powers $P_i = P_i / P_4$. While the normalized powers also vary with the load position, their mean values are not equal to the normalized powers $P_{oi}$ corresponding to a perfectly matched load. As a consequence, the accuracy of determining $P_{oi}$ may be affected by the signal source amplitude instability.

The present paper describes a procedure that, based on a different approach, is free of all of the above limitations. The method is not restricted to low reflection sliding terminations. It requires only normalized powers to be measured for each load setting. It is therefore insensitive to input power variations. The load settings (min. three) can be arbitrary and unknown. Hence, the sliding load is used with the same convenience and ease as in conventional four-port network analyzers.

The outline of the proposed method is as follows: As the first step, the calibration according to [1] (using a matched load and four unity-reflection standards) is performed, taking one position of the sliding load as the perfectly matched load. This results in incorrect, "biased" calibration constants. With such biased calibration and the stored normalized powers, the reflection coefficients of all of the sliding load settings are computed. The information is sufficient to correct the calibration constants.
2. Transformation of reflection coefficient by imperfect calibration

In this section, a general relation is derived defining the transformation between the actual reflection coefficient of a device under test (DUT) and the measured value when an incorrect calibration matrix is used. Analyzing the properties of the transformation for the case of calibration with an imperfect matched load standard, formulas are derived which will, in the following sections, enable to correct the calibration matrix.

If the actual reflection coefficients of the calibration standards differ from their supposed values the calibration will result in an incorrect - "biased" calibration matrix $D^b$ (superscript $b$ will be henceforth added to all such biased quantities). If $D^b$ is used in the measurement of a DUT with the actual reflection coefficient $\Gamma = x + jy$, the result will be an incorrect - measured reflection coefficient $\Gamma_m = x_m + jy_m$ which is given by the formula $w R^m = D^b P$ analogous to (1-4), i.e.

$$
\begin{bmatrix}
1 \\
x_m \\
y_m \\
M 
\end{bmatrix} =
\begin{bmatrix}
D^b \\
\end{bmatrix}
\begin{bmatrix}
p_1 \\
p_2 \\
p_3 \\
1 
\end{bmatrix}
$$

(2)

where $p_i$ are normalized powers corresponding to the actual reflection coefficient $\Gamma$. They are given by the correct (so far unknown) matrix $C$ from (1-1) or (1-2). Substituting (1-2) in (2) leads to the equation

$$
R^m = \frac{d}{w} D^b C R
$$

(3)

defining the transformation between the actual (contained in the column $R$) and measured (contained in $R^m$) reflection coefficients. Defining the transformation matrix as

$$
T = D^b C
$$

(4)

Eq. (3) becomes $R^m = \frac{d}{w} T R$, i.e.

$$
\begin{bmatrix}
1 \\
x_m \\
y_m \\
M 
\end{bmatrix} =
\begin{bmatrix}
t_{11} & t_{12} & t_{13} & t_{14} \\
t_{21} & t_{22} & t_{23} & t_{24} \\
t_{31} & t_{32} & t_{33} & t_{34} \\
t_{41} & t_{42} & t_{43} & t_{44} 
\end{bmatrix}
\begin{bmatrix}
1 \\
x \\
y \\
\rho^2 
\end{bmatrix}
$$

(5)

Note that while $\rho^2 = x^2 + y^2$, the corresponding relation for $M$ need not be valid: in general, $M \neq x_m^2 + y_m^2$. Similarly, $w \neq d$.

Next, let us consider a particular case of biasing the calibration which arises when an imperfect load

$$
\Gamma_L = x_L + jy_L = r_L \exp(j\varphi_L)
$$

(6)

is connected instead of the matched termination $\Gamma = 0$. (In the proposed method, $\Gamma_i$ is the reflection coefficient of one of the sliding load settings). Following the lines of the calibration described in [1], the scaling factors now will be

$$
Y_i = \left[p_i\right]_\Gamma = \Gamma_L \\
i = 1, 2, 3
$$

(7)

and will differ from the so far unknown true scaling factors

$$
Y_i = \left[p_i\right]_\Gamma = 0 \\
i = 1, 2, 3
$$

(8)

The substitution of (6) and (7) in the right hand and left hand columns of (1-1), respectively, leads to the relation

$$
Y_i = Y_i \epsilon_i \\
i = 1, 2, 3
$$

(9)

where

$$
\epsilon_i = \\
\frac{1 + 2x_i x_L - 2y_i y_L + z_i r_L^2}{1 + 2x_i x_L - 2y_i y_L + z_i r_L^2} \\
i = 1, 2, 3
$$

are error factors ($\epsilon_i = 1$ if $\Gamma_L = 0$). The biased scaling factors $Y_i^b$ enter the further calibration process. In the first place, they affect the quantities $Q_{ij}$ defined in Section 3.2 of [1], modifying them to $Q_{ij}^b = Q_{ij} \epsilon_i$. The quantities $Q_{ij}^b$ in turn enter Eq. (1-9) but their effect is manifested by merely multiplying Eq. (1-9) by a constant factor $e_i$. The solution to (1-11) will therefore not be affected. Consequently, the quantities $x_n, y_n$ characterizing the reference port, hence 4th row of matrices $B$ and $C$, will be determined correctly.

$Q_{ij}^b$ also enter the quantities $W_{ij}$ defined by (1-13), modifying them to $W_{ij}^b = W_{ij} \epsilon_i$ and, consequently, the solution of (1-12), (1-14) to

$$
x_i^b = \frac{x_i}{\epsilon_i} \\
y_i^b = \frac{y_i}{\epsilon_i} \\
z_i^b = \frac{z_i + 1}{\epsilon_i} - 1
$$

(10)

The proof can be made directly by substituting (10) in (1-12). The matrix $C^b = Y^b B^b$, with $B$ defined by (1-1) and (1-2), becomes

$$
C^b = \\
\begin{bmatrix}
Y_1 \epsilon_1 & 2Y_1 x_1 - 2Y_1 y_1 & Y_1 [z_1 + (1-\epsilon_1)] \\
Y_2 \epsilon_2 & 2Y_2 x_1 - 2Y_2 y_2 & Y_2 [z_2 + (1-\epsilon_2)] \\
Y_3 \epsilon_3 & 2Y_3 x_3 - 2Y_3 y_3 & Y_3 [z_3 + (1-\epsilon_3)] \\
1 & 2x_4 & -2y_4 & z_4 
\end{bmatrix}
$$

(11)
Note that 4th row as well as 2nd and 3rd columns of $T$ remained correct.

Now, the effect of imperfect matched load has been established. It can be shown (Appendix 1) that the transformation matrix $T$ assumes the form

$$
T = \begin{bmatrix}
  t_{11} & 0 & 0 & 1-t_{11} \\
  t_{21} & 1 & 0 & -t_{21} \\
  t_{31} & 0 & 1 & -t_{31} \\
  t_{41} & 0 & 0 & 1-t_{41}
\end{bmatrix}
$$

(12)

As seen, $T$ has only four independent elements. The next procedure will be as follows: In Section 3, the properties of the transformation (3) through the matrix (12) will be examined, yielding basis for both obtaining $t_0$ and correcting the calibration data. In Section 4, the elements of matrix $T$ are obtained from the sliding load data. Finally, in Section 5, the error factors $e_i$ are evaluated and the correction of the biased calibration matrices is undertaken.

3. Properties of transformation

Substituting (12) in (5), the relation between the actual and measured reflection coefficients is obtained

$$
\begin{bmatrix}
  1 \\
  x_m \\
  y_m \\
  M
\end{bmatrix}
= \begin{bmatrix}
  1-t_{11} & 0 & 0 \\
  t_{21} & 1 & 0 \\
  t_{31} & 0 & 1 \\
  t_{41} & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
  1 \\
  x \\
  y \\
  r^2
\end{bmatrix}
$$

(13)

The transformation has the following properties:

1. The actual reflection coefficient $\Gamma = x + iy = r e^{i\phi}$ is measured as $\Gamma_m = x_m + iy_m$ where

$$
x_m = \frac{x}{U} + \frac{t_{11}(1 - r^2)}{U}
$$

(14)

$$
y_m = \frac{y}{U} + \frac{t_{31}(1 - r^2)}{U}
$$

(15)

$$
U = \frac{w}{d} = t_{11}(1 - r^2) - r^2
$$

(16)

The quantity $M$ is given by

$$
M = \frac{t_{11} + (1 - t_{11})r^2}{U}
$$

(17)

2. A circle $\Gamma = r e^{i\phi}, \phi \in (-\pi, \pi)$ is transformed in the circle $\Gamma_m = \Gamma_e + R e^{i\phi}$. Its radius is

$$
R = \frac{r}{U}
$$

(18)

and its center $\Gamma_e = x_e + iy_e + R e^{i\phi_e}$ where

$$
x_e = \frac{t_{21}(1 - r^2)}{U}
$$

(19)

$$
y_e = \frac{t_{31}(1 - r^2)}{U}
$$

(20)

![Fig. 1](image)

Transformation of reflection coefficient from true to measured value by a calibration biased by imperfect matched load standard.

Apart from the change in radius, the circle transformation is of a translation type (Fig. 1), i.e., the phase angle with respect to the circle center remains unchanged:

$$
\phi = \text{arg}(\Gamma) = \text{arg}(\Gamma_m - \Gamma_e)
$$

(21)

3. The ideal matched load $\Gamma = 0$ is measured as $\Gamma_o = x_o + iy_o$ where

$$
x_o = \frac{t_{11}}{t_{11}}
$$

(22)

$$
y_o = \frac{t_{31}}{t_{11}}
$$

and the quantity $M_o$ is

$$
M_o = \frac{t_{41}}{t_{11}}
$$

(23)

This is an important point because the knowledge of $x_o, y_o, M_o$ enables us to find the error factors $e_i$ (see Section 5) and in turn correct the calibration matrix. Hence, the elements of matrix $T$ must be obtained. It will be accomplished in Section 4 using the sliding load data and the properties 1 and 2 of the transformation.
4. Elements of transformation matrix

Let the actual reflection coefficients of the calibration sliding load at its various \( n \geq 3 \) settings be

\[
\Gamma_k = r_k \exp(i\varphi_k) \quad k = 1, 2, \ldots n
\]

(the one used in determining \( Y'_c \) has been denoted \( \Gamma_c \) in Section 2). We assume that neither \( \Gamma_k \) nor \( \varphi_k \) are known. The values measured, i.e., those computed from (2) using the biased calibration matrix \( D^b \), are

\[
\Gamma_{mk} = x_{mk} + jy_{mk} \quad k = 1, 2, \ldots n
\]

If a circle is fitted [3] to the points \( \Gamma_{mk} \) (hence at least three load settings are necessary), its radius and center coordinates will have to satisfy (18) to (20) with \( r_k \) substituted for \( r \) and, from (16),

\[
U = t_{11} \left( 1 - \frac{r_c^2}{r_k^2} \right) - \frac{r_c^2}{r_k^2} \tag{24}
\]

Since the reflection coefficient \( \Gamma_L \) will be measured as a perfect match, the quantity \( M \) as calculated from (2) will be zero (a derivation see in Appendix 2). Then from (17)

\[
t_{11} \left( 1 - \frac{r_c^2}{r_k^2} \right) + \frac{r_c^2}{r_k^2} = 0 \tag{25}
\]

Eqs. (18) - (20), (24), (25) represent a set of five equations for six unknowns: \( t_{11}, t_{12}, t_{13}, t_{14}, r_c, U \). One more equation is obtained from the definition \( U = w/d \) in (16). Appendix 2 shows that \( w = 1 \) for \( \Gamma = \Gamma_L \). Using Eq. (1-3) for \( d \) then results in

\[
U = 1 + 2x_r r_c - 2y_r y_c + z_r^2
\]

Noting that \( 2x_r, -2y_r, z_r \) are 4th row elements of either matrices \( C \) and \( C^b \) and expanding \( x_r, y_r \) in terms of \( r_c, y_c \) yields

\[
U = 1 + C_{a1} r_c \cos(\varphi_c) + C_{a2} r_c \sin(\varphi_c) + C_{a4} r_c^2
\]

Argument \( \varphi_c \) can be expressed in terms of the fitted circle parameters using (21) with \( \Gamma_m = 0 \):

\[
\varphi_c = \arg(\Gamma_m - \Gamma_c) = \arg(-\Gamma_c) = \varphi_c - \pi
\]

Hence

\[
\cos(\varphi_c) = -\cos(\varphi_c) = -x_r/r_c,
\]

\[
\sin(\varphi_c) = -\sin(\varphi_c) = -y_r/r_c
\]

and

\[
U = 1 - \left( c_{a1} x_r + c_{a4} y_r \right) \frac{r_c}{R} + c_{a4} r_c^2 \tag{26}
\]

Eqs. (18) - (20), (24) - (26) are a complete set of equations to obtain all the unknown quantities. Eliminating \( U \) from (18) and (26), \( r_c \) is obtained as

\[
r_c = \frac{1}{\sqrt{C_{a4} \left[ h + \sqrt{h^2 - 1} \right]}} \tag{27}
\]

where

\[
h = \frac{1}{2R \sqrt{C_{a4}}} \left( 1 + c_{a1} x_r + c_{a4} y_r \right) \tag{28}
\]

(the proof see in Appendix 3). Then from (18) - (20), (24) - (25) we have straightforwardly

\[
t_{11} = \frac{r_c}{1 - r_c^2} \frac{1 - r_c R}{1 - r_c^2} \quad t_{12} = \frac{r_c}{R} \frac{x_r}{1 - r_c^2}
\]

\[
t_{13} = \frac{r_c}{1 - r_c^2} \frac{y_r}{1 - r_c^2} \quad t_{14} = \frac{r_c^2}{1 - r_c^2}
\]

At this point, our immediate goal has been achieved: the elements of the transformation matrix \( T \) have been obtained in terms of the radius \( R \) and center coordinates \( x_r, y_r \) of the circle fitted to the measured points corresponding to various arbitrary unknown settings of a sliding termination. The correction of the biased calibration constants is now enabled.

5. Correction

Substituting (29) to (22), (23), one obtains

\[
\begin{align*}
x_o &= \frac{x_r}{1 - r_c R} \\
y_o &= \frac{y_r}{1 - r_c R} \\
M_o &= \frac{r_c}{1 - r_c R}
\end{align*}
\]

Since these values represent the transform of the perfectly matched load by the biased calibration, Eq. (2) can be used to calculate the normalized powers corresponding to \( \Gamma = 0 \). According to (8), these normalized powers are equal to the correct scaling factors \( Y_i \). Hence, (2) can be written as

\[
\begin{bmatrix}
1 \\
x_o \\
y_o \\
M_o
\end{bmatrix} =
\begin{bmatrix}
\mathbf{D}^b
\end{bmatrix} \cdot
\begin{bmatrix}
Y_1 \\
Y_2 \\
Y_3 \\
1
\end{bmatrix}
\]
Left-multiplying it by \((D^b)^{-1}\) yields the following equation for \(Y_f\):

\[
\begin{bmatrix}
Y_1 \\
Y_2 \\
Y_3 \\
1
\end{bmatrix} = w \begin{bmatrix}
Y_1^b \\
Y_2^b \\
Y_3^b \\
1
\end{bmatrix} \begin{bmatrix}
x_o \\
y_o \\
M_0
\end{bmatrix}
\]

All matrices on the right-hand side are known; \(w\) is determined from the 4th row. The error factors \(e_i\) defined by (9), are then

\[
e_i = \frac{Y_i}{Y_1} = \frac{B_{11}^b + B_{12}^b x_o + B_{13}^b y_o + B_{14}^b M_0}{B_{11}^b + B_{21}^b x_o + B_{31}^b y_o + B_{41}^b M_0} \quad (31)
\]

The elements of \(C^b\) can be corrected by removing \(e_i\) from (11); the result is given in Appendix I, viz. Eq. (A2). Now the matrix \(C\) can be inverted to obtain the final correct calibration matrix \(D\). At this point, the calibration is completed.

Extensive experimental experience with an X-band waveguide six-port reflectometer [4] confirmed the usefulness of the method. Its effect is illustrated in Fig. 2. Figure 2 represents the measurement at \(f = 10\) GHz of a good quality R-100 waveguide sliding load taking one of its settings for the perfectly matched load. The circle is shifted from the origin by the systematic error which is approximately \(6.1 \cdot 10^{-1}\) in magnitude. This gives the system the effective directivity of \(44\) dB.

Figure 2 shows the measurement of the same load (at approximately the same settings) after the calibration described in this paper has been applied. The circle center is now offset by merely \(6.5 \cdot 10^{-1}\) which corresponds to the effective directivity of \(63\) dB. The procedure thus substantially improved the measurement accuracy.

6. Conclusions

A new procedure for calibrating the six-port reflectometer has been developed which uses a sliding termination as conveniently as in the conventional four-port network analyzers. The calibration procedure can be summed up as follows:

1. Measure and store normalized powers for 3 or more arbitrary positions of a sliding load (well spaced within half-wavelength) and four unity-reflection standards.
2. Perform the calibration [1] using the normalized powers for one (say the first) position of the sliding load as if it were a perfectly matched load.
3. Using the obtained (biased) calibration matrix, compute reflection coefficients of all the sliding load settings.
4. Fit a circle to these points; find its center coordinates \(x_o, y_o\) and radius \(R\).
5. Compute modulus \(r_i\) of the sliding load reflection coefficient from (27) and (28).
6. Compute \(x_o, y_o, M_0\) from (30). Compute \(e_i\) from (31).
7. Compute the correct matrix \(C\) using (A2). Invert \(C\) to obtain the correct calibration matrix \(D\). The calibration is completed.

Appendix 1: A form of transformation matrix

The transformation matrix is defined by (4) as

\[
T = D^b \cdot C
\]

where \(D^b\) is the biased calibration matrix and \(C\) is the true, so far unknown matrix defined in (1-2):

\[
C = Y B = \begin{bmatrix}
Y_1 & 2Y_1 x_1 & -2Y_1 y_1 & Y_1 z_1 \\
Y_2 & 2Y_2 x_2 & -2Y_2 y_2 & Y_2 z_2 \\
Y_3 & 2Y_3 x_3 & -2Y_3 y_3 & Y_3 z_3 \\
1 & 2x_4 & -2y_4 & z_4
\end{bmatrix} \quad (A1)
\]

Comparing (11) with (A1), the elements \(C_{ij}\) of matrix \(C\) can be expressed in terms of the elements \(C_{ij}^b\) of the biased matrix \(C^b\):

\[
C = \begin{bmatrix}
\frac{C_{11}^b}{e_1} & \frac{C_{12}^b}{e_1} & \frac{C_{13}^b}{e_1} & \frac{C_{14}^b}{e_1} & \frac{C_{15}^b}{e_1} \\
\frac{C_{21}^b}{e_2} & \frac{C_{22}^b}{e_2} & \frac{C_{23}^b}{e_2} & \frac{C_{24}^b}{e_2} & \frac{C_{25}^b}{e_2} \\
\frac{C_{31}^b}{e_3} & \frac{C_{32}^b}{e_3} & \frac{C_{33}^b}{e_3} & \frac{C_{34}^b}{e_3} & \frac{C_{35}^b}{e_3} \\
\frac{C_{41}^b}{e_4} & \frac{C_{42}^b}{e_4} & \frac{C_{43}^b}{e_4} & \frac{C_{44}^b}{e_4} & \frac{C_{45}^b}{e_4}
\end{bmatrix}
\]

(A2)
Eq. (4) then becomes the product

\[
T = \begin{bmatrix}
D_1^b & D_2^b & D_3^b & D_4^b \\
D_{11}^b & D_{12}^b & D_{13}^b & D_{14}^b \\
D_{21}^b & D_{22}^b & D_{23}^b & D_{24}^b \\
D_{31}^b & D_{32}^b & D_{33}^b & D_{34}^b \\
D_{41}^b & D_{42}^b & D_{43}^b & D_{44}^b \\
\end{bmatrix}
\times
\begin{bmatrix}
C_{11}^b & C_{12}^b & C_{13}^b & C_{14}^b - C_{11}^b \\
C_{21}^b & C_{22}^b & C_{23}^b & C_{24}^b - C_{21}^b \\
C_{31}^b & C_{32}^b & C_{33}^b & C_{34}^b - C_{31}^b \\
C_{41}^b & C_{42}^b & C_{43}^b & C_{44}^b - C_{41}^b \\
\end{bmatrix}
\]

As seen, 2nd and 3rd columns of matrix \(C\) are equal to those of matrix \(C^b\), which is inverse to \(D^b\); 2nd and 3rd columns of matrix \(T\) must therefore be part of the unit matrix, cf. (12). The first column of matrix \(C\) differs from that of matrix \(C^b\); the elements of 1st column of matrix \(T\) are therefore general quantities denoted \(t_{11}^b, t_{21}^b, t_{31}^b, t_{41}^b\).

The fourth column \(C_{44}\) of matrix \(C\) can be looked upon as a sum of 3 terms: \(C_{44} = C_{44}^b + C_{41} - C_{14}\). The first and second terms are columns of the biased matrix \(C^b\) and hence will contribute by the respective columns of the unit matrix. The third term is equal to 1st column of matrix \(C\) and will contribute the same way as to 1st column. The fourth column of matrix \(T\) therefore is

\[
T_{44} = D^b C_{44} = D^b C_{44}^b + D^b C_{41} - D^b C_{14}
\]

and, after evaluating,

\[
T_{44} = \begin{bmatrix}
t_{41}^b
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 1 & 0 & -1 & t_{11}^b
\end{bmatrix} = \begin{bmatrix}
1 - t_{11}^b
\end{bmatrix}
\]

Consequently, matrix \(T\) has the form of (12).

### Appendix 2: Measurement of the calibration load

The purpose of this appendix is to calculate, using Eq. (2), the quantities \(w\) and \(M\) for the case when the load with reflection coefficient \(\Gamma_L\) is connected as DUT. Since \(\Gamma_L\) is the reflection coefficient of the sliding load setting which was used in place of a matched load, the normalized powers \(P_1\) are according to (7) equal to the biased scaling factors \(Y_1^b = Y e_\phi\). Confronting this fact with (11), it is seen that the column \(P\) of powers is equal to 1st column \(C_{11}\) of matrix \(C^b = (D^b)^{-1}\). The product of \(D^b\) with \(P\) yields therefore 1st column of the unit matrix so that

\[
\begin{bmatrix}
1 \\
\vdots \\
1 \\
\end{bmatrix}
= \begin{bmatrix}
D^b \\
\vdots \\
D^b \\
\end{bmatrix}
\begin{bmatrix}
C_{11}^b \\
C_{21}^b \\
C_{31}^b \\
C_{41}^b \\
\end{bmatrix}
= \begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

Consequently \(x_m = 0, y_m = 0\) (which was expected),

\[
w = 1 \quad (A3)
\]

\[
M = 0 \quad (A4)
\]

### Appendix 3: Solving for \(r_L\)

Eliminating \(U\) from (18) and (26) with \(r = r_L\) in (18), the equation is obtained

\[
s + \frac{1}{s} = 2h \quad (A5)
\]

where

\[
s = r_L \sqrt{C_{44}}
\]

\[
h = \frac{1}{2 \sqrt{C_{44}}} \left(1 + C_{42} x_c + C_{43} y_c\right) \quad (A6)
\]

Eq. (A5) is a quadratic equation which has two mutually reciprocal solutions \(s_1, s_2 = 1/s_1\). In practical situation \(R << 1\) (low reflection sliding load) and \(|C_{44}| << 1\) (a good directivity reference port), hence \(h\) is a large positive quantity, \(h >> 1\). The solutions are therefore positive, one being small

\[
s_1 = \frac{1}{h + \sqrt{h^2 - 1}} \quad (A7)
\]

the other, \(s_2 = 1/s_1\), large. If \(R\) decreases to zero, \(h\) increases, and \(s_1\) (hence \(r_L\)) decreases, which corresponds to the physical situation. On the other hand, the second root \(s_2\) increases, which is contrary to the physical reality. Consequently, only \(s_1\) is a physically acceptable solution. Substituting (A6) to (A7), the final expression (27) is obtained.

### References


Calibrating the Six-Port Reflectometer ..., Part 1: Perfectly Matched Load

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A printing error occurred in Eq. (14) of the paper [1]. The formula should correctly read as follows:

\[
\begin{bmatrix}
  c_1 & s_1 & 1 \\
  c_2 & s_2 & 1 \\
  c_3 & s_3 & 1 \\
  c_4 & s_4 & 1
\end{bmatrix}
\begin{bmatrix}
  2x_1 & 2x_2 & 2x_3 \\
  -2y_1 & -2y_2 & -2y_3 \\
  z_1 & z_2 & z_3
\end{bmatrix}
= 
\begin{bmatrix}
  W_{11}-1 & W_{12}-1 & W_{13}-1 \\
  W_{21}-1 & W_{22}-1 & W_{23}-1 \\
  W_{31}-1 & W_{32}-1 & W_{33}-1 \\
  W_{41}-1 & W_{42}-1 & W_{43}-1
\end{bmatrix}
\]  

(14)

6th International Scientific Conference of Electrotechnical Faculty of Technical University of Košice

The 6th International Scientific Conference of Electrotechnical Faculty of TU of Košice was held at The TU Košice, in September 14-16, 1992.

Since 1982, this conference has taken place every two year, with guests from many countries over the world of a scientific exchange of experiences and also of contacting representatives of industry and scientific institutions. The program of the conference was intent in the following sections:

- Electronic computers and informatics
- Electroenergetics
- Cybernetics and artificial intelligence
- Physics
- Mathematics
- Radioelectronics
- High voltage technology
- Theoretical electrotechnology and electronic measurements

- Social sciences - Technology, civilisation, culture.

Over 250 papers of participants from 23 countries was presented during the conference.

The Radioelectronics section covers wide range of the field including following topics: Circuit theory, Electromagnetic field theory, Digital technology, Signal processing, Optoelectronics, Radioelectronics systems and Acoustoelectronics.

The papers has high professional level. The conference was held in the pleasant and creative atmosphere.

Ján Turán TU Košice