Inelastic finite element analysis of lateral buckling for beam structures

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Abstract

The inelastic lateral–torsional buckling resistance of real I-beam loaded by uniform moment was the subject of parametric studies using nonlinear finite elements, covering various initial imperfections as initial axis curvature, material properties, residual stress, etc. The ANSYS software package was used. The finite element SOLID185 was applied to study the inelastic failures of beams. The finite element analysis considers both geometrical and material nonlinearities. The static inelastic resistance was calculated for various beam lengths, and compared with elastic resistance and with resistances based on the Eurocode buckling curves.

Keywords: beam, inelastic stability, finite element, buckling, steel, imperfection, reliability, sensitivity.

1. Introduction

The European buckling curves are based on large experimental and parametric measurement programs as well as on analytical, numerical and probabilistic investigations. Numerical verification of Eurocode buckling curves is constantly the topic [1]. The theory of thin-walled beams as developed by Vlasov [2] was used in numerous calculations aimed at the reliability study of thin-walled beams [3-5]. Analytic approaches enable a transparent insight into the beam behaviour, nevertheless, only a limited number of imperfections can be taken into consideration [6,7]. Initial imperfections such as residual stress may reduce the buckling strength of a steel member. Modern calculation models based on the finite element method (FEM) enable to take into account all the unavoidable initial imperfections [8,9]. FEM makes possible to study many modes of failures and instability of the beams. The stability problems of the beams with imperfections such as geometrical and material imperfections with
residual stress should be investigated with use of both geometrical and material nonlinear solutions. Inelastic analysis should be applied especially if residual stress is present, and its values are significant.

In general, the computer modelling of stability loss of the real beam is a very complex problem, for the solution of which, large knowledge must be broadly applied to. The more is FEM advanced, the more information must be assigned to model inputs. Large information on materials, and on geometrical imperfections was investigated into by experimental programmes [10,11], and was used in many reliability studies [12,13]. The size and shape of residual stress are permanently the topics of discussions concerning the stochastic character of this quantity [7,14]. The subject of the present paper is the inelastic lateral–torsional buckling (LTB) static resistance of real I-beam loaded by uniform moment.

2. Calculation models

Inelastic failures of the beam are analysed by means of the finite element SOLID185. The objective is to model the real beam, and to calculate its static resistance so, as if the real loading test were carried out in the laboratory. The advanced geometrical and material nonlinear calculation model is a powerful instrument for a very accurate calculation of static resistance of the beam.

2.1. Geometrical Imperfections and Boundary Conditions

The cross-section geometry characteristics of double-symmetric hot-rolled beam I200 of steel grade 235 are shown in the Fig. 1. Important geometrical sectional characteristics are $h$, $b$, $t_1$, $t_2$ and $R_1$ and $R_2$.

![Fig. 1. Hot-rolled beam I200: (a) real cross-section, (b) idealized cross-section, (c) boundary conditions.](image)

The beam is considered as simply supported and loaded at equal bending moment of opposite ends. The initial geometrical imperfection of beams is considered according to the first eigenmode of LTB. It consists of initial displacement of axis $v_0$ and of initial rotation of cross-sections $\phi_0$. These imperfections are considered to be affine to the final shape as the functions sinus, $v_0$ being the curvature of the beam axis in the direction of major axis, i.e., in plane $xy$, and $\phi_0$, rotation of cross-sections along the beam length $L$, see Fig. 2.

$$v_0 = a_{v_0} \sin \left( \frac{\pi x}{L} \right), \quad \phi_0 = a_{\phi_0} \sin \left( \frac{\pi x}{L} \right)$$

(1)

If the beam is curved according to the first eigenmode, thus the amplitudes $a_{v_0}$ and $a_{\phi_0}$ are:
\[
a_{\phi} = \frac{M_{cr} L^2}{\pi^2 EI_z}
\]

\[
a_{\phi_0} = \frac{\pi^2 EI_z}{M_{cr} L^2 + 0.5\pi^2 hEI_z}
\]

where \(e_0\) is the amplitude of one half-wave of the sinus function relating to the upper edge of the cross-section, and is designed as \(L/1000\) [15,16], \(h\) is the cross-section height, \(E\) is the Young’s modulus of elasticity, \(I_z\) is the second moment of area around axis \(z\), \(L\) is the beam length, and \(M_{cr}\) is the elastic critical moment at LTB. The relations (1), (2) and (3) represent the starting point for writing two differential equations which describe the elastic LTB [17].

![Initial geometrical displacement and rotation of the beam axis.](image)

2.2. Finite Element Model 3D

Computational models were created in the Ansys software, using 3D elements SOLID185 [18]. SOLID185 is an 8-node element which can be used for 3D modelling of solid structures and has plasticity, hyperelasticity, stress stiffening, creep, large deflection, and large strain capabilities. It is defined by eight nodes having three degrees of freedom at each node: translations in the nodal \(x\), \(y\), and \(z\) directions. The element was set to be a homogeneous structural solid element. The enhanced strain formulation was considered. The enhanced strain formulation prevents shear locking in bending-dominated problems and volumetric locking in nearly incompressible cases. The formulation introduces a certain number of internal (and inaccessible) degrees of freedom to overcome shear locking, and an additional internal degree of freedom for volumetric locking (except for the case of plane stress in 2-D elements). All internal degrees of freedom are introduced automatically at the element level and condensed out during the solution phase of the analysis [18]. An example of the computational model is presented in Fig. 3. Initial imperfections are presented in magnified scale.

The bending moment on the edge cross-section of 3D model was created as a pair of forces in its nodes. Boundary conditions are set so that the edge cross-sections can warp, see Fig. 1 (c). The support in direction of axis \(x\) \(u_x = 0\) is introduced at one end only.
2.3. Residual Stress and Material Properties

In hot-rolled steel members, unsteady cross-sectional cooling down occurs after hot rolling. Cooling down occurs more rapidly at the flange edge and at the middle of the web. The primary tendency is the stabilization of volume changes during shrinkage. The subsequent slower cooling down and shrinkage of thermally more exposed internal cross-sectional parts in the contact regions of the flanges and the web results in compressive stress in the previously cooled down and volume stabilized regions. Shrinkage stress arises at the flange edge and in the web middle. Residual stress was modelled by means of the distribution of temperatures on the beam cross-section, see Fig. 4. The relation $\sigma_R=\Delta T E \alpha$ between temperature $\Delta T$ and residual stress $\sigma_R$ is illustrated in Fig. 4. $E=210$ GPa is Young modulus and $\alpha=1.2E-5$ K$^{-1}$ is linear expansion coefficient. An elastic-plastic stress-strain diagram without hardening according to the standard [19] is used for the computation. The value of yield strength $f_y$ is considered by nominal value 235 MPa.

3. The static resistance

The value of elastic resistance $M_R$ of the beam can be calculated according to analytical solution [5,6]. $M_R$ represents the bending moment at which the maximum value of the von Mises stress corresponds to yield strength $f_y$ of the steel. In plastic limit analysis of structural members subjected to bending, it is assumed that an abrupt transition from elastic to ideally plastic behaviour occurs at a certain value of moment, known as plastic moment $M_d=f_y W_{pl,y}$. When $M_d$ is reached, a plastic hinge is formed in the member. The plastic hinge allows large
rotations at constant plastic moment $M_{pl}$. The design resistance moment $M_{b,Rd}$ of the beam at LTB is determined from the relation $M_{b,Rd} = \frac{F_{LT}}{f_f} W_y f_M$ according to Standard Eurocode3. The cross-section 1200 is the cross-section of Class 1, and therefore the cross-section module $W_y$ can be determined as $W_y = W_{pl,y}$. The partial resistance factor is $\gamma_M=1.0$. The reduction factor for LTB $\chi_{LT}$ for the appropriate nondimensional slenderness $\lambda_{LT}$ can be determined from European buckling curve $b$, see Eurocode3. Cross-section characteristics of the idealized profile 1200 according to Fig. 1 (b) are given in Table 1.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-section height</td>
<td>$h$</td>
<td>0.200 m</td>
</tr>
<tr>
<td>Cross-section width</td>
<td>$b$</td>
<td>0.090 m</td>
</tr>
<tr>
<td>Web thickness</td>
<td>$t_1$</td>
<td>0.007 5 m</td>
</tr>
<tr>
<td>Flange thickness at quarter of the width</td>
<td>$t_2$</td>
<td>0.011 3 m</td>
</tr>
<tr>
<td>Second moment of area about axis y</td>
<td>$I_y$</td>
<td>21.235E-6 m$^4$</td>
</tr>
<tr>
<td>Second moment of area about axis z</td>
<td>$I_z$</td>
<td>1.188E-6 m$^4$</td>
</tr>
<tr>
<td>Torsion constant</td>
<td>$I_t$</td>
<td>1.187E-7 m$^4$</td>
</tr>
<tr>
<td>Warping constant</td>
<td>$I_{\omega}$</td>
<td>1.017E-8 m$^4$</td>
</tr>
<tr>
<td>Section modulus about axis y</td>
<td>$W_y$</td>
<td>21.235E-5 m$^3$</td>
</tr>
<tr>
<td>Section modulus about axis z</td>
<td>$W_z$</td>
<td>2.639E-5 m$^3$</td>
</tr>
<tr>
<td>Plastic section modulus about axis y</td>
<td>$W_{pl,y}$</td>
<td>24.684E-5 m$^3$</td>
</tr>
</tbody>
</table>

The computational model in the Ansys software is loaded increasingly, and calculated in geometrically nonlinear way by the Newton-Raphson method. The plastic resistance $M_{pl,Ansys}$ is defined as the maximum value of bending moment $M$, when the determinant of the stiffness matrix is non-zero, and the calculation converges. The plastic resistance $M_{pl,Ansys}$ is calculated without residual stress, and $M_{pl,Ansys,rs}$ is calculated with the influence of residual stress, see Fig. 4. The elastic resistance $M_{R,Ansys}$ is given by reaching of yield strength $f_y$ in any point of the beam. With regard to the plane symmetry of the beam along the plane passing through its centre, and in parallel with the $y_z$, reaching the yield strength takes place in one of cross-section tops in the middle of span. The linear regression was carried out to accurately quantify the elastic resistance for a narrow set of data including the value of acting moment and corresponding value of the von Mises stress near the yield strength. As the basic linear regression model, the polynomial of the seventh degree was applied, where the absolute error was still negligible.

4. Stress of the Beam under Limit State

The elastic resistances $M_{R,Ansys}$ is given by reaching the yield strength $f_y$ in any point of the beam, without occurrence of plasticization of the cross-section. The course of stress $\sigma_x$ in the middle span is illustrated in Fig. 5 (a).

![Fig. 5. (a) elastic vs. total plastic resistance; (b) $\sigma_x$ stress course in the web for nondimensional slenderness of 0.6 at reaching the $M_{pl,Ansys}$.](image-url)
It can be noticed that the plastic resistance of the cross-section subjected to bending is an important part of numerous optimization analyses [20]. The cross-section can theoretically plasticize totally according to Fig. 5 (a). However, the reality is so that even for the lowest considered values of slenderness, the cross-section of the beam need not plasticize fully at reaching the plastic moment $M_d$. Such a case of stress course $\sigma$, is, observed, e.g., for the slenderness $\bar{\lambda}_L=0.6$, presented in Fig. 5 (b). The von Mises stress which decides on the resulting value of resistance $M_{pl,Ansys}$, is depicted in Fig. 6 (a) and (b). Fig. 7 shows the von Mises stress of the beam with considering residual stress at reaching $M_{pl,Ansys,rs}$. The stress course in Fig. 7 is similar to that in Fig. 6, however, it is not identical.

Fig. 6. Course of the von Mises stress $\sigma_{vM}$ in the beam for nondimensional slenderness of 0.6 at reaching the $M_{pl,Ansys}$.

Fig. 7. Course of the von Mises stress $\sigma_{vM}$ in the beam for nondimensional slenderness of 0.6 at reaching the $M_{pl,Ansys,rs}$. 
5. Comparison of Resistances

The values of analytically computed resistances are depicted by the curve in Fig. 8. The diagram is completed by elastic critical moment $M_{cr}$ at LTB. The range of non-dimensional slenderness $\lambda_{LT}$ is from 0 to 2.1. In the present problem, non-dimensional slenderness $\lambda_{LT}$ is, in the present problem, dependent only on the beam length $L$ [9]. The calculation was limited by the beam length of 12 meters.

It is evident from the diagram in Fig. 8 that the absolute difference of plastic resistance $M_{pl,Ansys}$ and elastic $M_{R,Ansys}$ is increasing with decreasing value of slenderness. There takes place the use of plastic reserve of the cross-section. For the slenderness approximately higher that 1.4, this difference is negligible. The analytical elastic resistance $M_{R}$ [5,6] perfectly fits to elastic resistance $M_{R,Ansys}$, which was calculated using geometrically nonlinear FEM.

6. Conclusion

The paper has provided transparent insight into inelastic LTB. The 3D geometrically and material nonlinear FE model for computation of LTB resistance was presented in the paper. The deterministic influence of all imperfections was included into calculations. Studies have shown the influence of residual stress on the decrease of LTB resistance. The residual stress reduced the resistance by 6 % for $\lambda_{LT} = 1.2$, see $M_{pl,Ansys}$ and $M_{pl,Ansys,rs}$ in Fig. 8
and Fig. 9. The plastic resistance $M_{pl, Ansys, rs}$ calculated, including the influence of residual stress, perfectly fitted to design resistance $M_{b, Rd}$ determined according to Eurocode 3. In further studies, also random influence of all imperfections can be included into calculations. The presented 3D FE model can be applied to the probabilistic verification of reliability of steel beam structures designed according to the EUROCODE standards [13].

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References