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Study of Behaviour of Beams and Panels Based on Influence of Rigidity

O. Pešek^{a*} and J. Melcher^a

^a*Brno University of Technology, Faculty of Civil Engineering, Institute of Metal and Timber Structures, Veveří 331/95, Brno 602 00, Czech Republic*

Abstract

The aim of this paper is the elaboration of a study on the real behaviour of beams and thin panels considering the effect of large deflection. The classic plate theory was developed for panels whose deflections are lower than their depth. This is so-called a simple deflection theory. At thin panels of steel or structural glass the deflections are higher than their depth. Therefore, we have to apply the large deflection theory. Membrane effects influence the behaviour of thin panels. Typical rigid beams can be analyzed by a classic elastic theory. The results of large deflection theory give more accurate image of behaviour of non-rigid structures. Therefore, it is necessary to realize non linear calculations. The numerical models are realized using ANSYS software based on finite element method. Numerical models were realised using space and planar finite elements. They were analyzed by linear and nonlinear computation. The resulting computations were compared. Results using large deflection theory may provide more favourable base for designing of structural element and they correspond to actual beam and plate behaviour.

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Keywords: Structural glass; thin plate; finite element method; large deflection theory

1. Introduction

Recent architectural trends and technological developments brought changes in the use of glass in buildings. These include the use of large area glass panels, the use in staircases, partitions, railings, floors and roofs – the use of glass in parts of buildings which were from some traditionally materials. In many applications the traditional rules are based on the simplifying assumptions of the theory of small deflections cannot be extrapolated for the specific glass product for which they were devised.

* Tel.: +420-541-147-329 ; fax: +420-549-245-212

E-mail address: pesek.o@fce.vutbr.cz

2. Structural glass

A glass is an inorganic product of fusion, which has been cooled to a rigid condition without crystallization. Most of the glass used in construction is soda lime silica glass (SLSG). For fire protecting glazing and heat resistant glazing is borosilicate glass (BSG) used. One of the most important properties of glass is its excellent chemical resistance to many aggressive substances which makes glass one of the most durable materials in construction [1].

In Table 1 are summarized the most important physical properties of SLSG and BSG. The dynamic viscosity of glass is about 10^{20} dPa.s at room temperature (for comparison, the viscosity of water is 10^{-1} dPa.s and of honey 10^5 dPa.s). Glass shows an almost perfectly elastic, isotropic behavior and exhibits brittle fracture. It does not yield plastically, which is why local stress concentrations are not reduced through stress redistribution as is the case for other construction materials like steel [1].

Table 1. Structural glass material properties [1]

			Soda lime silica glass	Borosilicate glass
Density	ρ	kg/m ³	2500	2200 - 2500
Young's modulus	E	MPa	70 000	60 000 - 70 000
Poisson's ratio	ν	-	0,23	0,2

One of the most important properties of any structural material is the strength. The theoretical tensile (and compressive) strength of glass is exceptionally high and may reach 32 GPa (this value is based on molecular forces). The relevant tensile strength for engineering is much lower. The reason is that glass is brittle material. The tensile strength of glass depends very much on mechanical flaws on the surface. But flaws are not necessarily visible to the naked eye (effective flaw depth may be between 10^{-6} and 10^{-1} mm). A glass element fails as soon as the stress intensity due to tensile stress at the tip of one flaw reaches its critical value. The tensile strength of glass is not a material constant, but it depends on many aspects (on the condition of the surface, the size of glass element, the action history, the residual stress, the environmental conditions etc.) [1].

The compressive strength of glass is much larger than the tensile strength, because surface flaws do not grow or fail when element is in compression. The compressive strength is irrelevant for all structural applications, because an element's tensile strength is exceeded long before it is loaded to its compressive strength [1]. Structural tensile strength is illustrated in Fig. 1.

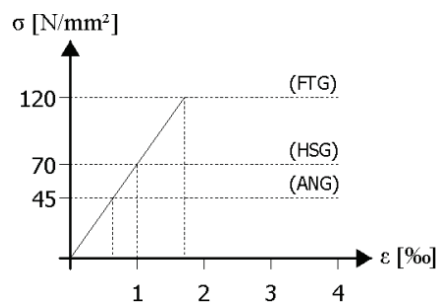


Fig. 1. Structural glass tensile strength [2]

Base glass product is float glass (annealed glass, ANG). By secondary processing thermal treatment we get heat strengthened glass (partly toughened glass, HSG) a fully tempered glass (tempered glass, FTG). Laminated (safety) glass is consist of two or more glass pane (it may be annealed glass, heat strengthened glass or fully tempered glass) which are join by PVB-foil (polyvinyl butyral).

3. Example of the beam made of structural glass

Parametric study was performed for the beam (example of staircase step) made of structural glass. The beam was loaded by continuous uniform planar load of $3,0 \text{ kNm}^{-2}$. Two types of static structure of beam were modeled. The first, beam was modeled as the simple beam and the second, beam was modeled as beam supported by fixed support in horizontal axe at both ends of the beam (statically indeterminate structure). The beam has rectangular cross section, constant along the length. Width of beam is 300 mm and thickness is variable in the range from 10 to 40 mm. The length of the beam is variable (span of the beam $L = 1,5 \text{ m}$; $2,0 \text{ m}$; $2,5 \text{ m}$ and $3,0 \text{ m}$).

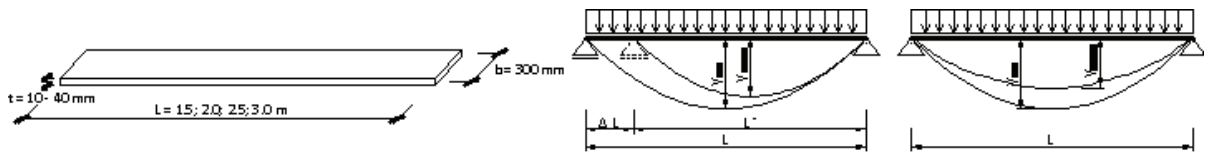


Fig. 2. (a) Beam geometry; (b) simple beam; (c) statically indeterminate structure

Material properties has been taken for soda lime silica glass (elastic isotropic material, Young's modulus $E = 70 \text{ GPa}$ and Poisson's ratio $\nu = 0,23$). The structure was modeled in software based on finite element method, by finite element SHELL181. Solution was elaborated by two methods – by small deflection theory (linear analysis) and by large deflection theory (nonlinear analysis).

In Fig. 3 symbol SB means simple beam, SIS - statically indeterminate structure, SLS - serviceability limit state – $L/400$. At beam modeled as a simple beam this difference between values of deflection is apparent in the deflection values greater than 100 mm. Deflection increases nonlinearly with thickness decreasing. When staircase step is modeled as statically indeterminate structure deflection increased approximately linearly at small thickness (Fig. 3).

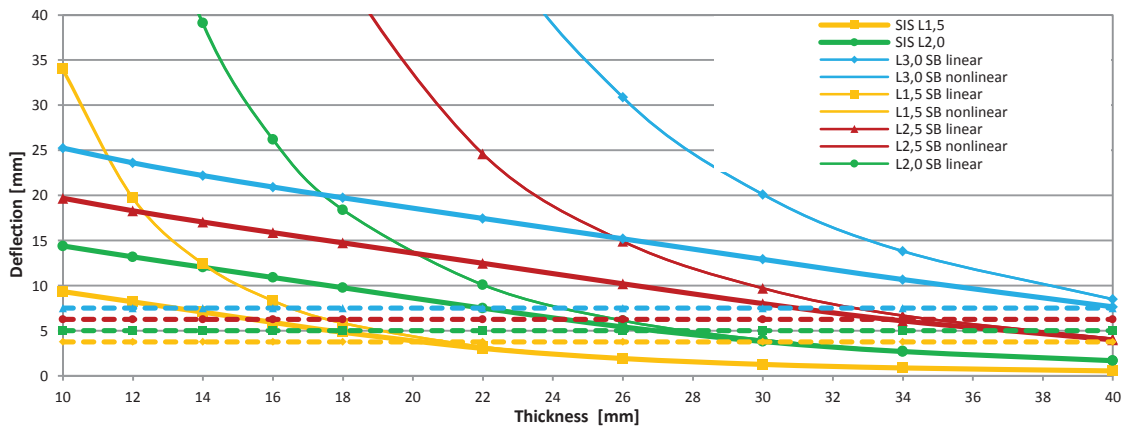


Fig. 3. Influence of thickness and supporting on deflection

The deflections of beam modeled as simple beam and statically indeterminate structure are similar for minimum thickness or higher determined by the serviceability limit state.

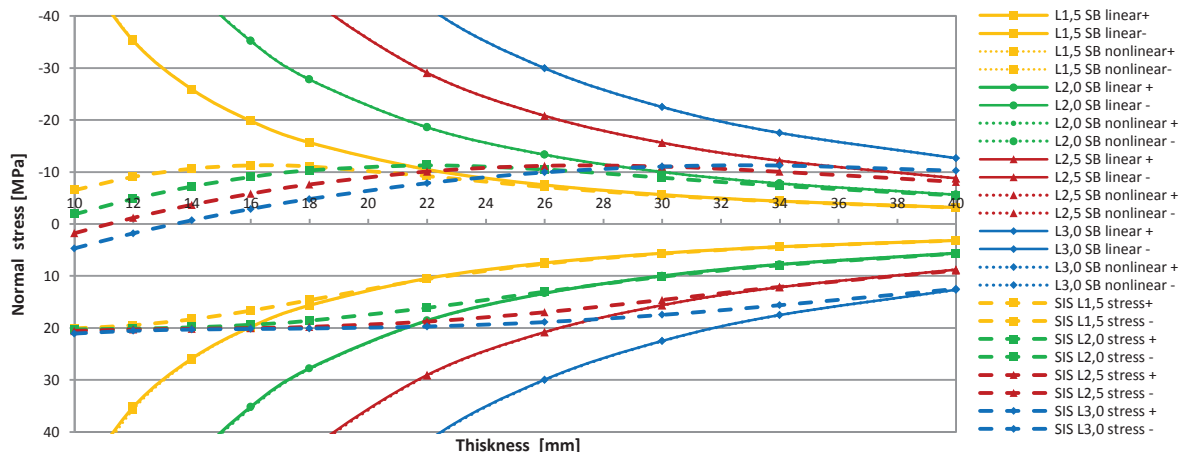


Fig. 4. Influence of thickness and supporting on normal stress

In Fig. 4. the normal stresses σ_x for all values of span modeled as simple beam and statically indeterminate structure calculated by linear and nonlinear solution are displayed. Tensile stresses on bottom surface calculated for simple beam and for statically indeterminate structure are almost identical for minimum (by SLS) and larger thickness. Minimum thickness is 22 mm for span 1,5m ($t_{min} \approx 28, 34$ and 40mm for span $L = 2.0, 2.5$ and 3m). These thicknesses correspond to tensile normal stress $\sigma_x \approx 12$ MPa. This is smaller than the value of structural glass tensile strength. This means that SLS is a decisive for design of analyzed beam made of structural glass with similar geometry. When the beam is modeled as statically indeterminate structure the stress on the upper surface can acquire positive values (tensile stress) for small thickness. It is caused by low value of rigidity. The structure (beam) behaves more as a rope than classic rigid beam.

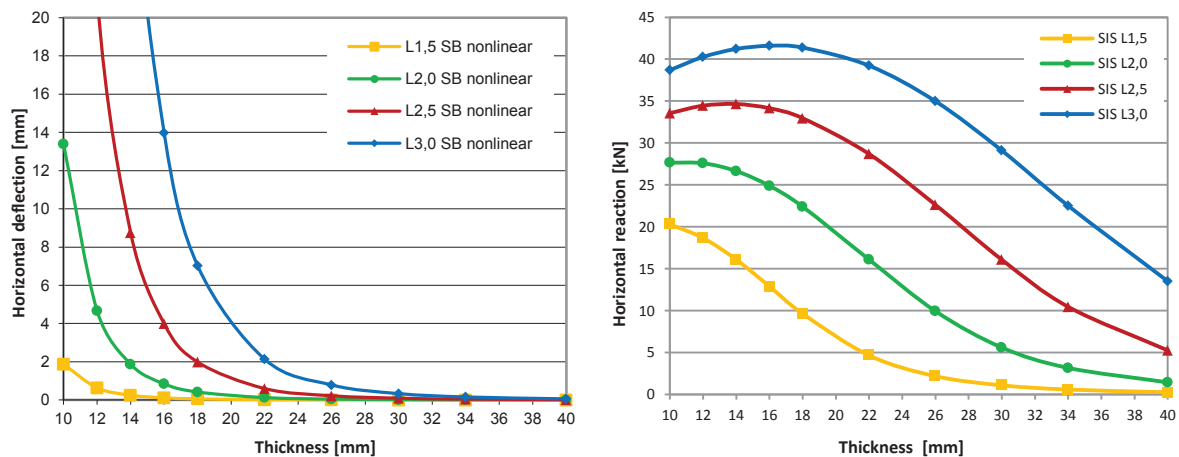


Fig. 5. Influence of thickness and supporting (a) on horizontal deflection; (b) on horizontal reaction for span of the beam $L = 1,5$ m

In Fig. 5. the influence of thickness on horizontal deflection (at simple beam) and on horizontal reaction (at statically indeterminate structure) is displayed. The values of horizontal reactions are relatively high at the statically indeterminate structure. The horizontal deflection has relatively low values for minimum thickness by SLS and is possible to admit those deflections to elastic fasteners.

4. Example of the plate made of structural glass

In this part of parametrical study has been analyzed influence of desk width on deflections and stresses. Span of the desk was 2 meters and desk thickness was 30 mm. The width of desk was variable from 300 to 3000 mm in steps of 300 mm. The plate (example of the staircase landing) is supported on two opposite sides and was modeled as simple beam by SHELL181 finite elements. Geometry of plate shows the Fig. 6.

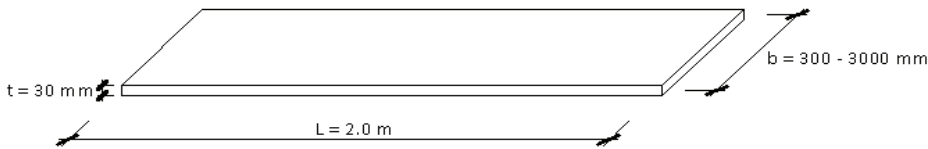


Fig. 6. Plate geometry

The plate was analyzed by linear and nonlinear solution. Deflections and stresses are constant along the width at simple beam theory. In reality, deflection and stresses are not constant along width of desk (Fig. 7, 8.). It should be noted that the difference of deflections between linear and nonlinear solutions is negligible and difference of deflections between simple beam theory and numerical modeling is in the order of some per cent. Deflections are biggest in the mid span at edge of the desk (Fig. 7a.) and those deflections are higher than at simple beam theory. On contrary, deflection at mid span at the center of width has lower value than at simple beam theory. In both cases the differences between results by simple beam theory and numerical modeling is greater with desk width increasing (Fig. 7b.).

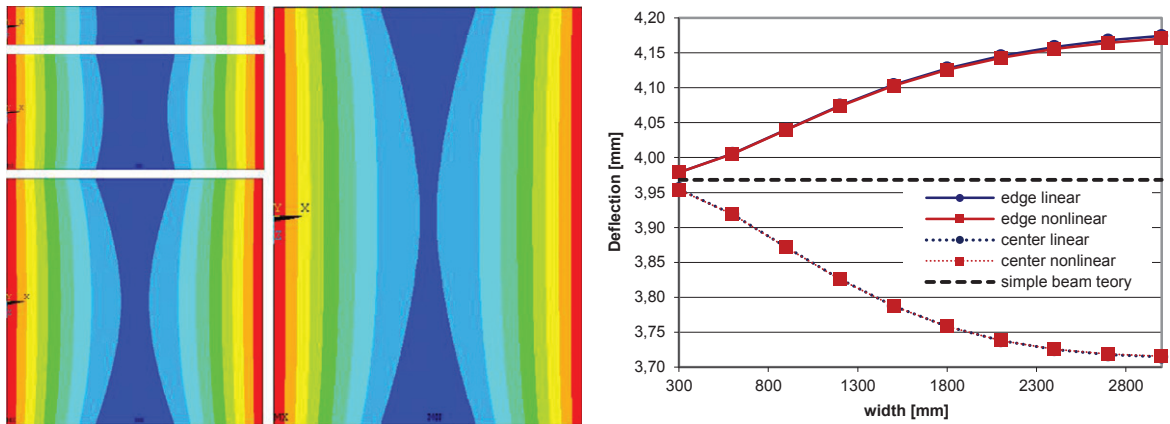


Fig. 7. Deflection on staircase desk (a) at width $b=300, 900, 1500, 2700 \text{ mm}$; (b) influence of width on deflection

Influence of desk width on normal stress σ_x is displayed on graph in Fig. 8. In Fig. 8a, the only positive values of normal stress at bottom surface of desk are shown. Maximum value of stress from linear solution is

smaller than maximum value of stress from nonlinear solution. The difference of results between linear and nonlinear solution is less than 1 percent. The highest value of normal stress is in the mid span at edge of desk and value of stress is higher than according to the simple beam theory. Stress at mid span in center of width is lower than the results of the simple beam theory. The difference between maximum value of tensile normal stress calculated by simple beam theory and finite element method is in the order of some per cent. What the width of desk is less the deformations are more similar to deformations of structure calculated by simple beam theory.

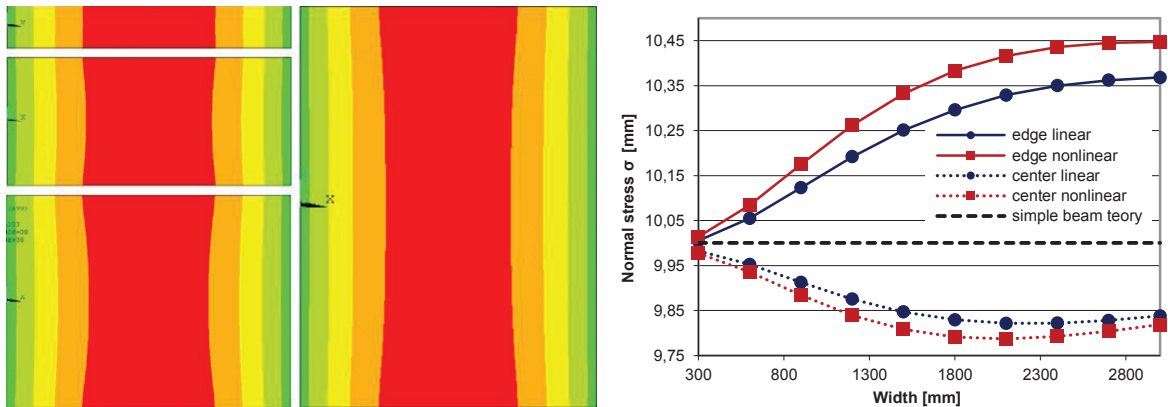


Fig. 8. Influence of width upon normal stress σ_x (a) bottom surface of desk; (b) graph

5. Conclusion

At thin members made of glass beams and plates components the results of small and large deflections theory shall be compared and verified. Because value of structural glass Young's modulus is relatively low serviceable limit state is decisive for design this structures. In this paper the appropriate study for thin beams and for thin plates has been presented. The results illustrate the influence of relation of component thickness to it span.

Acknowledgements

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