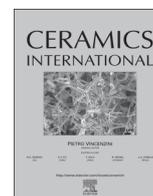




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The formation of feldspar strontian ($\text{SrAl}_2\text{Si}_2\text{O}_8$) via ceramic route: Reaction mechanism, kinetics and thermodynamics of the process

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ABSTRACT

The reaction mechanism, the equilibrium composition, the temperature range of stability of formed intermediates as well as the kinetics and thermodynamics of activated state during the formation of monoclinic strontium-aluminum-silicate feldspar strontian ($\text{SrAl}_2\text{Si}_2\text{O}_8$) via the ceramic route from the mixture of SrCO_3 , Al_2O_3 and SiO_2 is described in this work. Strontian does not appear up to the temperature of 1150°C and is the only stable phase at the temperature $\geq 1600^\circ\text{C}$. Three independent reactions lead to two parallel reaction pathways, i.e. the formation of strontian from single or binary oxides (1) and with Sr-gehlenite as the intermediate (2). Since the reaction rate constants ratio is higher than one ($k_1/k_2 > 1$), the first reaction route is favored according to the Wegscheider principle. The kinetics of chemical reaction of 1.5 order corresponding to the kinetic function $F_{2/3}((1-\alpha)^{-1/2}-1)$ was determined as the rate determining the mechanism of formation of strontian. The integral and differential methods show that the process requires average apparent activation energy of 229.3 kJ mol^{-1} . The determined average value of frequency factor is $2.1 \times 10^5\text{ s}^{-1}$.

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1. Introduction

Since aluminosilicate feldspars of fundamental formula MT_xO_8 (or $\text{M}_x\text{T}_y\text{O}_8$, where y reaches 4 and the parameter x does not fall below 0.975 [1]) are the most abundant minerals in the Earth's crust [2], these tectosilicates [3] are investigated due to the importance of their petrography [4,5], understanding of Earth's geology [5,6] and industrial importance in glass industry and production of ceramics [7–10]. T-sites are occupied by small strongly charged cations (typically Si^{4+} and Al^{3+}) and M-sites are occupied by larger weakly charged cations. Feldspars can be characterized by the content of all 92 naturally occurring elements [1]. Weathering and hydrothermal change of feldspar leads to the formation of minerals such as kaolinite [11,12], montmorillonite [13], illite [14] or mixed-layer clay minerals [15].

The feldspar crystal structure is composed of a 3D framework of corner-shared tetrahedra which are centered by Si^{4+} (Ge^{4+}) and Al^{3+} (Ga^{3+}). Large alkali (Na^+ , K^+) or alkaline-earth ions (Ca^{2+} , Sr^{2+} , Ba^{2+}) in the interstices balance the charge caused by different charges of tetrahedral Al^{3+} (Ga^{3+}) and Si^{4+} (Ge^{4+}) ions.

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Feldspars with smaller cations (Na^+ , Ca^{2+}) are triclinic, whereas larger cations (Sr^{2+} , Ba^{2+}) tend to support the monoclinic symmetry. Feldspars are known to undergo several phase transitions, which are of the order–disorder or displacive type. $\text{MeAl}_2\text{Si}_2\text{O}_8$ ($\text{Me}=\text{Ca}$, Sr , Ba) feldspars remain essentially ordered at all temperatures below the melting point [2,16–18].

Similar to anorthite ($\text{CaAl}_2\text{Si}_2\text{O}_8$, [19–22]), barium feldspar celsian ($\text{BaAl}_2\text{Si}_2\text{O}_8$, BAS, [23]) and strontium feldspar strontian ($\text{SrAl}_2\text{Si}_2\text{O}_8$, SAS, Fig. 1(a–c)) are the materials of great technological interest due to low thermal expansion coefficient ($2.29 \times 10^{-6}\text{ }^\circ\text{C}^{-1}$ for BAS [23] and $2.5 \times 10^{-6}\text{ }^\circ\text{C}^{-1}$ for SAS [24,25]), low and thermally stable dielectric constant, low dielectric losses, high mechanical strength, chemical inertness, and high melting temperature (1760°C for BAS [2,23,26] and 1650°C for SAS [23,27,28]), therefore are known as materials for the matrix of fiber-reinforced ceramic composites, protective coatings, electro-ceramics and refractories [23,28–34]. Other significant applications include the preparation of glass ceramics [23,35–37] and luminescent pigments of $\text{XAl}_2\text{Si}_2\text{O}_8:\text{M}^{n+}$ (where $\text{X}=\text{Ca}$, Sr , Ba and M^{n+} denotes ions of doped element) [38–44].

The strontium aluminate polymorphs of SAS are monocelsian (SM) and hexacelsian (SP). Sr-hexacelsian is known only as the synthetic product and Sr-monocelsian occurs as the mineral slawsonite (P_2/a) [23,25,45]. The reversible $\text{SM} \leftrightarrow \text{SP}$ transformation within the temperature range from 600 to 800°C causes volume changes of approximately 3% [24,25,28]. Celsian and Sr-celsian give rise to solid

[58,60–71] and Table 3). The plot of $\ln(g(\alpha)/T)$ versus T^{-1} gives straight line ($R^2 \rightarrow 1$) with the slope $-E_{ap}/R$ for the suitable choice of $g(\alpha)$. The intercept with y-axis ($\ln[AR/\Theta E_{ap}]$) enables then to calculate the value of frequency factor (A). The results were verified by the differential method based logarithmic form of generalized equation [72]

$$\ln \left[\frac{d\alpha/dt}{f(\alpha)} \right] = \ln \left[\frac{A}{\Theta} \right] - \frac{E_a}{RT} \quad (4)$$

The plot of $\ln[d\alpha/dt / f(\alpha)]$ versus T^{-1} gives straight line for proper $f(\alpha)$ function (Table 3).

3. Results and discussion

3.1. Investigation of behavior of raw meal upon thermal treatment

The HT-XRD plot in Fig. 2(a) shows the changes in phase composition during the thermal treatment of raw meal. The shift in quartz diffraction is caused by the transformation of low-quartz (α) to high-quartz (β) at the temperature of 575 °C. This reversible displacive (athermal) transition is deeply discussed in the following literature [73–75]. The diffraction lines of strontium carbonate disappear within the temperature range from 775 to 850 °C. In the same temperature interval the maximum intensity of diffraction lines of strontium oxide was reached. The amount of SrO then decreases with the formation of tristrontium aluminate ($\text{Sr}_3\text{Al}_2\text{O}_5$). With increasing temperature, tristrontium aluminate reacts with alumina to hexagonal strontium aluminate. The features of Sr_3A and SrO disappear at the temperature of 1125 °C.

The formation of strontium orthosilicate (SrSiO_4) at the temperature of 775 °C is responsible for decreasing intensity of diffraction lines of SiO_2 . At the temperature of 900 °C the features of strontium metasilicate appear. The diffraction lines of this phase increase with increasing temperature, while the intensity of diffraction lines of Sr_2SiO_4 decreases. The features of strontian appear at the temperature of 1150 °C. As the intensity of diffraction lines of feldspar increases with increasing temperature, the intensity of other components decreases (Fig. 2(b)). The evolution of phase composition of specimen during the thermal treatment to the temperature of 1660 °C is shown in Fig. 6(a).

The typical TG-DTA and heating microscopy plot is shown in Fig. 3. These results were supplemented by the results of EGA analysis (Fig. 4). The first endothermic effect at the temperature of 92 °C is accompanied by increasing intensity of CO_2 and H_2O on EGA. Drying, desorption of carbon dioxide and the first dehydration step of $\text{Sr}(\text{OH})_2 \cdot 8\text{H}_2\text{O}$ (formed via adsorption of air humidity on the recently formed surface of treated (milled) raw meal) to

$\text{Sr}(\text{OH})_2 \cdot 6\text{H}_2\text{O}$ [76] decreases the mass of sample by 0.06%. The second (formation of $\text{Sr}(\text{OH})_2 \cdot \text{H}_2\text{O}$ [76]) and final dehydration step to $\text{Sr}(\text{OH})_2$ shows a small endothermic peak at 180 °C. The both stages can be recognized on DTG at temperature of 170 and 210 °C. Polymorphic transformation of β - $\text{Sr}(\text{OH})_2$ to α - $\text{Sr}(\text{OH})_2$ shows a small endothermic peak at the temperature of 305 °C. Melting and decomposition of $\text{Sr}(\text{OH})_2$ takes place in the temperature range from 430 to 645 °C. The process shows peak temperature of 605 °C and mass of sample was reduced of 1.01%. The released water and its autodissociation in the hydroxide flux ($2\text{OH}^- \leftrightarrow \text{H}_2\text{O} + \text{O}^{2-}$, where water acts as a strong acid [77]) is followed by the reaction of melt with SrCO_3 and the increase of intensity of CO_2 and H_2O on EGA (Fig. 4).

Sharp endothermic peak at the temperature of 573 °C is caused by displacive transition between low (α)- and high (β)-quartz polymorphs (see Fig. 2). The endothermic nature of this transition slows down the reaction of molten phase with strontium carbonate, i.e. depression of the concentration of CO_2 on EGA has appeared (Fig. 4). The concentration of water vapor is increasing in the same time.

The thermal decomposition of strontium carbonate reduces the mass of sample by 11.2% within the temperature range from 685 to 1010 °C. The process increases the response of CO_2 on EGA (Fig. 4). The shape of peak caused by observed effects is affected by the transformation of orthorhombic (α - SrCO_3) to hexagonal (β) polymorph. The kinetics and the mechanism of this process were described in previous work [78]. The results of heating microscopy show that the specimen area increases by ~5% in the temperature interval limited by polymorphic transformation of SrCO_3 and the occurrence of strontian. The small endothermic peak at the temperature of 1160 °C indicates the formation of melt from which primary strontian starts crystallizing. The exothermic nature of crystallization process then leads to the exothermic peak at 1195 °C.

Other peaks appears the consequence of establishing of equilibrium composition, i.e. the formation of strontian as the single phase via reactions between the different intermediates (see Eq. (9)) and intermediates with single oxides (Eqs. (8) and (9)). The establishment of equilibrium composition leads to the formation of secondary strontian. The reaction mechanism of formation is different, but no structural difference between primary and secondary formed strontian is observed. According to the results of heating microscopy (Fig. 3(b)), the strontian starts to melt at the temperature of 1660 °C.

The scanning electron microscopy results of raw meal treated to temperatures marked in Fig. 3(b) are shown in Fig. 5. The results of HT-XRD (Fig. 2) indicate that the diffraction lines of strontian appear at the temperature of 1150 °C. The SEM analysis of sample treated to this temperature (a) shows the growth of nuclei of strontian. At the temperature of 1300 °C (b) the plate-like particles of strontian can be recognized. The structure consists of many narrow layers of twinned feldspar crystals (multiple lamellar [010] contact twins according to the "albite law" [79,80]), which endow the parts of the crystal with faintly striped appearance. The melt formed with increasing temperature enables the recrystallization and growth of large plate-like particles of tertiary strontian (c and d) which are stable up to the melting temperature. The process is accompanied by the expansion of specimen (see Fig. 3(b)).

Table 1

The derivation of ratio of rate constants for the formation of strontian (k_1) and Sr-gehlenite (k_2).

Temperature	[°C]	1150	1300	1400	1450
Strontian	[%]	4	36	69	83
Sr-gehlenite	[%]	0	6	7	0
$k_1/k_2 = w_1/w_2 > 1$		–	6	9.9	–

Table 2

Reaction thermodynamics for independent reactions Eqs. ((8)–(10)).

Temperature [°C]	1150	1200	1300	1400	1500	1600
Equation	$\Delta_r G^\circ$ [kJ mol ⁻¹]					
Eq. (8)	–3.017	–2.973	–2.806	–2.534	–2.161	–1.693
Eq. (9)	–42.214	–42.335	–42.517	–42.626	–42.670	–42.659
Eq. (10)	–25.872	–24.907	–23.034	–21.225	–19.463	–17.729

Table 3

The overview of results with applied kinetic functions [59,82–87] and the evaluation of the most probable reaction mechanism (marked by bold) for the formation of strontian.

Abr.	Kinetic function		Description	<i>n</i>	<i>R</i> ²	
	<i>g(α) = kt</i>	<i>f(α) = k - da/jdt</i>			Int.	Diff.
<i>F</i> _{1/3}	$1 - (1 - \alpha)^{2/3}$	$(3/2)(1 - \alpha)^{1/3}$	Chemical process or mechanism	One-third order	1/3	0.9482 0.4472
<i>F</i> _{3/4}	$1 - (1 - \alpha)^{1/4}$	$4(1 - \alpha)^{3/4}$	non-invoking equations	Three-quarters order	3/4	0.9774 0.8948
<i>F</i> _{2/3}	$(1 - \alpha)^{-1/2} - 1$	$2(1 - \alpha)^{3/2}$		One and half order	1 + 1/2	0.9998 0.9997
<i>F</i> ₂	$(1 - \alpha)^{-1} - 1$	$(1 - \alpha)^2$		Second order	2	0.9961 0.9903
<i>F</i> ₃	$(1 - \alpha)^{-2} - 1$	$(1/2)(1 - \alpha)^3$		Third order	3	0.9706 0.9647
<i>P</i> _{3/2}	$\alpha^{3/2}$	$(2/3)\alpha^{-1/2}$	Acceleratory rate equations ^a	Mampel's power law		0.9288 0.4271
<i>P</i> _{1/2}	$\alpha^{1/2}$	$2\alpha^{1/2}$			2	0.8701 0.5297
<i>P</i> _{1/3}	$\alpha^{1/3}$	$3\alpha^{2/3}$			3	0.7759 0.7199
<i>P</i> _{1/4}	$\alpha^{1/4}$	$4\alpha^{3/4}$			4	0.5752 0.7859
<i>E</i> ₁	$\ln \alpha$	α		Exponential law		0.9660 0.9050
<i>A</i> ₁ , <i>F</i> ₁	$-\ln(1 - \alpha)^b$	$(1 - \alpha)$	Sigmoidal rate equations or random	Kolmogorov–Johnson–Mehl–	1	0.9892 0.9712
<i>A</i> _{2/3}	$-\ln(1 - \alpha)^{2/3}$	$2/3(1 - \alpha)[-\ln(1 - \alpha)]^{1/3}$	nucleation and subsequent growth	Avrami–Erofeev equation ^c	1.5	0.9870 0.9385
<i>A</i> ₂	$-\ln(1 - \alpha)^{1/2}$	$2(1 - \alpha)[-\ln(1 - \alpha)]^{1/2}$			2	0.9841 0.8663
<i>A</i> ₃	$-\ln(1 - \alpha)^{1/3}$	$3(1 - \alpha)[-\ln(1 - \alpha)]^{2/3}$			3	0.9755 0.3516
<i>A</i> ₄	$-\ln(1 - \alpha)^{1/4}$	$4(1 - \alpha)[-\ln(1 - \alpha)]^{3/4}$			4	0.9774 0.1435
<i>A</i> ₄	$\ln(\alpha/(1 - \alpha))$	$\alpha(1 - \alpha)$		Prout–Tomkins equation ^d		– 0.0016
<i>R</i> ₁ , <i>F</i> ₀ , <i>P</i> ₁	α	$(1 - \alpha)^0$	Deceleratory rate equations based	Contracting disk		0.9182 0.0109
<i>R</i> ₂ , <i>F</i> _{1/2}	$1 - (1 - \alpha)^{1/2}$	$2(1 - \alpha)^{1/2}$	on boundary reactions	Contracting cylinder	2	0.9182 0.6923
<i>R</i> ₃ , <i>F</i> _{2/3}	$1 - (1 - \alpha)^{1/3}$	$2(1 - \alpha)^{2/3}$		Contracting sphere	3	0.9724 0.8464
<i>D</i> ₁	α^2	$1/2\alpha$	Deceleratory rate equations: based	Parabolic rate law ^e		0.9334 0.9050
<i>D</i> ₂	$\alpha + (1 - \alpha) \ln(1 - \alpha)$	$[-\ln(1 - \alpha)]^{-1}$	on diffusion mechanism	Valensi equation ^f		0.8182 0.8368
<i>D</i> ₃	$(1 - (1 - \alpha)^{1/3})^2$	$3/2(1 - \alpha)^{2/3}[1 - (1 - \alpha)^{1/3}]^{-1}$		Jander equation ^g		0.9195 0.9483
<i>D</i> ₄	$1 - (2\alpha/3) - (1 - \alpha)^{2/3}$	$3/2[(1 - \alpha)^{-1/3} - 1]^{-1}$		(C)G–B equation ^h		0.9635 0.8887
<i>D</i> ₅	$((1 - \alpha)^{-1/3} - 1)^2$	$3/2(1 - \alpha)^{4/3}[(1 - \alpha)^{-1/3} - 1]^{-1}$		ZLT equation ⁱ		0.9985 0.9990
<i>D</i> ₆	$((1 + \alpha)^{1/3} - 1)^2$	$3/2(1 + \alpha)^{2/3}[(1 + \alpha)^{1/3} - 1]^{-1}$		Anti-Jander equation [84]		0.9192 0.4172
<i>D</i> ₇	$1 + (2\alpha/3) - (1 + \alpha)^{2/3}$	$3/2[(1 + \alpha)^{-1/3} - 1]^{-1}$		Anti-GB equation [85]		0.9241 –
<i>D</i> ₈	$((1 + \alpha)^{-1/3} - 1)^2$	$3/2(1 + \alpha)^{4/3}[(1 + \alpha)^{-1/3} - 1]^{-1}$		Anti-ZLT equation ⁱ		0.9025 –
<i>G</i> ₁	$1/2(1 - \alpha)$	$1 - (1 - \alpha)^2$	Another kinetics equations with unjustified mechanism			0.8098 0.8701
<i>G</i> ₂	$1/3(1 - \alpha)^2$	$1 - (1 - \alpha)^3$				0.6945 0.7952
<i>G</i> ₃	$1/4(1 - \alpha)^3$	$1 - (1 - \alpha)^4$				0.5799 0.6868
<i>G</i> ₄	$1/2(1 - \alpha)[-\ln(1 - \alpha)]^{-1}$	$[-\ln(1 - \alpha)]^2$				0.9910 0.9859
<i>G</i> ₅	$1/3(1 - \alpha)[-\ln(1 - \alpha)]^{-2}$	$[-\ln(1 - \alpha)]^3$				0.9915 0.9971
<i>G</i> ₆	$1/4(1 - \alpha)[-\ln(1 - \alpha)]^{-3}$	$[-\ln(1 - \alpha)]^4$				0.9918 0.9995
<i>G</i> ₇	$4\{(1 - \alpha)[1 - (1 - \alpha)]^{1/2}\}^{1/2}$	$[1 - (1 - \alpha)^{1/2}]^{1/2}$				0.9413 0.5890
<i>G</i> ₈	$6\{(1 - \alpha)^{2/3}[1 - (1 - \alpha)]^{1/3}\}^{1/2}$	$[1 - (1 - \alpha)^{1/3}]^{1/2}$				0.9588 0.6065

^a The rate-determining step is the nucleation of new phase.

^b The equation is also termed as the first order Mampel law.

^c The general equation of this model is $kt = [-\ln(1 - \alpha)]^{1/n}$ and then $\alpha = 1 - \exp(-kt^n)$.

^d The equation $(\ln(\alpha/(1 - \alpha))) = k(t - t_{1/2})$ is based on the assumption of linearly growing nuclei branching into chains.

^e The Wagner's parabolic rate law: $\alpha^2 = 2kt$, where the rate of diffusion slows down with increasing thickness of product layer (frequently observed during the oxidation of metals [64]).

^f Valensi used Fick's first diffusion law to express the kinetics of diffusion through spherical and cylindrical shell [86].

^g The process is controlled by the diffusion through nonporous layer of product of spherical geometry [86,87].

^h Crank, Ginstling, and Braunshtein's equation [86,88].

ⁱ Zhuravlev, Lesotkin and Tempelman [68].

3.2. Investigation of reaction mechanism

The course of synthesis during the thermal treatment of raw meal, the temperature range of stability of formed intermediates, their further reactions and the formation of feldspar strontian are schematically drawn in Fig. 6(a). The main reaction pathway is initiated by thermal decomposition of SrCO₃. The surplus of SrO on the reaction interface with particles of SiO₂ and Al₂O₃ leads to the formation of Sr₃Al₃O₆ and Sr₂SiO₄ at first, but with increasing temperature SrAl₂O₄ and SrSiO₃ prevail. Feldspar strontian firstly appears at the temperature of 1150 °C.

Within the temperature range from 1250 to 1425 °C, the product of thermal treatment contains Sr-gehlenite (Sr₂Al₂SiO₇) as well. That means that the relationship between Sr-gehlenite and celsian is probably not the same like that of gehlenite (Ca₂Al₂SiO₇) and anorthite (CaAl₂Si₂O₈), where anorthite is formed from gehlenite [20,22]. The formation of Sr-gehlenite is probably caused by the lack of SiO₂ in the reaction zone and by the site saturation of strontium.

These results (Fig. 6(a)) indicate that the formation of strontian and Sr-gehlenite should be considered as parallel (competing)

reactions of the same order, with the rate constants *k*₁ and *k*₂, respectively. Increasing temperature supports the rate of diffusion of SiO₂ to the reaction zone and Sr-gehlenite is next transformed to strontian (*k*₃). Under isothermal conditions, the formation of Sr₂Al₂SiO₇ and its further transformation to SrAl₂Si₂O₈ are sequential reactions. Strontian remains a stable phase from the temperature of 1150 to the melting temperature of 1660 °C and quantitative X-ray diffraction analysis shows, that the concentration ratio of strontian (*w*₁) to gehlenite (*w*₂) is higher than one (Table 1). From that the reaction rate constant of formation of strontian being higher than that of gehlenite (*k*₁ > *k*₂) can be estimated (Wegscheider principle [81]).

The system contains six compounds (*N* = 6; Al₂O₃, SiO₂, SrAl₂O₄, SrSiO₃, Sr₂Al₂SiO₇ and SrAl₂Si₂O₈) at the temperature of 1350 °C which are formed from four elements (*M* = 4; Sr, Al, Si and O). The number and the stoichiometry of independent chemical reactions were solved using the phase diagram and the Gibbs stoichiometric rule [82,83]

$$R = N - h \quad (5)$$

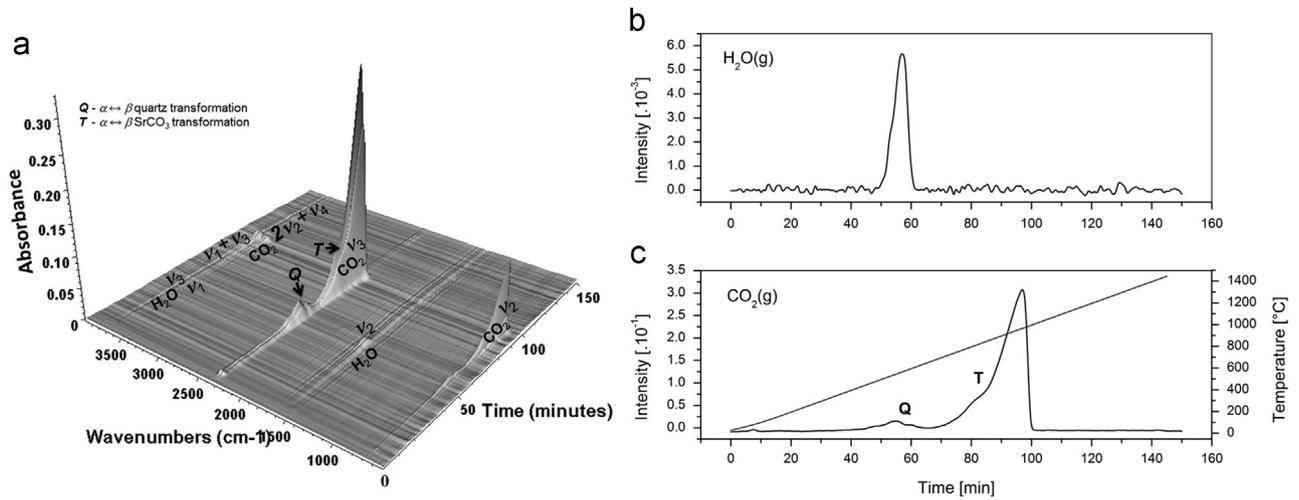


Fig. 4. EGA analysis of raw meal during the thermal treatment to 1450 °C (a) and time dependence of gaseous products released from the sample: water (b) and carbon dioxide (c).

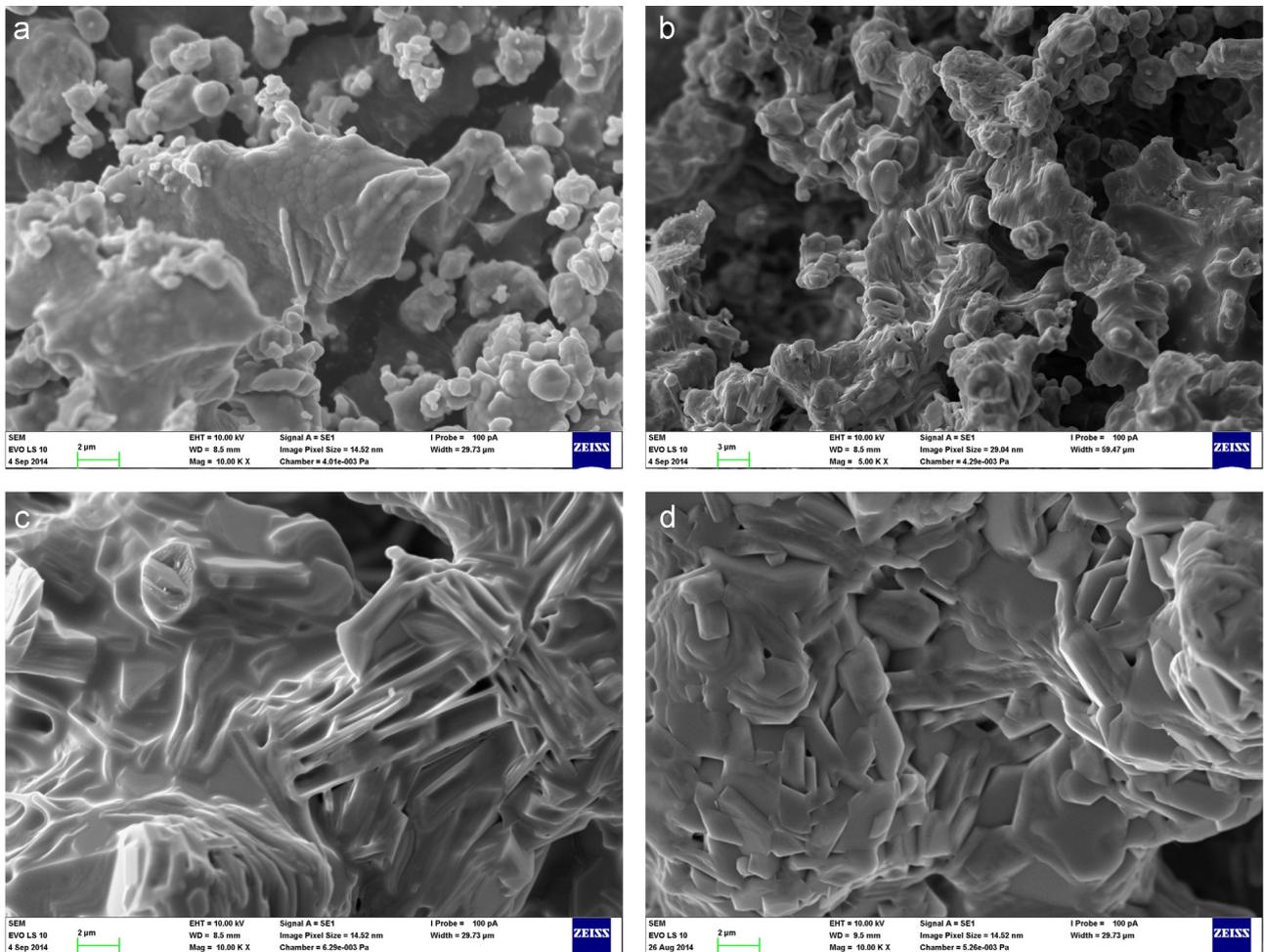


Fig. 5. SEM picture of strontian treated to the temperatures of 1150 °C (a), 1300 °C (b), 1400 °C (c) and 1660 °C (d).

chemical reaction of 3/2 order) as the most probable reaction mechanism of formation of strontian (Table 3) within the conversion interval (α) from 5% to 95%.

The kinetic plots for integral and differential method are shown in Fig. 7(a). Determined kinetic data are listed in Table 4. Both methods identify the same kinetic function ($F_{2/3}$) and the difference between determined values of apparent activation energy makes only 3.7%. The reconstruction of the process using determined kinetic

results is introduced in Fig. 7(b). Observed difference shows that the value of frequency factor is underestimated and overestimated for differential and integral method, respectively. Nevertheless, the average kinetic data are in a good agreement with the course of synthesis of strontian via applied ceramic method.

The calculation of the transition-state thermodynamic functions of investigated process is based on the Eyring (or Eyring–Polanyi) equation, which resembles the Arrhenius law [84,89] for the

temperature dependence of the rate constant ($k(T)$) [85–87,90]

$$k(T) = \kappa A \exp\left[-\frac{E_a}{RT}\right] = \kappa \frac{k_B T}{h} \exp\left[\frac{\Delta S^\ddagger}{R}\right] \exp\left[-\frac{\Delta H^\ddagger}{RT}\right]$$

$$= \kappa \nu \exp\left[-\frac{\Delta G^\ddagger}{RT}\right] = \kappa \nu K^\ddagger \quad (11)$$

where k_B , h and $\nu = k_B T/h$ are the Boltzmann ($1.381 \times 10^{-23} \text{ J K}^{-1}$), the Planck ($6.626 \times 10^{-34} \text{ J s}$) constant and the vibration frequency, respectively. The values of transmission coefficient κ are in many cases equal to unity [88,91]. The thermodynamic parameters of activated complex, including Gibbs free energy (ΔG^\ddagger), enthalpy (ΔH^\ddagger) and entropy (ΔS^\ddagger) of the process (initial state \leftrightarrow transition state [68,92]) were calculated using the relations [87,88,90]

$$\Delta H^\ddagger = E_{ap} - RT \quad (12)$$

$$\Delta S^\ddagger = R \left[\ln\left(\frac{hA}{k_B T}\right) - n \right] \quad [\kappa = 1] \quad (13)$$

where the reaction order n is about 1.5 (Table 3) and

$$\Delta G^\ddagger = \Delta H^\ddagger - T\Delta S^\ddagger \quad (14)$$

Calculated values of transition-state thermodynamic parameters are summarized in Table 4.

Assuming that both reactions are the same kinetics, Eq. (11) also provides the option to calculate the value of rate constant of the process at the temperature of 1300 and 1400 °C, i.e. for temperatures where k_1/k_2 ratios were determined (Table 1). This enables to calculate the reaction rate constant k_2 which is related to the formation of Sr-gehlenite. The results determined via both methods are introduced in Table 4.

4. Conclusion

The synthesis of feldspar strontian from raw meal composed of SrCO_3 , Al_2O_3 and SiO_2 powders via the ceramic route proceeds at the temperature ≥ 1150 °C. The main reaction pathway is initiated by the thermal decomposition of SrCO_3 , continues by the formation of binary oxide intermediates and primary strontian via the crystallization from non-equilibrium melt. Reactions between intermediates or the reaction of these intermediates with SiO_2 and Al_2O_3 lead to the formation of secondary strontian. The large plate-like particles of tertiary strontian were formed by recrystallization of primary and secondary strontian. Strontian becomes a single crystalline phase within the

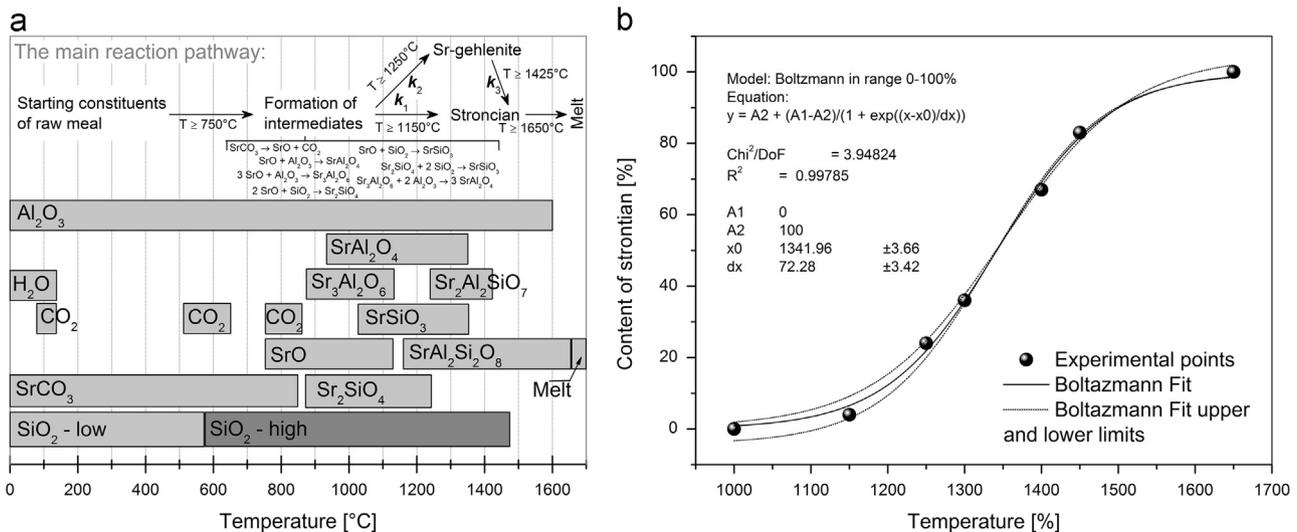


Fig. 6. The course of synthesis and the temperature range of stability of intermediates formed during the thermal treatment of raw meal (a) and the influence of temperature on the content of strontian (b).

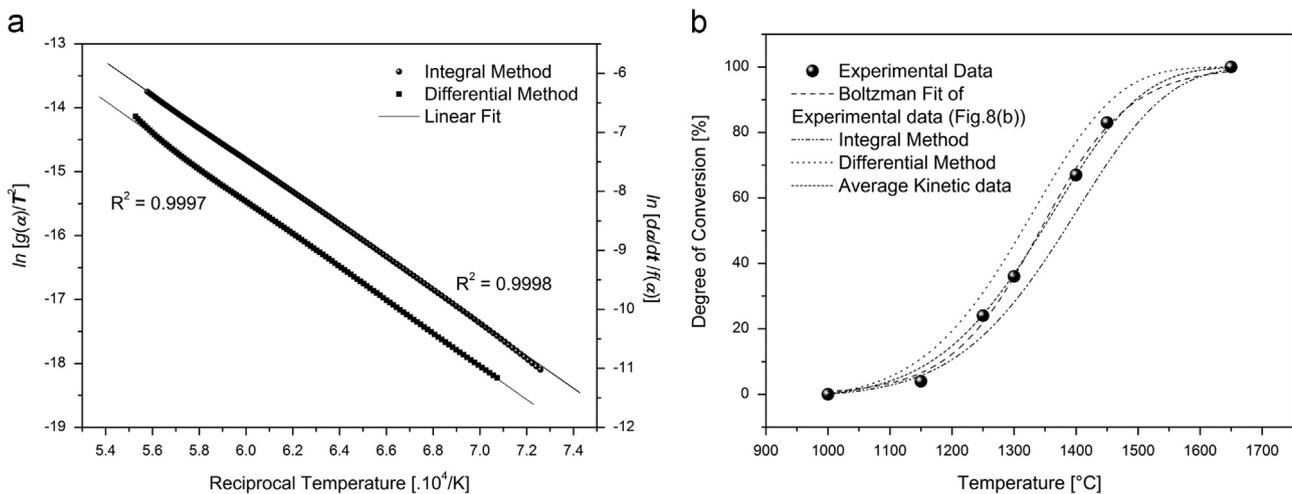


Fig. 7. Kinetic plots for integral and differential method (a) and the comparison of results calculated from assessed kinetic data with experiments (b).

Table 4

The overview of kinetic and thermodynamic parameters for activated state.

Parameter		Integral method	Differential method	Average
E_{ap}	[kJ mol ⁻¹]	225.1	233.5	229.3
A	[s ⁻¹]	2.4 · 10 ⁵	1.8 · 10 ⁵	2.1 · 10 ⁵
n	–	1½	1½	1½
ΔH^\ddagger	[kJ mol ⁻¹]	211.9 ± 0.2 [1000–1650 °C]	220.2 ± 0.2 [1000–1650 °C]	216.0 ^a
ΔS^\ddagger	[J (mol K) ⁻¹]	– 168.3 ± 0.1 [1000–1650 °C]	– 170.7 ± 0.1 [1000–1650 °C]	– 169.4 ^a
ΔG^\ddagger	[kJ mol ⁻¹]	480.8 ± 4.4 [1000–1650 °C]	493.2 ± 4.5 [1000–1650 °C]	486.9 ^a
$k_{1,1300\text{ °C}}$	[s ⁻¹ (mol m ⁻³) ¹⁻ⁿ] ^b	4.9 · 10 ⁻³	1.9 · 10 ⁻³	3.1 · 10 ^{-3a}
$k_{1,1400\text{ °C}}$		1.3 · 10 ⁻²	5.6 · 10 ⁻³	8.8 · 10 ^{-3a}
$k_{2,1300\text{ °C}}$		8.2 · 10 ⁻⁴	3.2 · 10 ⁻⁴	5.2 · 10 ^{-4a}
$k_{2,140\text{ °C}}$		1.3 · 10 ⁻³	5.7 · 10 ⁻⁴	8.8 · 10 ^{-4a}

^a The value was calculated from average data of apparent activation energy and frequency factor.^b As the dimension of rate constant depends on the reaction order n , the corresponding unit is [s (mol m⁻³)⁻¹].

temperature range from 1600 °C to the melting point temperature at 1660 °C. The analysis of phase composition and the Gibbs stoichiometric rule reveal three independent reactions, which lead to two parallel reaction pathways. Strontian can be formed via more favored reaction of single and binary oxides (k_1) or via the route using Sr-gehlenite as the intermediate (k_2).

The rate of one and half order reaction ($n=1\frac{1}{2}$) was recognized as the most probable mechanism (kinetic function $F_{2/3}:(1-\alpha)^{-1/2}-1$) of synthesis of feldspar strontian by both, integral and differential method. The values of apparent activation energy for integral and differential method were determined to be 225.2 and 233.5 kJ mol⁻¹, respectively. The comparison of experimental results with the course of the process calculated from average values of apparent activation energy (229.3 kJ mol⁻¹) as well as frequency factor (2.1×10^5 s⁻¹) provides a good agreement between those results. The average Gibbs free energy, enthalpy and entropy of activated complex were calculated to be 486.9 kJ mol⁻¹, 216.0 kJ mol⁻¹ and – 169.4 J (mol K)⁻¹, respectively.

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