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# KONSTRUKCE OPTIMÁLNÍ STRATEGIE ŘÍZENÍ ELEKTRICKÉHO VLAKU

# CONSTRUCTION OF THE OPTIMAL CONTROL STRATEGY FOR AN ELECTRIC-POWERED TRAIN

**Zkrácená verze Ph.D. Thesis**

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# Content

# 1 Introduction

## 1.1 Present state of the problem

The basic problem of the energy efficient train control was formulated and solved in some particular cases by Horn [8] in 1971 with use of the general form of the Pontryagin principle and relating mathematical tools. Since then, it has become a typical problem that can be solved with use of these means.

Many articles discussing this topic appeared especially during the nineties. The type of the optimal strategy consisting of four successive control levels (full power, speed holding, coasting and full braking) was introduced by Howlett et al. [13, 14]. Among articles dealing with various modifications of the basic problem we recall that Pudney et al. [26] considered a vehicle with discrete control settings and speed limits. Howlett et al. [11] discussed a track with non-zero gradient. Both of these assumptions were assumed by Cheng et al. [6] and Khmelnitsky [15]. Howlett and Pudney [12] summarized the above mentioned results. This theoretical background enabled the development of on-board computational systems (such as Metromiser or Freightmiser) for calculating of the efficient driving strategy which were successfully implemented in timetabled suburban and long-haul trains, e.g. in Brisbane or Toronto (see Yee et al. [27]). Let us note that some alternative approaches to these and relating problems were discussed e.g. by Han et al. [7], Howlett et al. [9, 10], Ko et al. [16], Li et al. [18], Liu et al. [19] and Pickhardt [20].

This thesis deals with the energy efficient train control problem and its modifications and introduces a different approach to developing the optimal control strategy along with exact calculation of the switching times and analysis of the solution based on the mathematical tools of nonlinear parametric programming.

First, we deal with the basic energy efficient train control problem introduced by Horn [8] in the following form. The aim is to minimize the objective functional

$$J = \int_0^T u^+(t)\, v(t)\, \mathrm{d}t \tag{1.1}$$

with respect to the system of differential equations

$$\begin{aligned} \dot{x}(t) &= v(t)\ , && (1.2)\\ \dot{v}(t) &= u(t) - r(v)\ , && (1.3) \end{aligned}$$

and boundary conditions

$$x(0) = 0\,,\ \ v(0) = 0\,, \tag{1.4}$$

$$x(T) = L\,,\ \ v(T) = 0\,. \tag{1.5}$$

The function $u^+$ is defined as follows

$$u^+(t) := \begin{cases} u(t)\,, & \text{for } u(t) > 0\,, \\ 0\,, & \text{for } u(t) \leq 0\,. \end{cases}$$

We assume that the control variable $u$ is a piecewise continuous function mapping the interval $[0, T]$ into $[-\alpha, \beta]$, where $\alpha, \beta > 0$ are given constants, and $r = r(v)$ is a differentiable function (with respect to $v$) with the properties $r, r' \geq 0$ and $r'(v)\,v$ is a nondecreasing function for $v \geq 0$. We shall illustrate our following considerations utilizing the linear and quadratic form of the resistance function $r$ (which satisfy the required properties). A generalization to the most common type of resistance function:

$$r(v) = bv + c(v)^2\,.$$

is only a technical matter.

Let us emphasize that the problem (1.1)–(1.5) describes the motion of a train along a straight level track of length $L > 0$ with minimal consumption of electric energy $J$. Without loss of generality let us further assume that the mass of the train $m = 1$. The phase coordinates $x$ and $v$ correspond to position and speed of the train, respectively. The given parameter $T$ represents the time that is available according to the timetable for the train to complete the track. The function $r$ represents the frictional resistance.

In the Section 2.2 we shall further assume a global speed constraint in the form

$$v(t) \leq v_m\,, \quad t \in \langle 0; T\rangle\,. \tag{1.6}$$

The given constant $v_m$ is the maximum allowed velocity of the train along the whole track.

Another modification of the basic problem will be introduced in the Section 2.3. We shall assume that the cost functional is in the form

$$J = \int_0^T (pu_\gamma v + q)\,\mathrm{dt} \quad \rightarrow \quad \min\,, \tag{1.7}$$

where function $u_\gamma$ satisfies

$$u_\gamma(t) := \begin{cases} u(t)\,, & \text{for } u(t) \geq 0\,, \\ \gamma u(t)\,, & \text{for } u(t) < 0\,. \end{cases}$$

Here, $0 < \gamma < 1$, $p, q > 0$ and $p + q = 1$ are given real input parameters. The intention in this case is to minimize the consumption of electrical energy as well as time of the journey (represented by parameter $T$) with prescribed weight parameters $p$ and $q$, respectively. The real parameter $\gamma$ represents the portion of the electrical energy that is being reloaded to the electrical circuit while braking.

Further, in the Section 2.4 we shall deal with the following system of differential equations

$$\dot{x}(t) = v(t) , \tag{1.8}$$

$$\dot{v}(t) = u(t) - r(v) + g . \tag{1.9}$$

To simplify our future considerations, we further assume that the constant $g$ satisfies $g \in (-\alpha, \beta)$ and $r(0) = 0$. Let us recall that the Equations (1.8) and (1.9) describe the motion of a train along a straight track with a constant gradient. Parameter $g$ represents the gravitational acceleration caused by the track gradient (obviously, $g > 0$ corresponds to downhill drive whereas $g < 0$ describes an uphill drive with a constant gradient).

Let us emphasize that the results presented in previous papers were more or less based on use of numerical methods for solving optimal control problems and thus the analysis of solution in the way introduced in this thesis could not have been performed. Most of the results presented in this thesis were introduced in the papers [21]–[24].

## 1.2 Utilized mathematical tools

Mathematical background utilized in the presented thesis consists especially in the mathematical theory of optimal control. We apply the Pontryagin principle and relating tools to develop the optimal control strategy for the presented problems and to derive relations for computation of the switching times. Further, we make use of the theory of nonlinear parametric programming in order to analyse the properties of the solution of introduced problems with respect to the input parameters. The mentioned theoretical results can be found in Bank [1], Brunovsky [3], Bryson et al. [4], Cermak [5] and Pontrjagin et al. [25].

## 1.3 Objectives of the thesis

The aim of this thesis is to develop the optimal control strategy for the energy-efficient train control problem (with emphasis on calculation of the switching times) and its analysis with respect to the values of the input parameters of the problem with use of nonlinear parametric programming. Further, we focus on the solution of some modifications of the basic energy optimal problem and discussion of its properties. We shall discuss an analytical solution of the presented problems which enable a more detailed analysis of the solution and more precise results. Moreover, we are going to derive an optimal control strategy for the time-energy optimal problem which introduces a completely different attitude to the energy optimal train control problem.

# 2 Main Results

This chapter summarizes the main results presented in the Ph.D. thesis. We present the solution of the introduced problems and its analysis with use of nonlinear parametric programming for individual optimal control problems.

## 2.1 Energy efficient train control

This section deals with the basic energy efficient train control problem (1.1)–(1.5) introduced by Horn [8]. This problem has been solved mainly with use of numerical methods. We describe an analytical approach that leads to development of an energy efficient train control with exact relations for computation of the switching times between driving modes and to introduction of the critical time as the key factor for determination of the optimal control strategy. Main results of this section were introduced in papers [22] and [24].

First, it is necessary to determine the value of the minimum time $T_{\min}$ that it is possible to complete the track within. Solving the corresponding minimum time problem we easily arrive at the well-known "bang-bang" control. Let us further assume that $T > T_{\min}$.

Let us recall the assertion which yields the energy efficient control strategy for the problem (1.1)–(1.5) (see e.g. Howlett [14]).

**Theorem 2.1.** *Let $(\hat{x}(t), \hat{v}(t); \hat{u}(t))$, $t \in \langle 0, T\rangle$ be the energy optimal solution of the problem (1.1)–(1.5). Then there exist $t_1$, $t_2$, $t_3$, where $0 < t_1 \le t_2 < t_3 < T$, such that*

$$\hat{u}(t) = \begin{cases} \beta & \text{for } 0 \le t < t_1\,, \\ r(\hat{v}(t)) \equiv const. & \text{for } t_1 \le t < t_2\,, \\ 0 & \text{for } t_2 \le t < t_3\,, \\ -\alpha & \text{for } t_3 \le t \le T\,. \end{cases}$$

The research of the author was directed mainly on the type of the relation between the switching times $t_1$ and $t_2$ (equality or sharp inequality) and other relating topics, especially the calculation of the switching times. *The type of the relation between $t_1$ and $t_2$ cannot be specified directly from Pontryagin principle.*

Let us assume that $t_1 = t_2$ and $r(v) = bv$. Then we obtain the equation for unknown $t_3$

$$Lb^2 + \alpha bT - \alpha bt_3 = \beta \ln\left(\frac{\alpha}{\beta}\,\mathrm{e}^{bT} - \frac{\alpha}{\beta}\,\mathrm{e}^{bt_3} + 1\right).$$

Consequently, the value of the switching time $t_1 = t_2$ is determined from the relation

$$t_1 = \frac{1}{b}\ln\left(\frac{\alpha}{\beta}\,\mathrm{e}^{bT} - \frac{\alpha}{\beta}\,\mathrm{e}^{bt_3} + 1\right)$$

and the value of the maximum speed $v_{\max}$ within the whole track according to the relation

$$v_{\max} = -\frac{\beta}{b}\left(\frac{\alpha}{\beta}\,\mathrm{e}^{bT} - \frac{\alpha}{\beta}\,\mathrm{e}^{bt_3} + 1\right)^{-1} + \frac{\beta}{b}\,.$$

Let us assume the relation $t_1 < t_2$. It is necessary to make use of the properties of the corresponding Hamilton function and Lagrange multipliers to conclude that

$$v\left(t_3\right) = \frac{v_{\max}}{2}\,.$$

The last equation represents the required third equation that we need to derive relations for calculation of the switching times for the case $t_1 < t_2$. Thus, we can derive the following equation for the unknown $t_2$

$$\left[\alpha\,\mathrm{e}^{b(T-t_2)} - 2\alpha - \beta\right] \ln\left[-\frac{\alpha}{\beta}\,\mathrm{e}^{b(T-t_2)} + \frac{2\alpha}{\beta} + 1\right] = Lb^2 + \alpha bT + \alpha bt_2 - \alpha \ln 2 - \alpha bt_2\,\mathrm{e}^{b(T-t_2)}$$

and relations for the remaining switching times $t_1$ and $t_3$ in the form

$$\begin{aligned} t_1 &= -\frac{1}{b}\ln\left[-\frac{\alpha}{\beta}\,\mathrm{e}^{b(T-t_2)} + \frac{2\alpha}{\beta} + 1\right]\,, \\ t_3 &= t_2 + \frac{1}{b}\ln 2\,. \end{aligned}$$

The value of the maximum velocity $v_{\max}$ can be determined based on the following relation

$$v_{\max} = \frac{\alpha}{b}\,\mathrm{e}^{b(T-t_2)} - \frac{2\alpha}{b}\,.$$

We can choose the optimal driving strategy based on the value of the cost functional $J$. With use of the relation (1.1) we can derive the relation for calculation of $J$

$$J = -\frac{\beta^2}{b^2} + \frac{\beta^2}{b}t_1 + \frac{\beta^2}{b^2}\,\mathrm{e}^{-bt_1} + b\left(v_{\max}\right)^2\left(t_2 - t_1\right)\,.$$

We easily choose the lower value (of course, if more than one of the two possible strategies $t_1 = t_2$, resp. $t_1 < t_2$, is feasible). Numerical calculations (based on algorithms from Bazaraa et al. [2]) show that the choice of the optimal control strategy depends only on the given value of the parameter $T$. In order to analyse the properties of the solution of the problem (1.1)–(1.5) with respect to the value of the parameter $T$ it is convenient to use the theory of nonlinear parametric programming with relating tools. To simplify the analysis let us assume that there exists a certain value $T_{\max}$, sufficiently large, with the property $T_{min} \leq T \leq T_{\max}$ and consider the case of the linear resistance function $r$.

Using the Theorem 2.1 we can easily rewrite the problem (1.1)–(1.5) into the following form of the nonlinear programming problem. We wish to minimize the objective function

$$J = \frac{\beta^2}{b^2}\left(bt_1 + \mathrm{e}^{-bt_1} - 1\right) + \frac{\beta^2}{b}\left(t_2 - t_1\right)\left(1 - \mathrm{e}^{-bt_1}\right)^2 \;\rightarrow\; \min \tag{2.1}$$

with respect to the equations

$$\alpha\left(\mathrm{e}^{b(T-t_3)}-1\right)=\beta\left(1-\mathrm{e}^{-bt_1}\right)\mathrm{e}^{b(t_2-t_3)}\,, \tag{2.2}$$

$$\alpha\left(t_3-T\right)+\beta\left(t_2-t_2\,\mathrm{e}^{-bt_1}+t_1\,\mathrm{e}^{-bt_1}\right)=bL \tag{2.3}$$

and inequalities

$$0\le t_1\le t_2\le t_3\le T\,. \tag{2.4}$$

Let us denote by symbol $M(T)$ the set of all feasible solutions of the given problem, i.e. the set of all $(t_1,t_2,t_3)$ satisfying the relations (2.2)–(2.4) for a given parameter $T$. Let us further introduce the following assumption:

**Hypothesis 2.1.** *The point-to-set mapping $M(T)$ is continuous in $T$ for all $T\ge T_{\min}$.*

Note that the validity of the Hypothesis 2.1 can be verified under specified values of the parameters $\alpha$, $\beta$, $b$ and $L$.

**Lemma 2.1.** *Let the Hypothesis 2.1 be fulfilled. Then the point-to-set mapping*

$$\psi(T):=\left\{(t_1,t_2,t_3)\in M(T)\,|\,J(t_1,t_2,t_3;T)=\varphi(T)\right\}\,,$$

*where*

$$\varphi(T):=\inf_{(t_1,t_2,t_3)\in M(T)}J(t_1,t_2,t_3;T)\,,$$

*is u.s.c.-B for every $T_{\min}\le T\le T_{\max}$.*

**Definition 2.1.** *A parameter $T$ is said to be the* critical time *of the problem (2.1)–(2.4) (we shall further denote it $T_{cr}$), if there exists an $\epsilon>0$ such that for $T=T_{cr}$ the nonlinear programming problem (2.1)–(2.4) has an optimal solution with the property $t_1=t_2$ and for $T\in(T_{cr},T_{cr}+\epsilon)$ the corresponding optimal solution satisfies $t_1<t_2$.*

**Lemma 2.2.** *Let $T_{cr}$ be the critical time of the problem (2.1)–(2.4) and let the Hypothesis 2.1 be fulfilled. Then $T_{cr}$ is the unique positive solution of the equation*

$$\alpha bT_{cr}+Lb^2+(\alpha+\beta)\ln\left(\frac{2\alpha+\beta}{\beta+\alpha\,\mathrm{e}^{bT_{cr}}}\right)=\alpha\ln 2\,. \tag{2.5}$$

**Theorem 2.2.** *Let $(t_1,t_2,t_3)$ be the optimal solution of the problem (2.1)–(2.4) and let the Hypothesis 2.1 be fulfilled. Then either $t_1=t_2$ for every $T\ge T_{\min}$ or there exists a unique value of $T_{cr}$ with the property that for $T\in\langle T_{\min},T_{cr}\rangle$ the optimal solution satisfies the relation $t_1=t_2$ and for $T>T_{cr}$ the property $t_1<t_2$ is fulfilled (moreover, the value $T_{cr}$ can be determined as the unique positive solution of the Equation (2.5)).*

## 2.2 Energy efficient train control with a speed constraint

This section is devoted to the description of the energy optimal driving strategy of an electric-powered train with a global speed constraint. Most of the results discussed in this section were introduced in the paper [23].

First, we need to specify the value of the maximal speed $v_{\max}$ of the train within the whole track under assumption of the basic problem (1.1)–(1.5) without any further constraints so that we determine whether the global speed constraint (1.6) is active ($v_{\max} \geq v_m$) or not. In the latter case, we may easily apply the results of the Section 2.1 (optimal strategy and the values of switching times) also for the case of the global speed constraint. The relevant relations for calculation of the value of $v_{\max}$ were presented in the Section 2.1. Let us therefore further assume that the relation $v_{\max} \geq v_m$ holds.

First, we have to determine the value of the minimal time $T^*_{min}$ that it is possible to complete the track within (involving the speed constraint (1.6)). In what follows, we assume that $T > T^*_{\min}$ and $v_{\max} > v_m$. It is possible to prove the following theorem.

**Theorem 2.3.** *Let $(\hat{x}(t), \hat{v}(t); \hat{u}(t))$, $t \in \langle 0, T\rangle$ be the energy optimal solution of (1.1)–(1.5) and (1.6). Let $r(v) = bv$ $(r(v) = c(v)^2)$. Then there exist $t_1$, $t_2$, $t_3$ such that*

$$\hat{u}(t) = \begin{cases} \beta & \text{for } t \in \langle 0, t_1) \ , \\ bv_m \quad (c(v_m)^2\,) & \text{for } t \in \langle t_1, t_2) \ , \\ 0 & \text{for } t \in \langle t_2, t_3) \ , \\ -\alpha & \text{for } t \in \langle t_3, T\rangle \ , \end{cases}$$

*where $0 < t_1 \leq t_2 < t_3 < T$.*

The case $t_1 = t_2$ corresponds to the relation $v_{\max} = v_m$. By integration of the Equations (1.2) and (1.3) on separate time intervals and involving the boundary conditions (1.4) and (1.5) it is easy to find the equations for calculation of the switching times $t_1$, $t_2$ and $t_3$ for both linear and quadratic resistance functions.
For $r(v) = c(v)^2$ we obtain the relation

$$t_1 = \frac{1}{\sqrt{\beta c}} \operatorname{arctanh}\left(\sqrt{\frac{c}{\beta}} v_m\right) .$$

Thereafter, we calculate the value of $t_3$ via the equation

$$\sqrt{\frac{c}{\alpha}} v_m \cot\left[\sqrt{\alpha c}\,(T - t_3)\right] - \ln v_m + \ln \sqrt{\frac{\alpha}{c}} \frac{\left|\cos\left[\sqrt{\alpha c}\,(T - t_3)\right]\right|}{\cot\left[\sqrt{\alpha c}\,(T - t_3)\right]} + cL$$
$$+ \sqrt{\frac{c}{\beta}} v_m \operatorname{arctanh}\left(\sqrt{\frac{c}{\beta}} v_m\right) = c v_m t_3 + 1 - \frac{1}{2} \ln\left(1 - \frac{c}{\beta} v_m^2\right)$$

and the value of $t_2$ from the relation

$$t_2 = t_3 + \frac{1}{c v_m} - \frac{1}{\sqrt{\alpha c}} \cot\left[\sqrt{\alpha c}\,(T - t_3)\right] .$$

The equations for computation of the switching time $t_3$ usually yield two different possible values of $t_3$. However, only one of them satisfies the relations $0 < t_1 \le t_2 < t_3 < T$.

## 2.3 Time-energy efficient train control

This section deals with the time-energy efficient train control, i.e. a problem where both the time and energy consumption ought to be minimised with prescribed weight coefficients. We assume a partial reloading of energy into electrical circuit while braking. Some basic features of the problem were discussed by Kundrat et al. [17] by use of a numerical approach. The results discussed in this section were presented in the paper [21].

The following theorems determine the character of the optimal control strategy.

**Theorem 2.4.** *Let* $(\hat{x}(t), \hat{v}(t); \hat{u}(t))$, $t \in \left\langle 0, \hat{T} \right\rangle$ *be the time-energy optimal solution of (1.7) and (1.2)–(1.5). Then*

$$\hat{u}(t) = \begin{cases} \beta & \text{for } \lambda_2(t) - p\hat{v}(t) > 0\,, \\ r(\hat{v}) \equiv const. & \text{for } \lambda_2(t) - p\hat{v}(t) = 0\,, \\ 0 & \text{for } \lambda_2(t) - p\hat{v}(t) < 0 \quad \wedge \quad \lambda_2(t) - p\gamma\hat{v}(t) > 0\,, \\ -\alpha & \text{for } \lambda_2(t) - p\gamma\hat{v}(t) < 0\,. \end{cases}$$

**Theorem 2.5.** *Let* $(\hat{x}(t), \hat{v}(t); \hat{u}(t))$, $t \in \left\langle 0, \hat{T} \right\rangle$ *be the time-energy optimal solution of (1.7) and (1.2)–(1.5). Then there exist* $t_1$, $t_2$, $t_3$, *where* $0 < t_1 \le t_2 < t_3 < \hat{T}$, *such that*

$$\hat{u}(t) = \begin{cases} \beta & \text{for } 0 \le t < t_1\,, \\ r(\hat{v}) \equiv const. & \text{for } t_1 \le t < t_2\,, \\ 0 & \text{for } t_2 \le t < t_3\,, \\ -\alpha & \text{for } t_3 \le t \le \hat{T}\,. \end{cases}$$

Let us now determine the values of the switching times $t_1$, $t_2$ and $t_3$ and the value of the total driving time $T$. Of course, this determination is possible if the type of resistance function is specified. We emphasize that for unspecified driving time the Hamilton function satisfies the relation $H \equiv 0$ for $t \in \langle 0, T \rangle$. Further, $\lambda_1(t) \equiv C_1 = const.$ on $\langle 0, T \rangle$.

Suppose that the relation $t_1 < t_2$ holds. Then, for the resistance function $r = cv^2$ we obtain the relation for optimal value of $v_{\max}$

$$v_{\max} = \sqrt[3]{\frac{q}{2pc}}\,.$$

Further, we may arrive at the following cubic equation for calculation of the velocity $v(t_3)$

$$-q + 3pcv_{\max}^2 v(t_3) - p\gamma c\,[v(t_3)]^3 = 0$$

with a single feasible root. Consequently, we obtain the following relations for calculation of the switching times $t_1$, $t_2$, $t_3$ and the total driving time $T$:

$$
\begin{aligned}
t_1 &= \frac{1}{\sqrt{\beta c}} \operatorname{arctanh}\left(\sqrt{\frac{c}{\beta}} \cdot \sqrt[3]{\frac{q}{2pc}}\right) , \\
t_2 &= t_1 + \frac{1}{c v_{\max}} \ln \left| \cos \arctan\left[\sqrt{\frac{c}{\alpha}} v\left(t_3\right)\right]\right| + \frac{L}{v_{\max}} \\
&\quad - \frac{1}{c v_{\max}} \ln\left[\frac{v_{\max}}{v\left(t_3\right)} \cosh\left(\sqrt{\beta c} t_1\right)\right] , \\
t_3 &= t_2 + \frac{1}{c}\left[v^{-1}\left(t_3\right) - v_{\max}^{-1}\right] , \\
T &= t_3 + \frac{1}{\sqrt{\alpha c}} \arctan\left[\sqrt{\frac{c}{\alpha}} v\left(t_3\right)\right] .
\end{aligned}
$$

In the case $t_1 = t_2$ we need to determine the values of three unknown parameters $t_1 = t_2$, $t_3$ and $T$. We arrive at the following relation for calculation of the value $v\left(t_3\right)$:

$$
\begin{aligned}
& q\left[1 + \left(\frac{c}{\alpha} + \frac{c}{\beta} \mathrm{e}^{2cL}\right) v^2\left(t_3\right)\right]^{\frac{3}{2}} + pcv^3\left(t_3\right) \mathrm{e}^{3cL} - \\
& \left[1 + \left(\frac{c}{\alpha} + \frac{c}{\beta} \mathrm{e}^{2cL}\right) v^2\left(t_3\right)\right] \cdot \left[q \,\mathrm{e}^{cL} + p\gamma c v^3\left(t_3\right) \mathrm{e}^{cL}\right] = 0 .
\end{aligned} \tag{2.6}
$$

The value of the maximal velocity $v_{\max} = v\left(t_1\right)$ can be calculated from the relation

$$
v_{\max} = \frac{v\left(t_3\right) \mathrm{e}^{cL}}{\sqrt{1 + \left(\frac{c}{\alpha} + \frac{c}{\beta} \mathrm{e}^{2cL}\right) v^2\left(t_3\right)}} > v\left(t_3\right) .
$$

The last inequality determines which of the roots of the Equation (2.6) it is necessary to choose in order to obtain a feasible solution of the problem. The equations for the determination of the values of switching times in case $t_1 = t_2$ are as follows:

$$
\begin{aligned}
t_1 = t_2 &= \frac{1}{\sqrt{\beta c}} \operatorname{arctanh}\left(\sqrt{\frac{c}{\beta}} \cdot v_{\max}\right) , \\
t_3 &= t_1 + \frac{1}{c}\left[v^{-1}\left(t_3\right) - v_{\max}^{-1}\right] , \\
T &= t_3 + \frac{1}{\sqrt{\alpha c}} \arctan\left[\sqrt{\frac{c}{\alpha}} v\left(t_3\right)\right] .
\end{aligned}
$$

We have determined the values of the switching times $t_1$, $t_2$, $t_3$ and the total driving time $T$ for both possible types of driving strategy which follow directly from the Pontryagin principle. We can choose again the optimal case based on the value of the cost functional $J$. This value can be calculated according to the following relation

$$
\begin{aligned}
J = & \frac{p\beta}{c} \ln \cosh\left(\sqrt{\beta c} t_1\right) + pc\left[\sqrt{\frac{\beta}{c}} \tanh\left(\sqrt{\beta c} t_1\right)\right]^3 \cdot \left(t_2 - t_1\right) + \\
& \frac{p\gamma\alpha}{c} \ln\left|\cos\left[\sqrt{\alpha c}\left(T - t_3\right)\right]\right| + qT .
\end{aligned} \tag{2.7}
$$

We easily choose the lower value (of course, if more than one of the two possible strategies $t_1 = t_2$, resp. $t_1 < t_2$, is feasible). As it is obvious from the numerical results, we can conjecture that there exists a certain value of the input parameter $p$ (that we will further call critical parameter and denote as $p_{cr}$) such that for $p \leq p_{cr}$ the optimal solution satisfies the relation $t_1 = t_2$ whereas for $p > p_{cr}$ it holds $t_1 < t_2$ (if the remaining input parameters $\alpha$, $\beta$, $\gamma$, $L$ and $c$ are fixed). Let us verify this conjecture and determine the value of $p_{cr}$ with use of the theory of nonlinear parametric programming (see Bank [1]). For the following considerations we will assume the resistance function $r = cv^2$ again.

First, we rewrite the original optimal control problem (1.7) and (1.2)–(1.5) by use of the Theorem 2.5 into the form of a nonlinear programming problem. We wish to minimize the objective function (2.7) with respect to the equalities

$$\begin{aligned} &\ln\left|\sqrt{\beta c}\,(t_3 - t_2)\,\frac{\sinh\left(\sqrt{\beta c}t_1\right)}{\cos\left[\sqrt{\alpha c}\,(T - t_3)\right]} + \frac{\cosh\left(\sqrt{\beta c}t_1\right)}{\cos\left[\sqrt{\alpha c}\,(T - t_3)\right]}\right| \\ &+ \sqrt{\beta c}\,(t_2 - t_1)\tanh\left(\sqrt{\beta c}t_1\right) - cL = 0\,, \end{aligned} \tag{2.8}$$

$$\sqrt{\alpha}\tan\left[\sqrt{\alpha c}\,(t_3 - T)\right]\cdot\left[\sqrt{\beta c}\,(t_3 - t_2) + \coth\left(\sqrt{\beta c}t_1\right)\right] + \beta = 0 \tag{2.9}$$

and inequalities

$$0 \leq t_1 \leq t_2 \leq t_3 \leq T\,. \tag{2.10}$$

We shall denote by $M\,(p)$ the set of all feasible solutions of the specified nonlinear programming problem, i.e. the set of all $(t_1, t_2, t_3, T)$ satisfying (2.8)–(2.10) for a given $p$. It is easy to see that the point-to-set mapping $M\,(p)$ is continuous in $p$ for all $p \in \langle 0, 1)$ (the set of feasible solutions of the problem actually does not depend on $p$).

**Lemma 2.3.** *The point-to-set mapping*

$$\psi\,(p) := \left\{(t_1, t_2, t_3, T) \in M\,(p)\,|\,J\,(t_1, t_2, t_3, T; p) = \varphi\,(p)\right\}\,,$$

*where*

$$\varphi\,(p) := \inf_{(t_1,t_2,t_3,T)\in M(p)} J\,(t_1, t_2, t_3, T; p)\,,$$

*is upper semicontinuous (according to Berge) for every* $0 \leq p \leq p_{\max} < 1$, $p_{\max} \in (0, 1)$.

**Definition 2.2.** *A parameter* $p$ *is said to be the critical parameter of the problem (2.7)–(2.10) (and we shall further denote it as* $p_{cr}$*) if there exists an* $\epsilon > 0$ *such that for* $p = p_{cr}$ *the nonlinear programming problem (2.7)–(2.10) has an optimal solution with property* $\hat{t}_1 = \hat{t}_2$ *and for* $p \in (p_{cr}, p_{cr} + \epsilon)$ *the corresponding optimal solution satisfies* $\hat{t}_1 < \hat{t}_2$.

**Lemma 2.4.** *Let* $p_{cr}$ *be the critical parameter of the problem (2.7)–(2.10). Then*

$$p_{cr} = \frac{1}{2cv_{cr}^3 + 1}\,, \qquad \textit{where} \qquad v_{cr} = \sqrt{\frac{1 - \eta^2}{\frac{c}{\beta} + \frac{c}{\alpha}\,\mathrm{e}^{-2cL}}} \tag{2.11}$$

*and $\eta$ is the unique solution, satisfying the relation $\eta > \mathrm{e}^{-cL}$, of the equation*

$$2\eta^3\,\mathrm{e}^{3cL} - 3\eta^2\,\mathrm{e}^{2cL} + \gamma = 0\,.$$

**Theorem 2.6.** *Let $\left(\hat{t}_1, \hat{t}_2, \hat{t}_3, \hat{T}\right)$ be the optimal solution of the problem (2.7)–(2.10). Then either $\hat{t}_1 = \hat{t}_2$ for every $p \in (0,1)$ or there exists a unique value $p_{cr}$ with the property that for $p \in (0, p_{cr})$ the optimal solution satisfies $\hat{t}_1 = \hat{t}_2$ and for $p \in (p_{cr}, 1)$ the relation $\hat{t}_1 < \hat{t}_2$ is fulfilled. Moreover, the value $p_{cr}$ can be found via the Equation (2.11).*

## 2.4 Optimal train control on a track with non-zero gradient

This section deals with the energy efficient train control under additional assumption of a non-zero track gradient. Let us note that most of the results discussed in this section have not been published yet and will be a subject of author's further investigation. In this section we are going to present the optimal control strategy for the problem (1.1), (1.4), (1.5), (1.8) and (1.9). First, we have to determine the value of the minimum time $T_{\min}$ again that it is possible to complete the track within (corresponding to "bang-bang" control). Let us further assume that the relation $T > T_{\min}$ is satisfied for given $T$.

We can prove the following theorem.

**Theorem 2.7.** *Let $(\hat{x}(t), \hat{v}(t); \hat{u}(t))$, $t \in \langle 0, T\rangle$ be the energy optimal solution of (1.1), (1.4), (1.5), (1.8) and (1.9). Then for $g \leq 0$ it holds*

$$\hat{u}(t) = \begin{cases} \beta & \text{for } \lambda_2(t) - \hat{v}(t) > 0\,, \\ r(\hat{v}) - g \equiv \text{const.} & \text{for } \lambda_2(t) - \hat{v}(t) = 0\,, \\ 0 & \text{for } \lambda_2(t) - \hat{v}(t) < 0 \quad \wedge \quad \lambda_2(t) > 0\,, \\ -\alpha & \text{for } \lambda_2(t) < 0\,. \end{cases} \tag{2.12}$$

*where $\lambda_2$ is defined by the corresponding adjoint system. For $g > 0$ there exists a certain value $T > T_{\min}$ (which we shall further denote as $T_c$) such that for $T < T_c$ the previous relation (2.12) is fulfilled, whereas for $T \geq T_c$ the optimal solution satisfies*

$$\hat{u}(t) = \begin{cases} 0 & \text{for } 0 \leq t < t_c \text{ (coasting)}, \\ -\alpha & \text{for } t_c \leq t < T_c \text{ (full braking)}, \\ -g & \text{for } T_c \leq t < T \text{ (standstill)}, \end{cases}$$

*where $0 < t_c < T_c \leq T$.*

The value of the time $T_c$ can be determined as the solution of the minimum time problem under assumption $u \in \langle -\alpha, 0\rangle$. Therefore,

$$T_c = \frac{1}{b}\ln\omega\,,$$

where $\omega$ satisfies the equation

$$\alpha\, \mathrm{e}^{Lb^2/\alpha} \cdot \omega^{(\alpha-g)/\alpha} - (\alpha - g) \cdot \omega - g = 0\,.$$

Let us note that for $g > 0$ and $T \geq T_c$ the optimal solution described in the previous theorem satisfies $J = 0$ and for $T > T_c$ is not unique.

It can be easily shown that the value of the switching time $t_c$ can be determined via the following relation for $r(v) = bv$

$$t_c = T \cdot \left(1 - \frac{g}{\alpha}\right) + \frac{bL}{\alpha}\,.$$

The following theorem specifies the optimal order of the driving modes for all values of the input parameters except for the case $g > 0$ and $T \geq T_c$.

**Theorem 2.8.** *Let $(\hat{x}(t), \hat{v}(t); \hat{u}(t))$, $t \in \langle 0, T\rangle$ be the energy optimal solution of (1.1), (1.4), (1.5), (1.8) and (1.9). Then for $g \leq 0$ there exist $t_1$, $t_2$, $t_3$, where $0 < t_1 \leq t_2 < t_3 < T$, such that*

$$\hat{u}(t) = \begin{cases} \beta & \text{for } 0 \leq t < t_1\,, \\ r(\hat{v}) - g \equiv \text{const.} & \text{for } t_1 \leq t < t_2\,, \\ 0 & \text{for } t_2 \leq t < t_3\,, \\ -\alpha & \text{for } t_3 \leq t \leq T\,. \end{cases}$$

*The assertion of this theorem is valid for $g > 0$ and $T < T_c$ as well (where the value $T_c$ was specified in the Theorem 2.7).*

Let us now determine the values of the switching times under the assumption $r(v) = bv$. First, let us suppose that $t_1 < t_2$. Optimal value of the velocity $v(t_3)$ satisfies the relation

$$v(t_3) = v_{\max} \cdot \frac{bv_{\max}}{2bv_{\max} - g}\,.$$

Consequently, we obtain the following relation for calculation of the velocity $v_{\max}$:

$$\begin{aligned} &(bv_{\max} - g + \alpha) \cdot \ln \frac{(\alpha - g) \cdot (bv_{\max} - g)}{(\alpha - g) \cdot (2bv_{\max} - g) + b^2 v_{\max}^2} - \alpha \cdot \ln \frac{bv_{\max} - g}{2bv_{\max} - g} \\ &= (\beta + g - bv_{\max}) \cdot \ln \frac{\beta + g - bv_{\max}}{\beta + g} + bL^2 - b^2 v_{\max} T \end{aligned}$$

and derive the following relations for calculation of the switching times:

$$\begin{aligned} t_1 &= -\frac{1}{b} \ln \left(1 - \frac{b}{\beta + g} v_{\max}\right)\,, \\ t_2 &= T + \frac{1}{b} \ln \frac{(\alpha - g) \cdot (bv_{\max} - g)}{(\alpha - g) \cdot (2bv_{\max} - g) + b^2 v_{\max}^2}\,, \\ t_3 &= T - \frac{1}{b} \ln \left[1 + \frac{b^2 v_{\max}^2}{(\alpha - g) \cdot (2bv_{\max} - g)}\right]. \end{aligned}$$

In the case $t_1 = t_2$ we arrive at the following relation for calculation of the switching time $t_1$

$$\alpha^\alpha \,\mathrm{e}^{Lb^2+\alpha bT-bgT} = \left[(\alpha-g)\,\mathrm{e}^{bT} - \beta\,\mathrm{e}^{bt_1} + \beta + g\right]^\alpha \cdot \mathrm{e}^{\beta bt_1}$$

and the equation for the determination of the value of the remaining switching time $t_3$

$$t_3 = \frac{1}{b}\ln\left[(\alpha-g)\,\mathrm{e}^{bT} - \beta\,\mathrm{e}^{bt_1} + \beta + g\right] - \frac{1}{b}\ln\alpha\,.$$

We can choose the optimal driving strategy again based on the value of the cost functional $J$. This value can be calculated according to the following relation

$$J = \beta\left(\frac{\beta+g}{b^2}\,\mathrm{e}^{-bt_1} + \frac{\beta+g}{b}t_1 - \frac{\beta+g}{b^2}\right) + (bv_{\max} - g)\,v_{\max}\,(t_2 - t_1)\;.$$

We easily choose the lower value (of course, if more than one of the control strategies $t_1 = t_2$, resp. $t_1 < t_2$, is feasible). Again, the numerical calculations show that the choice of the optimal control strategy depends only on the given value of the entry parameter $T$. A similar analysis to that introduced for the basic energy efficient train control problem in the Section 2.1 can be performed in this case as well. The resulting relation for calculation of the critical time under assumption of analogical condition to the Hypothesis 2.1 is as follows

$$T_{cr} = \frac{1}{b}\ln\frac{(\beta+g)}{(\alpha-g)}\cdot\frac{(bv_{cr}-g)^2 + \alpha\,(2bv_{cr}-g)}{(bv_{cr}-g)\cdot(\beta+g-bv_{cr})}\,,$$

where $v_{cr}$ can be determined according to the following equation

$$\left(\frac{bv_{cr}-g}{2bv_{cr}-g}\right)^\alpha \cdot \left(\frac{\beta+g-bv_{cr}}{\beta+g}\right)^{\beta+g} \cdot \left[\frac{(\alpha-g)\cdot(bv_{cr}-g)}{(\alpha-g)\cdot(2bv_{cr}-g)+b^2v_{cr}^2}\right]^{g-\alpha} = \mathrm{e}^{-b^2L}\;.$$

The behaviour of the optimal strategy will be a subject of author's further investigation.

# 3 Conclusion

The thesis describes the character of the optimal control strategy and the way of calculation of the switching times for the energy-efficient train control problem and its modifications. We performed an analysis of the solution for the presented mathematical models with use of nonlinear parametric programming. We introduced the concept of the critical time (or critical parameter) and explained its significance as the deciding factor for developing of the optimal control strategy.

We presented the basic energy-efficient train control problem under assumption of standard types of resistance function as well as some of the natural generalizations of the problem. We introduced and analysed the problem with speed constraint and discussed the problem with a non-zero track gradient. We formulated and completely solved the time-energy efficient train control problem which represents a different view on this area.

The emphasis was put mainly on exact form of solutions where the application of numerical methods is restricted only on solving algebraic equations. Let us note that most of the results presented in this thesis represent a different approach towards solving this problem than introduced in previous papers. This approach enabled a detailed analysis of the solution with use of analytical means.

The energy-efficient train control problem can be generalized or modified in several ways. The enhanced models can be more or less complicated than those presented in this thesis. However, the general behaviour of the solution of such problems will remain similar. The introduced optimal driving modes will be present in most of the models what was proved by implementation of the results on real railway or suburban traffic with positive results. The critical time (or critical parameter) and the relating analysis with use of nonlinear parametric programming can be applied on several models as well.

The natural generalizations and extensions to our results can be achieved especially for the speed constraints or track gradient. We may assume e.g. local speed constraints. The general form of the track gradient can be represented by the function $g(x)$ describing varying profile of the track. There will be performed a further investigation of the behaviour of the problem with constant track gradient as well. We may also further investigate steep inclines (declines) as it was discussed by Cheng et al. [6] or Howlett et al. [11]. Moreover, a combination of the restrictions and further assumptions may be applied. Further, there might be used another types of resistance functions, e.g. exponential form of the function $r(v)$.

Most of the input parameters presented in this thesis are not constant in real situations. Usually, we may observe stochastical behaviour with a mean value and a certain standard deviation based on the corresponding probability distribution. This can be applied e.g. for the maximum allowable accelaration of the train, for resistance function $r$ or constant $\gamma$ and results in a completely different approach to the problem.

The main aim of this thesis was to present an exact form of the solution for the energy efficient train control problem and its modifications where it is applicable. However, most of the problems mentioned in this section lead us to use some more or less sophisticated numerical methods or methods of artificial intelligence which was out of the scope of this thesis and will be a subject of author's future investigation.

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# Abstract

The Ph.D. thesis deals with the description of the nature of optimal driving strategy for an electric-powered train as well as the calculation of switching times of optimal driving regimes for standard types of resistance function. We apply the Pontryagin principle and related tools of optimal control theory to develop the optimal driving strategy and to derive equations for computation of switching times and the corresponding speed profiles. Besides the basic form of the energy efficient train control problem we consider also its modifications including the global speed constraint, track gradient as well as time-energy efficient train control. Moreover, we analyse also the solution with use of the theory of nonlinear parametric programming. The emphasize is put on exact forms of solutions with a minimal use of numerical methods.