A COMPARISON OF EDGE-BASED AND REGION-BASED SEGMENTATION PERFORMED WITH PDES

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Abstract: The paper discusses the level set segmentation method, which employs the solution of the partial differential equation (PDE) describing the multidimensional function whose zero slice in plane *xy* determines the contour of the object. One of the possible approaches to the contour evolution is using the image edges; the edges define the boundaries of the segmented objects. A major disadvantage of this procedure consists in the often unclear or noisy edges, a condition that may lead to incomplete segmentation. Such a drawback related to the curve evolution is solvable through utilizing regions with similar brightness levels; consequently, the actual segmentation cannot be affected by unclear transitions between the objects and the background. The final comparison then shows that both methods find specific application within the segmentation process.

Keywords: Level set, PDE, edge, region, segmentation

1. INTRODUCTION

In general terms, image segmentation consists in separating the objects from one another or from the background; the expected result is then embodied in a description of separated regions corresponding to the individual image objects. A wide range of segmentation methods are available, including, for example, thresholding, region clustering, or watershed; this paper nevertheless focuses on techniques that exploit partial differential equations, and the group of such instruments comprises, among others, the active contours and level set approaches.

Active contours utilize the evolution of an explicitly set parametric curve, an evolution influenced by image forces directing the curve to the object edges. Here, a problem occurs when the topology of the segmented objects is changed, namely, when curve splitting or merging is to be solved. For this reason, it then appears more advantageous to use the level set method, which utilizes the implicit definition of the curve as a zero slice in the *xy* plane, performed via a multidimensional function. This, or the level set, function is then characterized within the present article, where two relevant evolution options are compared; the options consist in the edge- and region-based methods.

2. LEVEL SET SEGMENTATION

The level set technique was proposed by S. Osher a J. A. Sethian in their late 1980s study [1]. The principle of the method relies on forming a level set function, $\phi(x, y, t)$, which has one dimension more than the parametric curve $C(x, y)$. This curve is a zero slice in the *xy* plane of the level set function (zero level set); thus, we have $C(x, y)=\{(x,y)\in \mathbb{R}^2: \phi(x, y, t)=0\}$. By forming the level-set function via utilizing the image energies, we can separate the objects. The formula describing the evolution of the level set function ϕ is comprised in Eq. (1) below:

$$\frac{\partial \phi}{\partial t} = -\nabla \phi \cdot \boldsymbol{v} \,, \tag{1}$$

where $\boldsymbol{v}$ expresses the evolution speed of the function ϕ and t is the time. The evolution speed is usually expressed by the force acting in the direction of the normal vector $\boldsymbol{n} = \dfrac{\nabla \phi}{|\nabla \phi|}$. The discussed equation (1) can then be modified as follows:

$$\frac{\partial \phi}{\partial t} = -\nabla \phi \cdot \upsilon = -\nabla \phi \cdot F \cdot \boldsymbol{n} = -\nabla \phi \cdot F \cdot \frac{\nabla \phi}{|\nabla \phi|} = F \cdot |\nabla \phi| \,. \tag{2}$$

Such a modified form (2) is commonly solved by means of the finite difference method.

2.1. EDGE BASED SEGMENTATION

Caselles et al. [3] propose a method for evolving the level set function via the image gradient (edges); this technique utilizes the edge indication function g, which assigns small values to a large gradient in a smoothed image. We then have

$$g = \frac{1}{1 + |\nabla I_\sigma|^2} \,, \tag{3}$$

where I_σ is an image treated with a Gaussian filter. Using the function g, the level set function can be written as follows:

$$\frac{\partial \phi}{\partial t} = g |\nabla \phi| div \left(\frac{\nabla \phi}{|\nabla \phi|} \right) + \nabla g \nabla \phi \,, \tag{4}$$

where the expression $\nabla g \nabla \phi$ ensures the direction of the contour C (zero level-set) towards the image edges. At this point, let us note that a number of other similar methods with different parameters have been published, but all of these employ edge-based representation of the image gradient ∇I.

2.2. REGION BASED SEGMENTATION

The above-discussed technique to develop the level-set function by means of the image gradient (edges) is satisfactory when the boundaries of the image are clearly defined. In reality, however, a substantial amount of images exhibit fluent transition of brightness levels between the objects in image, and the use of edges for segmentation is thus completely ineffective.

The problem of blurred edge boundaries can be eliminated through the region based approach, a solution proposed by Chan and Vese [4]. In the referenced paper, the authors employ the Mumford-Shah segmentation technique [5], where the initial phase consists in obtaining a model of homogeneous areas in the given image; such a model facilitates the determination of region boundaries even with no sharp edges in the original image. Let us define a curve C in the area Ω, which separates the regions Ω_1 and Ω_2. We have that C is a zero slice in the plane xy of level set functions ϕ; assume that Ω_1 is an object and Ω_2 the background. Each region is approximately homogeneous and has a certain value u^1 and u^2 for Ω_1 and Ω_2. To determine the relationship to a given area of the function ϕ, the Heaviside function H_ϕ: is used; we then have

$$H_\phi = \begin{cases} 1, & if \ \phi > 0, (x \in \Omega_1) \\ 0, & if \ \phi \le 0, (x \in \Omega_2). \end{cases} \tag{5}$$

As the presented function is not derivable due to discontinuity, H_ϕ is, in the region close to zero, replaced with a substituent such as a sigmoid function. Consequently, the derivation H_ϕ is zero

everywhere except at the boundary between the regions Ω_1 and Ω_2. Now we can define the energy functional, whose minimization enables us to derive the development of the level set function.

$$E(\mu_i, \Omega_i) = \int_{\Omega_1} \left(I(\mathbf{x}) - \mu_1 \right)^2 d\mathbf{x} + \int_{\Omega_2} \left(I(\mathbf{x}) - \mu_2 \right)^2 + v \left| \partial \Omega_1 \right| =$$

$$= \int_{\Omega} \left(I(\mathbf{x}) - \mu_1 \right)^2 - \left(I(\mathbf{x}) - \mu_2 \right)^2 H_\phi + \left(I(\mathbf{x}) - \mu_2 \right)^2 d\mathbf{x} + v \int_{\Omega} \left| \nabla H_\phi \right| d\mathbf{x}, \tag{6}$$
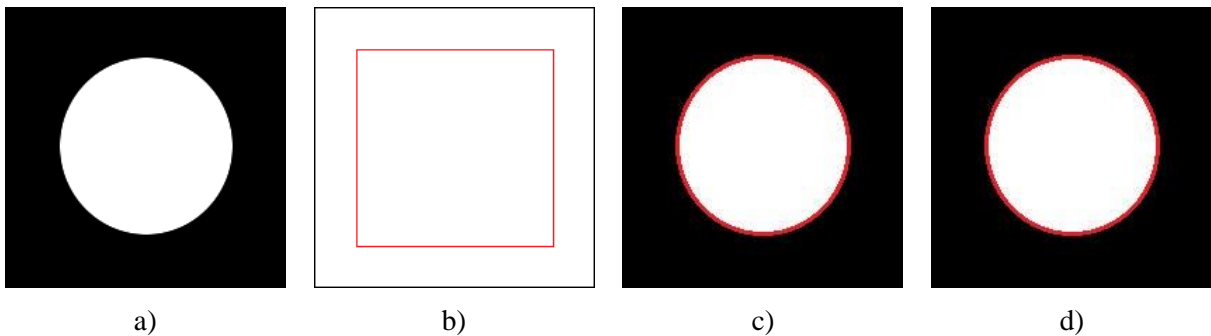
where $\boldsymbol{x}$ is the vector of the coordinates (x, y); $I(\boldsymbol{x})$ is the input image; v denotes the regularization constant; and μ_1 and μ_2 express the arithmetic mean of the values of the region inside, or outside respectively, the curve C. By minimizing the potential from the above formula (6), we obtain the PDE to solve the level-set function in time. The equation is as follows:

$$\frac{\partial \phi}{\partial t} = \delta(\phi) \left(v \, div \left(\frac{\nabla \phi}{|\nabla \phi|} \right) + \left(I(\mathbf{x}) - \mu_2 \right)^2 - \left(I(\mathbf{x}) - \mu_1 \right)^2 \right), \tag{7}$$

where $\delta(\phi)$ denotes the derivative of the Heaviside function (as mentioned above, this can be, for example, a sigmoid); the constant v is, as in formula (6), a regularization one; and the meaning of μ_1 and μ_2 is identical with that in the said formula (6).

3. APPLICATION OF THE METHODS

This chapter describes the actual application of the edge- and region-based method to illustrative images, which show the differences between the resulting segmentations. The first test image consists of a white circle on a black background; the related segmentation is shown in Fig. 1. The initial curve was selected to be identical for both techniques (Figure 1 b), and the methods also exhibit the same segmentation results. In the given context, it then follows from this that the procedures yield such identical results if two homogeneous areas are separated by a sharp edge (a large gradient). The times required to run the edge- and region-based techniques on the applied PC[1] corresponded to 0.72 s and 2.24 s, respectively; these times, however, constitute only rough values, which may change depending on a concrete implementation and its optimization. Yet, as can be inferred from the proposed data, the region-based method is generally more time-intensive compared to its counterpart procedure.
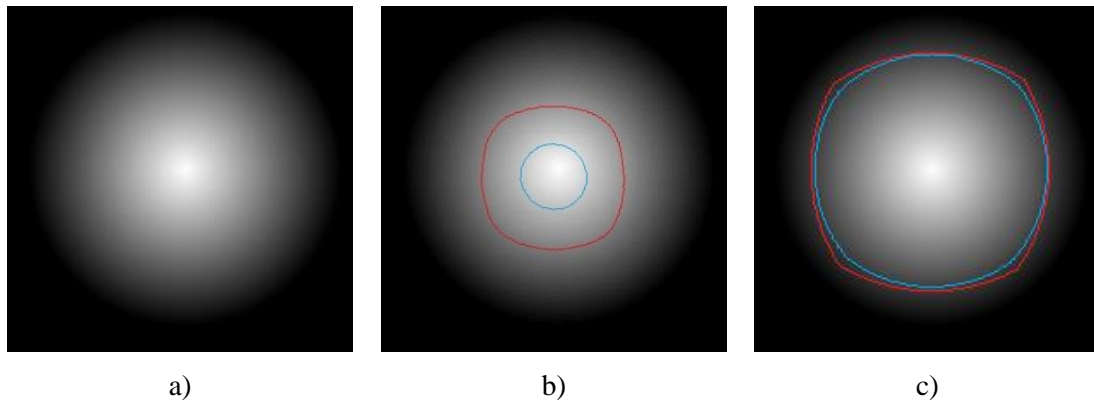

| a) | b) | c) | d) |

Figure 1: The segmentation of a simple shape: a) The input image; b) the initial contour; c) edge-based segmentation (100 iterations); and d) Chan-Vese segmentation (100 iterations).

Another case is presented in Fig. 2, where the input image again consists in a circle; the boundaries of this shape are nevertheless markedly blurred, thus constituting a fluent transition from the background to the object. In this situation, too, the initial curve is selected to be identical for both techniques (Figure 1 b). The described image configuration is nevertheless characterized by
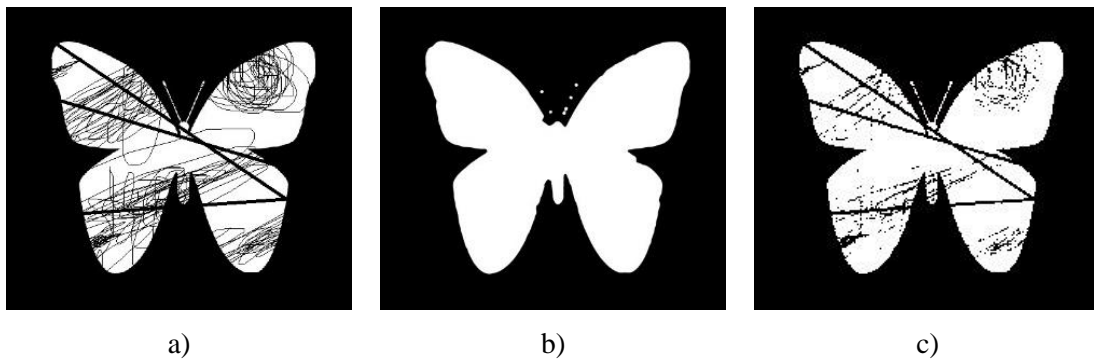
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[1] CPU: i5-4460 3.2 GHz, 8 GB RAM, 64bit Win7, implemented in Matlab 2013b.

complete failure of the edge-based method. To illustrate the procedure's inapplicability, Figure 2 b) shows the curve after 200 (red) and 300 (blue) iterations, indicating that the curve gradually diminishes toward the centre of the image; then, after more than 300 iterations, the curve disappears entirely, and the segmentation finally fails due to the absence of image edges, which otherwise serve as the stop function for the development of the level-set function. Conversely, region-based segmentation remains fully effective, similarly to its performance in the previous image (Fig. 1). Figure 2 displays the results obtained through segmentation via the region-based Chan-Vese method after 300 (red) and 800 (blue) iterations.

The relevant portion of Figure 2 represents an unsuccessful application of the edge-based method. But, in certain conditions, this approach offers major advantages over the other option discussed. In this connection, Figure 3 shows an example of segmenting a damaged object; the related segmentation results for the edge- and region-based procedures are indicated in Figure 3, b) and c). After being treated with the edge-based method, the object assumes a shape almost equal to the form it had before the damage occurred; thus, the actual operation could be most aptly defined as image restoration. As regards region-based segmentation, it is true that individual parts of the object are separated, but such a result in not desirable for the given task.

a)   b)   c)

Figure 2: The segmentation of a shape with smooth transition of the edges: a) the input image; b) edge-based segmentation (200 iterations/red, 300 iterations/blue); c) Chan-Vese segmentation (300 iterations/ red, 800 iterations/blue).

a)   b)   c)

Figure 3: The segmentation of a disrupted object: a) the original image; b) edge-based segmentation; c) region- based segmentation.

4. CONCLUSION

In the opening sections, the article discusses the level set segmentation method, whose general framework is introduced to support the subsequent presentation of two relevant solution approaches, namely, the edge-based and region-based (Chan-Vese) procedures.

The capabilities of the techniques are then compared using several test images. According to the results of the related test cycles, the edge-based approach can be used exclusively when the image objects are separated by a detectable edge and, as such, is not applicable at fluent transitions. This technique, however, still performs more than satisfactorily in segmenting disrupted images and is usable for image restoration (Figure 3 b).

The region-based variant of the level-set method finds application also in images that contain unclear edges, creating a model of homogeneous areas based on close brightness levels. The technique can be utilized in separating objects with fluent brightness level transitions at the boundaries; however, compared to the edge-based method, this approach is computationally more intensive.

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