ACCELERATED HYPERBOLIC NILT METHOD USED FOR FREQUENCY–DEPENDENT TRANSMISSION LINE SIMULATION

Nawfal Al–Zubaidi R–Smith

Doctoral Degree Program (3), FEEC BUT

E-mail: alzubaidi@phd.feec.vutbr.cz

Supervised by: Lubomír Brančík

E-mail: brancik@feec.vutbr.cz

Abstract: In this paper a proposed hyperbolic numerical inverse Laplace transform method is presented and utilized to simulate waves along a frequency dependent lossy transmission line. The method is accelerated via the quotient–difference (qd) algorithm of Rutishauser, which speeds up the convergence of residual infinite series. In general, Laplace transform is very useful in the solution of partial differential equations describing transient behaviour of linear dynamical systems, but the difficulty arises in obtaining their originals analytically. This difficulty can be overcome by using the numerical method. The paper presents an application of a proposed qd–NILT method to simulate voltage and/or current waveforms along frequency–dependent transmission lines. The technique was algorithmized in the Matlab language and experimentally verified.

Keywords: numerical inversion, Laplace transform, transmission line, quotient–difference, Matlab.

1 INTRODUCTION

Telegraphic equations describing transmission lines with distributed parameters are represented by first order partial differential equations in space–time domain [1]. Applying Laplace transform, with respect to time, simplifies the solution but the difficulty arises when attempting to return the results back to the time–domain analytically, especially when considering lossy or more sophisticated transmission lines analytical approaches become impossible [2]. In this paper a numerical techniques is presented which is accelerated with the quotient–difference algorithm [3].

Numerical Inverse Laplace Transform (NILT) methods are ranked among potential methods for time domain simulations [4], [5]; for instance the analysis of the transient phenomena in systems with distributed parameters [6], applications of electromagnetic transient simulations [7], computation of transient profiles along power transmission lines [2], or even more sophisticated systems. Generally, NILT methods are widely used in several scientific areas, mainly for the solution of respective differential equations. In the electrical engineering field, it is normally applied for solving the transient process of linear time-invariant systems [5–7]. Specifically, when some scientific problem is solved by using the Laplace domain solution $F(s)$, the solution process is greatly simplified, but the unavoidable step is to obtain the result in the objective time domain $f(t)$ [4]. In many situations, this is the most difficult or even impossible part to obtain by the standard process of using the inverse Laplace transform tables. For instance, when solving systems with distributed parameters we end up with Laplace transforms in the irrational or transcendental forms. This emphasizes the importance of introducing the NILT methods as a powerful tool to deal with similar situations [1]. The hyperbolic NILT method described in this paper yields a result with the possibility to increase its accuracy based on the number of arithmetic operations or namely, on the cost of the elapsed calculation time [8]. The obtained objective function from the NILT operation, i.e. the f_{en} is considered as if the function $f(t)$ would have passed through a nearly ideal low pass filter [8]. In previous works the Euler-transform was applied on the hyperbolic NILT method as a convergence acceleration enhancement to the method which resulted with a very high improvement to the proposed NILT method [8], [9]. For further reading in reference [9] several tests and error analysis for proposed NILT with the Euler-transform convergence acceleration case are presented.

2 THEORETICAL FOUNDATIONS AND RECAPITULATION OF NILT METHOD

2.1 PRINCIPLE FORMULA AND BASIC ASSUMPTIONS

The hyperbolic NILT method is based on approximating the Laplace transform inverse kernel e^{st} in the inverse Laplace transform definition Bromwich integral [3], [4], [8]

$$f(t) = \frac{1}{2\pi \mathrm{j}} \int_{c-\mathrm{j}\infty}^{c+\mathrm{j}\infty} F(s) e^{st} ds, \tag{1}$$

considering the following basic assumptions:

- $F(s)$ is regular for $\mathrm{Re}\{s\} > 0$,
- When $|s| \to \infty$ then $F(s) \to 0$,
- $F^*(s) = F(s^*)$.

2.2 APPROXIMATE FORMULAE

A resumption of the original hyperbolic NILT presented in [3], [8], and [9] is described in this section.

The Laplace transform inverse kernel e^{st} in (1) is approximated by the following hyperbolic relations:

$$K_{sh}(st,a) = e^{a} / 2\sinh(a - st), \tag{2}$$

$$K_{ch}(st,a) = e^{a} / 2\cosh(a - st). \tag{3}$$

The reciprocal hyperbolic functions are expressed by the infinite sum of rational functions in $z = (a - st)$,

$$\frac{1}{\sinh z} = \frac{1}{z} + 2z \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 \pi^2 + z^2}, \tag{4}$$

$$\frac{1}{\cosh z} = 2\pi \sum_{n=0}^{\infty} \frac{(-1)^n (n + 0.5)}{(n + 0.5)^2 \pi^2 + z^2}. \tag{5}$$

The devised formulas for both approximations are: [8]

$$f_{sh}(t,a) = \frac{e^a}{2t} F\left(\frac{a}{t}\right) + \frac{e^a}{t} \sum_{n=1}^{\infty} (-1)^n \mathrm{Re}\left\{ F\left(\frac{a}{t} + \frac{\mathrm{j}n\pi}{t} \right) \right\}, \tag{6}$$

$$f_{ch}(t,a) = \frac{e^a}{t} \sum_{n=1}^{\infty} (-1)^n \mathrm{Im}\left\{ F\left(\frac{a}{t} + \frac{\mathrm{j}(n - 0.5)\pi}{t} \right) \right\}. \tag{7}$$

Further enhancement of the method is achieved by considering the arithmetic mean:

$$f_{en}(t,a) = \frac{e^a}{2t} \left(\frac{1}{2} F\left(\frac{a}{t}\right) + \sum_{n=1}^{\infty} (-1)^n \left(\mathrm{Re}\left\{ F\left(\frac{a}{t} + \mathrm{j}n\frac{\pi}{t} \right) \right\} + \mathrm{Im}\left\{ F\left(\frac{a}{t} + \mathrm{j}\left(n - \frac{1}{2}\right)\frac{\pi}{t} \right) \right\} \right) \right). \tag{8}$$

It is conceived in [8], that the absolute error of formula (8) can be obtained by replacing the hyperbolic functions in (2) and (3) by their definition exponential functions and rearranging into the form of infinite geometric series. From the absolute error derived it is noticed that with a proper choice of the parameter a then the absolute error should be effectively minimized.

3 CONVERGENCE ACCELERATION BY QUOTIENT-DIFFERENCE TECHNIQUE

To practically simulate the NILT method, the infinite series are ought to be truncated up to a certain number of terms. To reduce the effect of truncation of infinite series the quotient-difference algorithm (Qd) is utilized [qd], which is a very useful technique to accelerate the convergence of infinite series, and has shown to be a technique with a high stability [3], [10]. Fundamentally, only $2P + 1$ terms are used as input data for the Qd computation. Accordingly, the finite series in (6) is evaluated up to n_{sum} terms, while the infinite series part can now be expressed in a finite form as

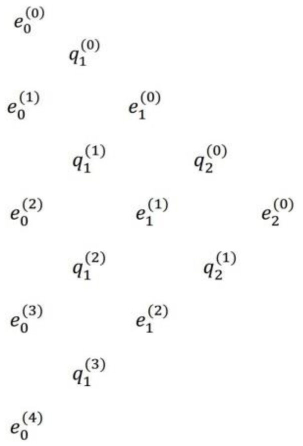
$$\sum_{n_{sum}+1}^{\infty} (-1)^n \operatorname{Re}\{F_{tn}\} \approx u(z,P) = \sum_{n_{sum}+1}^{n_{sum}+1+qd} z^n Q_n , \tag{9}$$

where qd=$2P$+1. The sum $u(z,P)$ is replaced by the corresponding continued fraction [10]

$$v(z,P) = d_0 /(1 + d_1 z /(1 + \cdots + d_{2P} z)), \tag{10}$$

which gives more precision to the result of the infinite sum than $u(z,P)$.

The lozenge diagram for the Qd algorithm computation is shown in Figure 1. The formulae for computing the Qd diagram are more detailed presented in [10].



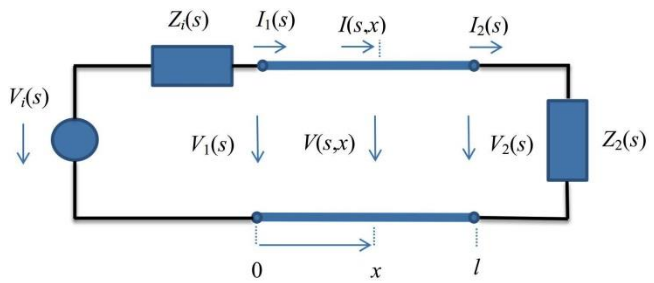


Figure 1: Quotient-difference diagram **Figure 2:** Transmission line Laplace model

After obtaining the Qd final part the result is then added to the truncated series summation, which improves the accuracy and efficiency of the proposed NILT method.

4 TRANSMISSION LINE APPLICATION

In this section, the proposed accelerated one-dimensional NILT method is used to simulate voltage/current waveforms on a frequency–dependent transmission line. Figure 2 proposes a Laplace–domain model of a uniform lossy transmission line. This model is obtained by using the Laplace transform of one variable to the telegraphic equations (kind of first order partial differential equations) [1]. When considering zero initial voltage and current distributions a result is:

$$-\frac{dV(x,s)}{dx} = Z_0(s) I(x,s) , \tag{11}$$

$$-\frac{dI(x,s)}{dx}=Y_0(s)V(x,s)\,, \tag{12}$$

where $Z_0(s) = R_0 + sL_0$ and $Y_0(s) = G_0 + sC_0$ are per-unit-length (p. u. l.) series impedance and shunt admittance, respectively, with R_0 ,L_0 ,G_0 , and C_0 are p. u. l. parameters that represent resistance, inductance, conductance and capacitance, respectively. After incorporating boundary conditions, then the Laplace transform of the solution is:

$$V(s,x)=V_i(s)\frac{Z_c(s)}{Z_i(s)+Z_c(s)}\cdot\frac{e^{-\gamma(s)x}+\rho_2(s)e^{-\gamma(s)[2l-x]}}{1-\rho_1(s)\rho_2(s)e^{-2\gamma(s)l}}\,, \tag{13}$$

$$I(s,x)=V_i(s)\frac{1}{Z_i(s)+Z_c(s)}\cdot\frac{e^{-\gamma(s)x}-\rho_2(s)e^{-\gamma(s)[2l-x]}}{1-\rho_1(s)\rho_2(s)e^{-2\gamma(s)l}}\,, \tag{14}$$

where l is length of the transmission line, $Z_c(s)$ and $\gamma(s)$ (the characteristic impedance and propagation constant respectively) are obtained by:

$$Z_c(s)=\sqrt{\frac{Z_0(s)}{Y_0(s)}},\ \ \gamma(s)=\sqrt{Z_0(s)\cdot Y_0(s)}\,, \tag{15}$$

It is known that at high frequency applications skin effect can play an important role by increasing resultant losses of the transmission line. The skin effect influence on transmission lines will be considered here to include frequency dependences of primary parameters. In general, skin effect has a more considerable impact than the polarization effect on the surrounding medium.

As is stated in [11], the skin effect above a certain frequency has an important impact on such applications. This is quite difficult to incorporate into a direct time-domain solution, but it is much easier by including in a frequency response calculation. Basically [11]:

$$Z(s)=R_0+sL_0+K\sqrt{s}\,, \tag{16}$$

where the latter term $K\sqrt{s}$ represents high–frequency internal resistance and including high–frequency inductive reactance. Consequently, the shunting admittance is given as:

$$Y(s)=G_0+sC_0\,. \tag{17}$$

Equations (16) and (17) are then substituted in (15) instead of $Z_0(s)$ and $Y_0(s)$,respectively. The reflection coefficients at the beginning and end of the line $\rho_1(s)$, $\rho_2(s)$ equal:

$$\rho_1(s)=\frac{Z_i(s)-Z_c(s)}{Z_i(s)+Z_c(s)},\ \ \rho_2(s)=\frac{Z_2(s)-Z_c(s)}{Z_2(s)+Z_c(s)}\,. \tag{18}$$

At this point by implementing the proposed hyperbolic Qd-NILT it is possible to obtain the time domain voltage and current waveforms at preselected points on the transmission line.

Let us consider a uniform lossy transmission line with the following characteristics:

l=3 m, R_0 =0.35 Ω/m, L_0=265 nH/m, C_0 =95 pF/m, G_0 =0.1 mS/m, Z_i =10 Ω, Z_2 =2.5 kΩ. The transmission line is excited with the voltage wave: $v_i(t)=\sin^2\left(\frac{\pi\cdot t}{2\times10^{-9}}\right)$, for $0\le t\le 2\times10^{-9}$ and $v_i(t)$=0 elsewhere. The intention of utilizing this sinusoidal waveform as an excitation wave is to simulate high frequency signals due to our interest of studying the frequency dependence transmission lines. Primarily, the Laplace transform of the excitation voltage waveform is obtained as: $V_i(s)=\frac{2\pi^2\left(1-\exp\left(-2\times10^{-9}s\right)\right)}{s\left(\left(2\times10^{-9}s\right)^2+4\pi^2\right)}$. By using the proposed hyperbolic Qd–NILT it is possible to obtain the voltage/current waveforms on any specified point along the line as can be seen in Figure 3 and Figure 4, respectively. Two cases are studied the frequency independence of transmission line, i.e. K=0, and frequency dependence of the transmission line i.e. $K = 2.5 \cdot 10^{-4}\ \Omega\sqrt{\mathrm{s}}/\mathrm{m}$.

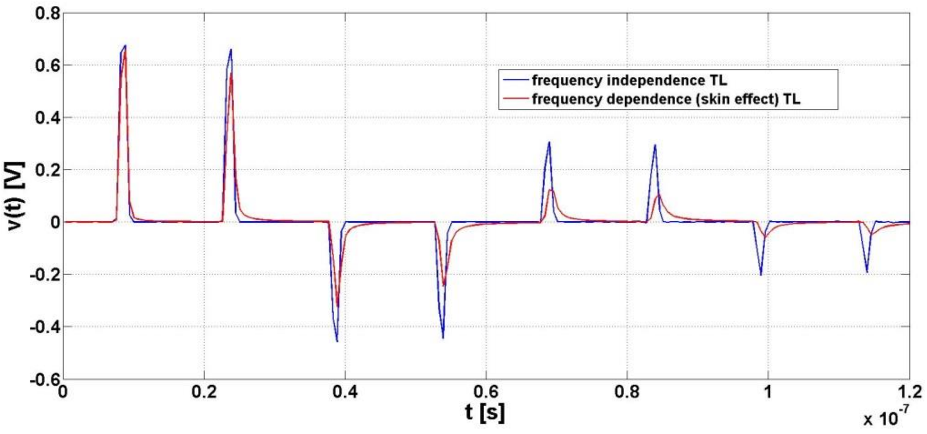


Figure 3: Voltage waveform at $l/2$ via hyperbolic Qd–NILT

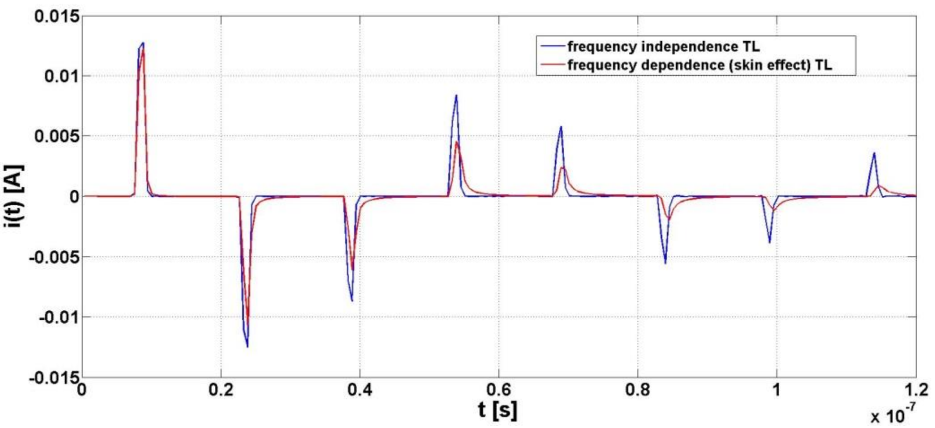


Figure 4: Current waveform at $l/2$ via hyperbolic Qd–NILT

As can be seen in Figure 3 and Figure 4 the voltage and current time–domain waveforms, respectively, are numerically obtained via the proposed accelerated hyperbolic Qd–NILT. The point of interest for the measurement along the transmission line can be simply preselected and analysed. It can be noticed from Figures 3, 4 that the frequency dependence skin effect simulation has an important role in high frequency applications. The further the waveform propagates on the TL the higher impact of skin effect is noticed, e.g. after 20 nanoseconds it can be clearly seen in Figure 3.

5 CONCLUSION

In this paper a proposed 1D hyperbolic NILT accelerated by means of the quotient–difference algorithm of Rutishauser is briefly presented. The presented technique is efficiently utilized in a practical application of electrical engineering, namely, the time–domain simulation of lossy frequency–dependence transmission line. The simulation of the voltage and current waveforms were successfully implemented and compared with frequency–independence transmission line. Using Matlab environment the CPU time duration for performing the simulation was approximately 540 msec for the voltage part. As for future work it is interesting to simulate the impact of skin effect on multi-dimensional transmission lines. Moreover, the current focus is to expand the hyperbolic Qd–NILT into higher dimensions to have the ability solve more sophisticated systems, e.g. applications that are mathematically described by more than one variable.

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