Comparison of Sensitivity Properties of Selected Models of Dynamical Systems

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Abstract

In this paper the piecewise-linear (PWL) autonomous dynamical systems are described. The sensitivity properties for all models are calculated (analytically and numerically). We will start with circuit model of 2nd-order system, which relative eigenvalue sensitivity of characteristic polynomials with respect to all circuit parameters change is calculated. Using the cascade models we can proceed to the third- and higher-order models and their relative sensitivity we can obtain easily from the lower-order models. In the last part of this paper the sensitivity of Chua's circuit is compared with the sensitivity of the third-order elementary canonical models.

Keywords

piecewise-linear dynamical system, state model, canonical model, Chua's circuit, sensitivity analysis

1. Introduction

The state models of circuits that we will study are autonomous. These nonlinear dynamical systems can be described by the system of 1st order ordinary differential equations (ODE) generally:

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}) \qquad \mathbf{x}(0) = \mathbf{x}_0 \qquad (1\text{-}1)$$

where $\mathbf{x} \equiv \mathbf{x}(t) \in \Re^n$ is called the state vector, the function $\mathbf{F}(\mathbf{x}(t)) \equiv \mathbf{F}(\mathbf{x})$ is an autonomous vector field, i.e. the vector field dependent only on state not on time, $\mathbf{x}_0$ are the initial condition and $\dot{\mathbf{x}} \equiv \dot{\mathbf{x}}(t)$ denote the derivative $\mathbf{x}(t)$ with respect to time. The vector field $\mathbf{F}(\mathbf{x})$ for the analysis of piecewise-linear dynamical systems is described by:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b} \qquad \mathbf{x}(0) = \mathbf{x}_0 \qquad (1\text{-}2)$$

where $\mathbf{b}$ is the column vector, $\mathbf{A}$ the state matrix.

Piecewise-linear (PWL) analysis means to divide the state space of a nonlinear dynamical system into a set of separate affine regions and to study each region separately and the result means "to glue" these pieces together.

The driving-point characteristic, which can have a different nonlinearity-gradient, simulates the nonlinearity in circuit [6] as shown in Fig.1, i.e.

$$i_R = G_b v_R + \frac{1}{2}(G_b - G_a)\cdot\left(|v_R + E| - |v_R - E|\right)$$

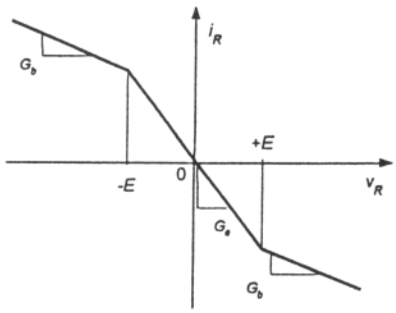

Fig. 1 Symmetric PWL function of nonlinear resistor

The gradient of PWL function has influence on dynamic behavior [7] as will be show in next chapters. PWL autonomous dynamical system belonging to Class C of vector field in $\Re^n$ can be described by state equation in matrix form [1]:

$$\dot{\mathbf{x}} = \mathbf{A}_n\mathbf{x} + \mathbf{b}h(\mathbf{w}^T\mathbf{x}) \qquad (1\text{-}3)$$

where $\mathbf{A_n} \in \Re^{n\times n}$, vectors $\mathbf{b}, \mathbf{w} \in \Re^n$ and the simplest form of PWL function is:

$$h(\mathbf{w}^T\mathbf{x}) = \frac{1}{2}\left(|\mathbf{w}^T\mathbf{x} + 1| - |\mathbf{w}^T\mathbf{x} - 1|\right) \qquad (1\text{-}4)$$

The function is continuous, odd-symmetric and partitiones the vector field $\Re^n$ by two parallel planes into the inner region and two outer regions.

The dynamical behavior of such systems is described by two sets of eigenvalues μ_k, ν_k representing two characteristic polynomials associated with corresponding regions [1], i.e. for:

-inner region $\rightarrow |w^T x| \langle 1$

$$\mathrm{P}(s) = \det(\mathbf{1}s - \mathbf{A}_0) = (s-\mu_1)(s-\mu_2)\cdots(s-\mu_n) = \\ = s^n - p_1 s^{n-1} + p_2 s^{n-2} + \cdots + (-1)^{n-1} p_n$$

-outer regions $\rightarrow |w^T x| \geq 1$

$$\mathrm{Q}(s) = \det(\mathbf{1}s - \mathbf{A}) = (s-\nu_1)(s-\nu_2)\cdots(s-\nu_n) = \\ = s^n - q_1 s^{n-1} + q_2 s^{n-2} + \cdots + (-1)^{n-1} q_n$$

(1-5a,b)

where $\mathbf{A_0} = \mathbf{A} + \mathbf{b}\mathbf{w}^T$, $\mathbf{1}$ is a unity matrix and coefficients q_k, p_k are the equivalent eigenvalue parameters.

2. Second-order Systems

The simplest models suitable for sensitivity analysis are the 2nd-order circuits. The state matrix $\mathbf{A}$ has general form $\mathbf{A_I}$ or a special form $\mathbf{A_{II}}$ [4], which will be used for next calculations:

$$\mathbf{A}_I = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \mathbf{A}_{II} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & 0 \end{bmatrix} \tag{2-1}$$

The form of characteristic polynomial is:

$$(s - \nu_1)\cdot(s - \nu_2) = s^2 - s\cdot q_1 + q_2 \tag{2-2}$$

The analytical results of relative sensitivity will be show in the next Chapter.

3. Third-order Systems

3.1 Chua's Circuit

The so-called ***Chua's circuit*** family represents the dynamical systems, which are canonical in the sense of the minimum of free parameters needed for their design [3]. The dimensionless Chua's equations [7] can be rewritten to the matrix form (1-3), where:

$$\mathbf{A} = \begin{bmatrix} -k\alpha(b+1) & k\alpha & 0 \\ k & -k & k \\ 0 & -k\beta & -k\gamma \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} k\alpha(b-a) \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{3-1}$$

Dimensionless Chua's equations represent the general, compact and universal ODE form suitable for the comparison with other ODE equivalent.

Many other state models can be derived utilizing the linear topological conjugacy. The topological conjugacy expresses the mutual relations between two qualitatively equivalent systems having the same eigenvalues. The topological conjugacy conditions are in [7].

3.2 Canonical ODE Models

These models are linearly conjugate to Chua's oscillator. They are canonical with respect to the behavior of the associated vector field, with respect to the number of circuit parameters, i.e. they contain minimum number of elements necessary. They have elementary relation between equation parameters and the equivalent eigenvalue parameters.

The state matrix of ***first canonical ODE model*** has form with equivalent eigenvalue parameters p_k, q_k where the main parameters are identical with the equivalent eigenvalue parameters for outer and inner region respectively:

$$\mathbf{A} = \begin{bmatrix} q_1 & -1 & 0 \\ q_2 & 0 & -1 \\ q_3 & 0 & 0 \end{bmatrix} \qquad \mathbf{A}_0 = \begin{bmatrix} p_1 & -1 & 0 \\ p_2 & 0 & -1 \\ p_3 & 0 & 0 \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} p_1 - q_1 \\ p_2 - q_2 \\ p_3 - q_3 \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{3-2}$$

Dual case to the first one represents ***second canonical ODE model***:

$$\mathbf{A} = \begin{bmatrix} q_1 & q_2 & q_3 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} p_1 - q_1 \\ p_2 - q_2 \\ p_3 - q_3 \end{bmatrix} \tag{3-3}$$

The forms of partial transform matrix $\mathbf{K}$, resultant transform matrix $\mathbf{T}$ and corresponding integrator–based circuit are in [8].

3.3 The Block-diagonal and Block-triangular Forms

The other state models are derived using linear topological conjugacy.

The ***block-diagonal*** $\mathbf{A}_{BD}$ ***and block-triangular*** $\mathbf{A}_{BT}$ ***forms*** of 3rd-order systems are based on the decomposition of the state matrix $\mathbf{A}$, i.e.:

$$\mathbf{A}_{BD} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_3 \end{bmatrix} \qquad \mathbf{A}_{BT} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{0} \\ \mathbf{A}_2 & \mathbf{A}_3 \end{bmatrix}$$

$$\mathbf{A}_1 = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \mathbf{A}_2 = a_{31} \quad \mathbf{A}_3 = a_{33} \tag{3-4}$$

Let $\nu_{1,2} = \nu' \pm j\nu''$ denote the complex conjugate eigenvalues and ν_3 the real eigenvalue. Then for the two cases can be written:

- Elementary Canonical Submatrix

$$\mathbf{A} = \left[\begin{array}{cc|c} \nu_1 + \nu_2 & -1 & 0 \\ \nu_1\nu_2 & 0 & 0 \\ \hline 0 & 0 & \nu_3 \end{array}\right] \tag{3-5a}$$

- Complex Decomposed Submatrix

$$\mathbf{A}=\left[\begin{array}{cc|c}\nu' & -\nu'' & 0\\ \nu'' & \nu' & 0\\ \hline 0 & 0 & \nu_3\end{array}\right] \qquad (3\text{-}5b)$$

For both cases is valid:

$$\mathbf{w}^T=\begin{bmatrix}1 & 0 & 1\end{bmatrix} \quad \text{and} \quad \mathbf{b}^T=\begin{bmatrix}b_1 & b_2 & b_3\end{bmatrix} \qquad (3\text{-}6)$$

The forms of partial transform matrix **K,** resultant transform matrix **T** and corresponding integrator–based circuit are in [8].

4. Fourth-order Systems

The higher-order models are based on decomposition of state matrix **A**. We can use the so-called cascade models [4]. The submatrices of state matrix **A** have form (generally):

$$\mathbf{A}_1=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix} \qquad \mathbf{A}_2=\begin{bmatrix}a_{13} & a_{14}\\ a_{23} & a_{24}\end{bmatrix}$$

$$\mathbf{A}_3=\begin{bmatrix}a_{33} & a_{34}\\ a_{43} & a_{44}\end{bmatrix} \qquad (4\text{-}1)$$

By comparing the individual polynomial coefficients with state matrix **A** we obtain conditions for new models of 4th-order [4]. Their forms are:

$$\mathbf{A}_1=\begin{bmatrix}\nu_1+\nu_2 & 1\\ \nu_1\cdot\nu_2 & 0\end{bmatrix} \qquad \mathbf{A}_3=\begin{bmatrix}\nu_3+\nu_4 & 1\\ \nu_3\cdot\nu_4 & 0\end{bmatrix} \qquad (4\text{-}2)$$

and the complex decomposed form for $\nu_{1,2}=\nu_1'\pm j\nu_1''$ and $\nu_{3,4}=\nu_3'\pm j\nu_3''$

$$\mathbf{A}_1=\begin{bmatrix}\nu_1' & \nu_1''\\ \nu_1'' & \nu_1'\end{bmatrix} \qquad \mathbf{A}_3=\begin{bmatrix}\nu_3' & \nu_3''\\ \nu_3'' & \nu_3'\end{bmatrix} \qquad (4\text{-}3)$$

5. Sensitivity Analysis

The sensitivity properties are very important for realization of all circuit models.

5.1 Eigenvalue Sensitivity

The relative sensitivity of state matrices eigenvalues parameters on all elements parameters change of the modeled circuit is determined by definition in [2]. For circuit elements $a_{i,j}$ of state matrix **A** the sensitivity has form:

$$S_r(\nu_k.a_{i,j})=\frac{a_{i,j}}{\nu_k}\cdot\frac{\partial\nu_k}{\partial a_{i,j}} \qquad \text{for } i,j,k=1.2..n$$

(5-1a)

$$\sum_{i,j=1}^{n}S_r(\nu_k,a_{i,j})=1 \qquad (5\text{-}1b)$$

for case $\mathbf{b}\neq\mathbf{0}$ [3] is

$$S_r[\mu_k,(a_{i,j}+b_i)]=\frac{(a_{i,j}+b_i)}{\mu_k}\cdot\frac{\partial\mu_k}{\partial(a_{i,j}+b_i)}=$$
$$=\frac{a_{i,j}+b_i}{b_i}\cdot S_r(\mu_k,b_i)=\frac{a_{i,j}+b_i}{a_{i,j}}\cdot S_r(\mu_k,a_{i,j}) \qquad (5\text{-}2)$$

As a special case is derived relative sensitivity of eigenvalues, separately for real part ν' and imaginary part ν'' of eigenvalues of the state matrix. Let $\nu_{1,2}=\nu'+j\nu''$. The relative sensitivity is given:

- for real part

$$S_r(\nu',a_{i,j})=\frac{a_{i,j}}{\nu'}\cdot\left(\frac{\partial\nu'}{\partial a_{i,j}}\right)=\frac{S'}{\nu'}(a_{i,j}) \qquad (5\text{-}3a)$$

- for imaginary part

$$S_r(\nu'',a_{i,j})=\frac{a_{i,j}}{\pm\nu''}\cdot\left(\frac{\partial\nu''}{\partial a_{i,j}}\right)=\frac{S''}{\pm\nu''}(a_{i,j}) \qquad (5\text{-}3b)$$

- For 2nd-order system relative sensitivity has the form:

$S_r(\nu_k,a_{11})=\dfrac{a_{11}}{\nu_k}\dfrac{\nu_k-a_{22}}{2\nu_k-q_1}$	$S_r(\nu_k,a_{22})=\dfrac{a_{22}}{\nu_k}\dfrac{\nu_k-a_{11}}{2\nu_k-q_1}$
$S_r(\nu_k,a_{12})=S_r(\nu_k,a_{21})=\dfrac{-a_{12}}{\nu_k}\dfrac{a_{21}}{2\nu_k-q_1'}$	

- The analytically derived results (for 3rd-order models) are presented in [3]. The best results we get for the block-diagonal and block-triangular form of state matrices in complex-decomposed form. These results are confirmed numerically too.

- The results for the block-diagonal form of the state matrix (for outer regions $\mathbf{D}_{-1,+1}$) generally:

for $k = 1,2$

$S_r(\nu_k,a_{11})=\dfrac{a_{11}}{\nu_k}\dfrac{\nu_k-a_{22}}{2\nu_k-q_1'}$	$S_r(\nu_k,a_{22})=\dfrac{a_{22}}{\nu_k}\dfrac{\nu_k-a_{11}}{2\nu_k-q_1'}$
$S_r(\nu_k,a_{12})=$ $=S_r(\nu_k,a_{21})=\dfrac{a_{12}}{\nu_k}\dfrac{a_{21}}{2\nu_k-q_1'}$	$S_r(\nu_k,a_{33})=0$

for k = 3 $\qquad S_r(\nu_k,a_{33})=1$

Real part sensitivity	*Imaginary part sensitivity*
$S_r(\nu',a_{11})=\dfrac{1}{2}\dfrac{a_{11}}{\nu'}$	$S_r(\nu'',a_{11})=-\dfrac{1}{2}\dfrac{a_{11}(\nu'-a_{22})}{\nu''^2}$
$S_r(\nu',a_{12})=S_r(\nu',a_{21})=0$	$S_r(\nu'',a_{12})=$ $=S_r(\nu'',a_{21})=-\dfrac{a_{12}a_{21}}{2\nu''^2}$
$S_r(\nu',a_{22})=\dfrac{1}{2}\dfrac{a_{22}}{\nu'}$	$S_r(\nu'',a_{22})=-\dfrac{a_{22}(\nu'-a_{11})}{2\nu''^2}$

• The analytical results of relative sensitivity of dimensionless parameters of Chua's oscillator have a complicated form. The sum of all eigenvalue sensitivities is not 1 (opposite to canonical state models). Most sensitive is the state model to gradient of the PWL function. For other ODE equivalents the sensitivity is more equally spread between the particular terms. It means, that these circuits are not so much sensitive to the parameters change.

The relative sensitivities for small different change of equivalent parameters are calculated too. The results proved regularity of the piecewise-linear method.

Tab.I Sets of equivalent eigenvalue parameters for relative sensitivity calculation

	q1	q2	q3	p1	p2	p3	
set1	-1,168	0,846	-1,295	0,090	0,43	0,653	fig.2
set2	-10,288	-1,200	-2,719	0,363	1,06	0,277	fig.3
set3	-1,910	0,141	-2,387	-0,122	11,93	7,655	fig.4
set4	-2,400	-0,710	-3,270	0,800	100,21	20,018	fig.5

Tab.II Calculated sensitivities for elementary canonical model

	a11(ν)	a12(ν)	a21(ν)	a23(ν)	a31(ν)
set1	0,533	0,056	-0,300	0,355	0,355
set2	0,971	0,013	-0,011	0,002	0,002
set3	0,608	0,124	-0,020	0,144	0,144
set4	0,606	0,151	0,060	0,092	0,092
	a11+b1	a21+b2	a31+b3	a12(μ)	a23(μ)
set1	0,035	-0,229	0,474	0,246	0,474
set2	0,089	-0,982	0,958	-0,024	0,958
set3	-0,006	-0,902	0,937	0,035	0,937
set4	0,002	-1,002	1,001	-0,001	1,001

Tab.III Calculated sensitivities for Chua's oscillator

	α(ν)	β(ν)	γ(ν)	b(ν)	sum
set1	14,065	5,298	-0,043	-28,103	-8,782
set2	13,624	2,570	2,217	24,875	43,286
set3	-0,453	-0,004	0,061	0,011	-0,384
set4	-0,273	0,000	0,002	0,000	-0,271
	α(μ)	β(μ)	γ(μ)	a(μ)	sum
set1	20,160	6,082	0,121	124,263	150,626
set2	887,592	133,753	796,181	-7050,098	-5232,572

Tab.IV Calculated sensitivities for block-diagonal state model

	a11	a12	a11re	a11im	a12re	a12im
set1	(-0,061i)	0,5+0,030i	1,000	-0,004	0,000	0,502
set2	(-0,039i)	0,5+0,069i	1,000	-0,019	0,000	0,510
set3	(-0,195i)	0,5+0,098i	1,000	-0,038	0,000	0,519
set4	(-0,3i)	0,5+0,15i	1,000	-0,090	0,000	0,545

Tab.V Calculated sensitivities for block-diagonal state model (min. nonzero parameters)

	a11	a12	a11re	a11im	a12re	a12im
set1	(-0,061i)	0,5+0,030i	1,000	-0,004	0,000	0,502
set2	(-0,039i)	0,5+0,069i	1,000	-0,019	0,000	0,510
set3	(-0,195i)	0,5+0,098i	1,000	-0,038	0,000	0,519
set4	(-0,3i)	0,5+0,15i	1,000	-0,090	0,000	0,545

Tab.VI Block-diagonal matrix (complex-decomposed)

	a11	a12	a11re	a11im	a12re	a12im
set1	0,002-0,030i	0,498+0,030i	0,500	0,000	0,000	0,500
set2	0,009-0,068i	0,491-0,068i	0,500	0,000	0,000	0,500
set3	0,018+0,094i	0,482-0,094i	0,500	0,000	0,000	0,500
set4	0,041+0,138i	0,459-0,138i	0,500	0,000	0,000	0,500

5.2 Equivalent Eigenvalue Sensitivity

The relative sensitivity of state matrices equivalent eigenvalues parameters on all elements parameters change of the modeled circuit is determined by eq. (5-1) in form:

for $k = 1,2,3$:

$$S_r(q_1, a_{i,j}) = \frac{1}{\sum_k \nu_k} \cdot \left[\nu_k S_r(\nu_k, a_{ij})\right]$$

$$S_r(q_3, a_{i,j}) = \sum_k S_r(\nu_k, a_{ij})$$

$$S_r(q_2, a_{i,j}) = \frac{1}{q_2} \cdot \begin{pmatrix} \nu_1 \cdot (\nu_2 + \nu_3) S_r(\nu_1, a_{ij}) + \\ + \nu_2 \cdot (\nu_1 + \nu_3) S_r(\nu_2, a_{ij}) + \\ + \nu_3 \cdot (\nu_1 + \nu_2) S_r(\nu_3, a_{ij}) \end{pmatrix} \quad (5\text{-}4)$$

The relative sensitivity of state matrices equivalent eigenvalues parameters on all elements parameters change of the 3rd-order modeled circuit of modeled systems is show in next.

1. *Elementary canonical model* of first type

$Sr(q_1, a_{11}) = 1$	$Sr(q_2, a_{11}) = 0$	$Sr(q_3, a_{11}) = 0$
$Sr(q_1, a_{12}) = 0$	$Sr(q_2, a_{12}) = 1$	$Sr(q_3, a_{12}) = 1$
$Sr(q_1, a_{21}) = 0$	$Sr(q_2, a_{21}) = 1$	$Sr(q_3, a_{21}) = 1$
$Sr(q_1, a_{31}) = 0$	$Sr(q_2, a_{31}) = 0$	$Sr(q_3, a_{31}) = 1$

2. *Block diagonal state matrix*

$Sr(q_1, a_{11}) = \frac{a_{11}}{q_1}$	$Sr(q_2, a_{11}) = \frac{a_{11}(a_{22} + a_{33})}{q_2}$	$Sr(q_3, a_{11}) = \frac{a_{11}a_{22}a_{33}}{q_3}$
$Sr(q_1, a_{12}) = 0$	$Sr(q_2, a_{12}) = \frac{-a_{12}a_{21}}{q_2}$	$Sr(q_3, a_{12}) = \frac{-a_{12}a_{21}a_{33}}{q_3}$
$Sr(q_1, a_{21}) = 0$	$Sr(q_2, a_{21}) = \frac{-a_{12}a_{21}}{q_2}$	$Sr(q_3, a_{21}) = \frac{-a_{12}a_{21}a_{33}}{q_3}$
$Sr(q_1, a_{22}) = \frac{a_{22}}{q_1}$	$Sr(q_2, a_{22}) = \frac{a_{22}(a_{11} + a_{33})}{q_2}$	$Sr(q_3, a_{22}) = \frac{a_{11}a_{22}a_{33}}{q_3}$
$Sr(q_1, a_{33}) = \frac{a_{33}}{q_1}$	$Sr(q_2, a_{33}) = \frac{a_{33}(a_{11} + a_{22})}{q_2}$	$Sr(q_3, a_{33}) = \frac{-a_{33}(a_{12}a_{21} - a_{11}a_{22})}{q_3}$
$\sum Sr(q_1, a_{ij}) = 1$	$\sum Sr(q_2, a_{ij}) = 2$	$\sum Sr(q_3, a_{ij}) = 3$

The relative sensitivity of higher order models (eg. 4th-order model) we obtain as addition of the sensitivities of corresponding models of lower-order systems (2nd and 3rd-order).

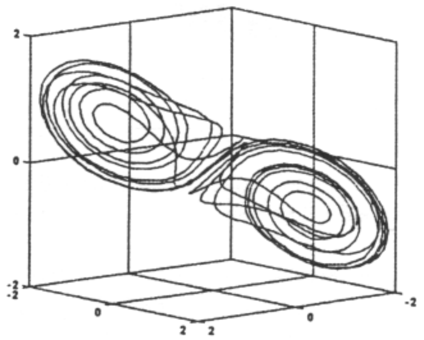

Fig.2 Chaotic attractor for equivalent eigenvalue in Tab.I, set 1

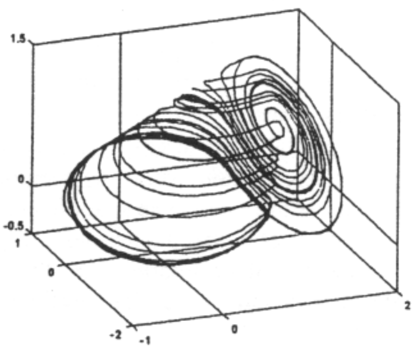

Fig.3 Chaotic attractor for equivalent eigenvalue in Tab.I, set 2

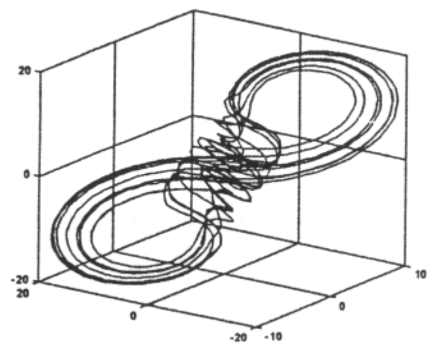

Fig.4 Chaotic attractor for equivalent eigenvalue in Tab.I, set 3

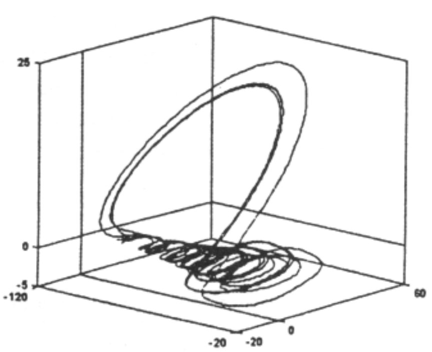

Fig.5 Chaotic attractor for equivalent eigenvalue in Tab.I, set 4

6. Conclusions

The relative sensitivity is important for finding the most acceptable chaotic phenomena model with the least number of nonzero parameters in state matrix and having the best stability for realization.

From the detail analytical and numerical results of relative sensitivity for new canonical models and the block-diagonal and block-triangular forms we see, that these models can be the best for practical realization. The sensitivity properties of Chua's circuit were numerically calculated to compare the results with results of the third-order elementary canonical models.

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