

# Class of Recursive Wideband Digital Differentiators and Integrators

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**Abstract.** *New designs of recursive digital differentiators are obtained by optimizing a general fourth-order recursive digital filter over different Nyquist bands. In addition, another design of recursive digital differentiators is also obtained by optimizing the specified pole-zero locations of existing recursive digital differentiator of second-order system. Further, new designs of recursive digital integrators are obtained by inverting the transfer functions of designed recursive digital differentiators with suitable modifications. Thereafter, the zero-reflection approach is discussed and then applied to improve the phase responses of the designed recursive digital differentiators and integrators. The beauty of finally obtained recursive digital differentiators and integrators is that they have nearly linear phase responses over wideband and also provide the choice of suitable recursive digital differentiator and integrator according to the importance of accuracy, bandwidth and the system simplicity.*

## Keywords

Digital differentiator, digital integrator, linear phase, recursive, wideband.

## 1. Introduction

The differentiators and integrators are highly useful devices to determine the time derivatives and integrals of measured signals. These devices are used extensively in the fields of signal processing, bio-medical engineering, radar engineering, control systems, accumulator analysis, waveform shaping etc.

In radars, the velocity and acceleration of objects are determined from the position measurements using differentiators [1]. The differentiators are also used to obtain higher-order derivatives of bio-medical data [2]. The frequency responses of an ideal differentiator and an ideal integrator are given by (1) and (2) respectively.

$$H_d(\omega) = j\omega, \quad (1)$$

$$H_i(\omega) = 1/(j\omega) \quad (2)$$

where  $j = \sqrt{-1}$  and  $\omega$  is the angular frequency in radians

per second. Initially, a simple circuit of resistance and capacitance is used to obtain differentiators and integrators. Further, the serial resistance-capacitance circuits, in conjunction with operational amplifiers have been used to design differentiators and integrators for more bandwidth [3]. In [4], Al-Alaoui has proposed a novel differential differentiator to solve the destabilizing effects of traditional differential differentiators. Later, he has also proposed a differential integrator with a built in high-frequency compensation [5]. In [6], he has developed low-frequency differentiators and integrators for bio-medical and seismic signals.

Several techniques have been developed to design digital differentiators and integrators by using recursive or non-recursive discrete-time systems in the study of digital signal processing (DSP) [7]-[34]. In general, the recursive digital differentiators have been obtained by inverting the transfer functions of derived recursive digital integrators with suitable modifications [7]. Initially, the recursive digital integrators have been designed by performing a simple linear interpolation between the magnitude responses of classical rectangular, trapezoidal and the Simpson digital integrators [7]-[12]. Further, a linear programming optimization approach is proposed to design recursive digital integrators [13]. Later, Al-Alaoui has proposed several designs of recursive digital differentiators by using various techniques [14]-[17]. In [18], [19]; Tseng has proposed new approaches to design recursive digital integrators by using fractional delay filters.

Initially, Ngo has proposed recursive wideband digital integrators and differentiators of third-order systems based on Newton-Cotes integration rule [20]. These designs approximate the ideal integrator or differentiator over the full Nyquist band with nearly 5% maximum percentage relative errors (MPREs) in magnitude responses. It is interesting to note that the Ngo integrator and differentiator have not more than 39.4° and 12.2° absolute phase errors from the ideal linear phase responses over wideband. Further, Gupta-Jain-Kumar (GJK) have proposed recursive wideband digital integrators and differentiators for nearly 3% MPREs in magnitude responses using the linear interpolation of three different digital integration techniques [21], [22]. It is noted that the GJK integrator and differentiator have nearly 34.2° and 11° maximum phase deviations from the ideal linear phase responses over wideband

[21]. Thereafter, Upadhyay has proposed recursive digital differentiators for 2% MPREs in magnitude responses over wideband [23]. It is interesting to note that the Upadhyay differentiators have comparatively more linearity in phase responses over wideband. In [24], Al-Alaoui has proposed 2-segment, optimized 3-segment and optimized 4-segment recursive digital integrators and differentiators for low relative errors in magnitude responses. Recently, Upadhyay-Singh (US) have designed new recursive digital differentiators and integrators for 0.48% MPREs in magnitude responses over the full Nyquist band except near to  $\omega = \pi$  [25].

It is noticed that in general, all the existing recursive wideband digital differentiators have not comparable magnitude and phase responses as in case of existing non-recursive digital differentiators [26]-[34]. However, recursive digital differentiators are more computationally efficient and easily convertible to recursive digital integrators. These reasons give more priority to use recursive digital differentiators as compared to non-recursive digital differentiators. Therefore, the design of recursive digital differentiators and integrators for highly accurate magnitude responses with good linearity in phase responses over wideband is the main issue in current research environment.

The practical implementation of digital signal processing systems is easily possible only up to the operating frequency of few hundreds of MHz by using the electronic circuits such as delay, adder, multiplier etc. Therefore, Hsue et al. have been proposed several techniques to design microwave differentiators and integrators using equal-length non-uniform transmission lines [2], [35]-[37].

In this paper, a class of new recursive digital differentiators and integrators is proposed by using the second-order and the fourth-order discrete-time systems. The beauty of the proposed class is that the finally obtained differentiators and integrators have good linearity in phase responses over wideband.

This paper is organized as follows: new designs of recursive digital differentiators and integrators are obtained in section 2. Further, the zero-reflection approach is discussed in section 3. In section 4, the zero-reflection approach is applied to improve the linearity in phase responses of designed recursive digital differentiators and integrators. The comparisons of the proposed and existing recursive digital differentiators and integrators are given in section 5. In last, the conclusions are given in section 6.

## 2. Design of Recursive Digital Differentiators and Integrators

### 2.1 Designs of Second-order Systems

In [25], Upadhyay-Singh have proposed a recursive wideband digital differentiator by optimizing the pole-zero

locations of existing recursive digital differentiator of second-order system [23]. The transfer function of US digital differentiator is given in (3). Now the error function  $E$  i.e. defined in (4), is minimized by optimizing the specified locations of one pole i.e.  $z = -0.571$  and one zero i.e.  $z = -1.99$  of US wideband digital differentiator  $H_{\text{usd}}(z)$  [25] over the full Nyquist band ( $\omega_c = \pi$ ). This minimization requires a general transfer function  $H(z)$ , which is defined in (5). The transfer function of recursive digital differentiator thus obtained is given in (6).

$$H_{\text{usd}}(z) = \frac{0.5805(z-1)(z+1.99)}{T(z+0.571)(z+0.11)}, \quad (3)$$

$$E = \int_0^{\omega_c} \left( |H(e^{j\omega}) - \omega \right)^2 d\omega, \quad (4)$$

$$H(z) = \frac{0.5805(z-1)(z+1.99 + \Delta z_2)}{T(z+0.571 + \Delta p_1)(z+0.11)} \quad (5)$$

where  $\Delta z_2$  and  $\Delta p_1$  are the optimization parameters. The bounds of  $\Delta p_1$  confirm the stability of the designed recursive digital differentiator.

Now, the corresponding recursive digital integrator is obtained by inverting the transfer function of the designed recursive digital differentiator  $H_{\text{dil}}(z)$  with suitable modifications as in [7]. The transfer function of the recursive digital integrator thus obtained is given in (7).

$$H_{\text{dil}}(z) = \frac{0.5805(z-1)(z+1.987)}{T(z+0.569)(z+0.11)}, \quad (6)$$

$$H_{\text{il}}(z) = \frac{T(z+0.569)(z+0.11)}{(0.5805 \times 1.987)(z+0.5033)(z-1)}. \quad (7)$$

### 2.2 Designs of Fourth-order Systems

First, consider a general transfer function of fourth-order recursive discrete-time system in coefficient form i.e. defined in (8), which is used to obtain new recursive digital differentiators according to the importance of accuracy and bandwidth. Now, this general transfer function is optimized to minimize the error function  $E$  i.e. already defined in (4), over different Nyquist bands by using the coefficient optimization technique as in [23]. The stability of optimized systems is confirmed by applying the pole-reflection approach [7].

It is observed that one of the optimized transfer functions is highly accurate over wideband, while another is useful over the full Nyquist band with good accuracy. Further, it is also observed that the both optimized transfer functions have comparatively more relative error near to  $\omega = 0$ . Therefore, the pole-zero optimization approach can be applied to match the magnitude response at  $\omega = 0$  as in [25]. It is noticed that the nature of all poles and zeros of the first optimized transfer function is only real, while for second is two real and two complex conjugate for poles

and zeros each. The general transfer functions required for pole-zero optimization are defined in (9) and (10) according to the nature of poles and zeros.

$$H_1(z) = \frac{K(1 + \alpha_1 z^{-1} + \alpha_2 z^{-2} + \alpha_3 z^{-3} + \alpha_4 z^{-4})}{T(1 + \beta_1 z^{-1} + \beta_2 z^{-2} + \beta_3 z^{-3} + \beta_4 z^{-4})}, \quad (8)$$

$$H_2(z) = \frac{K'(z-1 + \Delta a_1)(z + a_2 + \Delta a_2)}{T(z + b_1 + \Delta b_1)(z + b_2 + \Delta b_2)} \times \frac{(z + a_3 + \Delta a_3)(z + a_4 + \Delta a_4)}{(z + b_3 + \Delta b_3)(z + b_4 + \Delta b_4)}, \quad (9)$$

$$H_3(z) = \frac{K'(z-1 + \Delta a_1)(z + a_2 + \Delta a_2)(z^2 + \alpha'_1 z + \alpha'_2)}{T(z + b_1 + \Delta b_1)(z + b_2 + \Delta b_2)(z^2 + \beta'_1 z + \beta'_2)} \quad (10)$$

where  $K'$ ,  $a_{2-4}$ ,  $b_{1-4}$ ,  $\alpha'_{1,2}$  and  $\beta'_{1,2}$  depend upon the coefficient optimization;  $\Delta a_{1-4}$  and  $\Delta b_{1-4}$  are the optimization parameters for pole-zero optimization. The bounds of  $\Delta b_{1-4}$  confirm the stability of designed recursive digital differentiators. It is interesting to note that the bounds of  $\Delta a_1$  must be closure to zero to obtain the low relative error at  $\omega = 0$ . Now, the general transfer functions  $H_2(z)$  and  $H_3(z)$  are optimized to minimize the error function  $E$  i.e. already defined in (4), over different Nyquist bands using the optimization parameters  $\Delta a_{1-4}$  and  $\Delta b_{1-4}$ . Further, the scaling coefficient  $K'$  is also used to minimize the MPREs in magnitude responses over wideband. The transfer functions of recursive digital differentiators for  $\omega_c = 0.9\pi$  and  $\omega_c = \pi$  thus obtained are given in (11) and (12) respectively.

$$H_{d2}(z) = \frac{0.16556(z-1)(z+5.4945)(z+1.2788)(z-0.1019)}{T(z+0.8239)(z+0.295)(z-0.1014)(z+0.0365)} \quad (11)$$

$$H_{d3}(z) = \frac{0.5449(z-0.9971)(z+0.6384)(z^2-1.2174z+2.1232)}{T(z+0.6842)(z+0.1402)(z^2-0.5806z+0.4703)} \quad (12)$$

Now, the corresponding recursive digital integrators are obtained by inverting the transfer functions of the designed recursive digital differentiators  $H_{d2,3}(z)$  with suitable modifications as in [7]. The transfer functions of recursive digital integrators thus obtained are given in (13) and (14) respectively.

$$H_{i2}(z) = \frac{T(z+0.8239)(z+0.295)(z-0.1014)(z+0.0365)}{1.1633(z-1)(z+0.182)(z+0.782)(z-0.1019)} \quad (13)$$

$$H_{i3}(z) = \frac{T(z+0.6842)(z+0.1402)(z^2-0.5806z+0.4703)}{1.1569(z-0.9971)(z+0.6384)(z^2-0.5734z+0.4709)} \quad (14)$$

### 3. Zero-reflection Approach

It is noticed that the pole-reflection approach preserves the magnitude response of any recursive digital

filter, but affects the phase response of that filter [7]. The preserving nature of magnitude response also exists in the zero-reflection; it means that the zero-reflection approach can be easily used to improve the phase response of any digital filter. Let a general transfer function of a stable recursive digital filter  $H_0(z)$  of third-order system, which has two complex conjugate zeros at  $z = z_1, z_1^*$  and one real non-unity zero at  $z = a$ . The transfer function  $H_0(z)$  can be easily modified by reflecting the non-unity zeros either from outside the unit-circle to inside or vice-versa with suitable modifications, so that the magnitude response of resultant transfer function  $H_m(z)$  should be same as of the original transfer function  $H_0(z)$ . The procedure to find the resultant transfer function is clearly explained in (15).

$$H_m(z) = \frac{a|z_1|^2(z-(1/a))(z-(z_1/|z_1|^2))(z-(z_1^*/|z_1|^2))}{(z-p_1)(z-p_2)(z-p_3)} \quad (15)$$

where  $p_1, p_2$  and  $p_3$  are the general pole locations of original stable system  $H_0(z)$ . It can be easily seen that the zero-reflection approach can be used to improve the phase response of any discrete-time system. Let a digital filter has two real non-unity zeros, then the zero-reflection approach can be applied on both zeros one by one and also simultaneously on both zeros. This will result in to three additional digital filters of same magnitude responses, but having the different phase responses. Therefore, it is obvious that the improvement in phase response is always possible and this possibility will be more in case of higher-order discrete-time system.

## 4. Design of Linear Phase Recursive Digital Differentiators and Integrators

### 4.1 Designs of Second-order Systems

Now the zero-reflection approach as discussed in the previous section is applied to improve the phase responses of the designed recursive digital differentiator and integrator of second-order systems, which are given in (6) and (7) respectively. The designed differentiator in (6) has only one non-unity zero and the integrator in (7) has two non-unity zeros, it means that one another design of the recursive digital differentiator and three other designs of recursive digital integrators can be easily obtained by applying the zero-reflection approach over (6) and (7). The transfer functions of recursive digital differentiator and integrators thus obtained are given in (16) and (17-19) respectively.

$$H_{d4}(z) = \frac{(0.5805 \times 1.987)(z-1)(z+0.5033)}{T(z+0.569)(z+0.11)}, \quad (16)$$

$$H_{i4}(z) = \frac{0.569T(z+1.7575)(z+0.11)}{(0.5805 \times 1.987)(z+0.5033)(z-1)}, \quad (17)$$

$$H_{i5}(z) = \frac{0.11T(z+0.569)(z+9.0909)}{(0.5805 \times 1.987)(z+0.5033)(z-1)}, \quad (18)$$

$$H_{i6}(z) = \frac{(0.569 \times 0.11)T(z+1.7575)(z+9.0909)}{(0.5805 \times 1.987)(z+0.5033)(z-1)}. \quad (19)$$

Fig. 1 shows the phase responses of the designed recursive digital differentiators and integrators of second-order systems, which are given in (6), (7) and (16-19). From Fig. 1, it is observed that the designed recursive digital differentiator  $H_{d4}(z)$  and integrators  $H_{i1,5}(z)$  have not more than  $6^\circ$  absolute phase errors from the ideal linear phase responses over wideband. Further, it is also observed that the designed integrator  $H_{i4}(z)$  has good approximation of ideal phase response ( $-90^\circ$ ) up to the Nyquist frequency  $\omega = 0.7\pi$ .

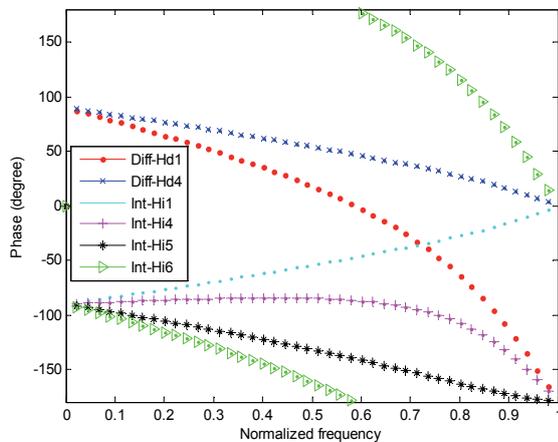


Fig. 1. Phase responses of the designed recursive digital differentiators and integrators of second-order systems.

### 4.2 Designs of Fourth-order Systems

As in case of second-order systems, the zero-reflection approach is also applied to improve the phase responses of designed digital differentiators and integrators of fourth-order systems, which are given in (11-14). The designed differentiator in (11) has three non-unity zeros and integrator in (13) has four non-unity zeros, it means that seven additional designs of recursive digital differentiators and thirteen additional designs of recursive digital integrators can be easily obtained by applying the zero-reflection approach over (11) and (13) respectively. However, the designed differentiator in (12) and integrator in (14) have two real non-unity zeros and two complex conjugate zeros. It means that seven additional designs of recursive digital differentiators and seven additional designs of recursive digital integrators can be easily obtained by applying the zero-reflection approach over (12) and (14) respectively. It is observed that few of the obtained designs after applying the zero-reflection approach have good linearity in phase responses over wideband. The transfer functions of such recursive digital differentiators and integrators corresponding to  $\omega_c = 0.9\pi$  or (11, 13) and  $\omega_c = \pi$  or (12, 14) are given in (20-22) and (23, 24) respectively.

$$H_{d5}(z) = \frac{0.21172(z-1)(z+5.4945)(z+0.782)(z-0.1019)}{T(z+0.8239)(z+0.295)(z-0.1014)(z+0.0365)}, \quad (20)$$

$$H_{d6}(z) = \frac{1.1633(z-1)(z+0.182)(z+0.782)(z-0.1019)}{T(z+0.8239)(z+0.295)(z-0.1014)(z+0.0365)}, \quad (21)$$

$$H_{i7}(z) = \frac{T(z+0.8239)(z+3.3898)(z-0.1014)(z+0.0365)}{3.94334(z-1)(z+0.182)(z+0.782)(z-0.1019)}, \quad (22)$$

$$H_{d7}(z) = \frac{1.1569(z-0.9971)(z+0.6384)(z^2-0.5734z+0.4709)}{T(z+0.6842)(z+0.1402)(z^2-0.5806z+0.4703)}, \quad (23)$$

$$H_{i8}(z) = \frac{T(z+0.6842)(z+7.1327)(z^2-0.5806z+0.4703)}{8.2512(z-0.9971)(z+0.6384)(z^2-0.5734z+0.4709)}. \quad (24)$$

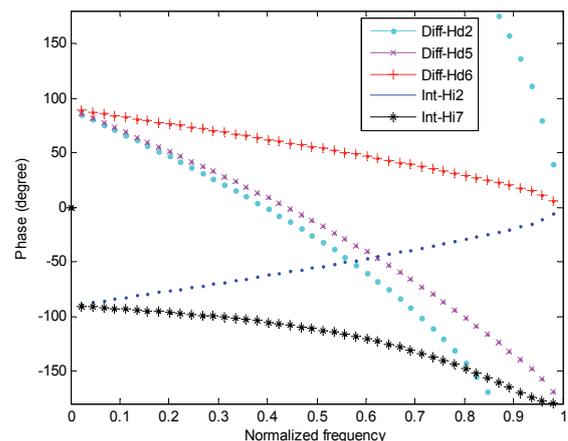


Fig. 2. Phase responses of the designed recursive digital differentiators and integrators of fourth-order systems for  $\omega_c = 0.9\pi$ .

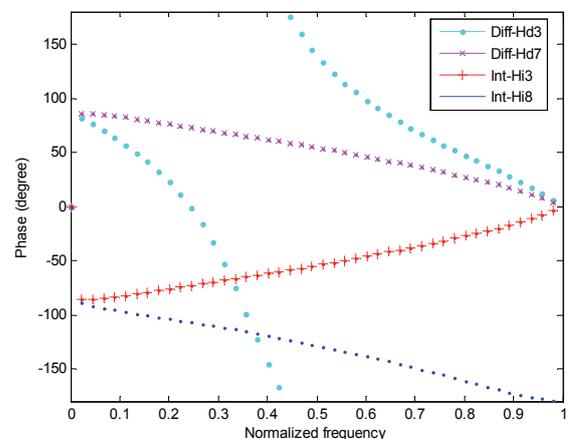


Fig. 3. Phase responses of the designed recursive digital differentiators and integrators of fourth-order systems for  $\omega_c = \pi$ .

Fig. 2 shows the phase responses of the designed recursive digital differentiators and integrators of fourth-order systems ( $\omega_c = 0.9\pi$ ), which are given in (11, 20, 21) and (13, 22) respectively. Further, Fig. 3 shows the phase responses of the designed recursive digital differentiators and integrators of fourth-order systems ( $\omega_c = \pi$ ), which are given in (12, 23) and (14, 24) respectively. From Fig. 2, it is observed that the designed recursive digital differentiator  $H_{d6}(z)$  and integrator  $H_{i2}(z)$  have not more than  $5^\circ$  maximum phase deviations from the ideal linear phase responses over wideband. Further from Fig. 3, it is observed that the designed recursive digital differentiator  $H_{d7}(z)$  and integrators  $H_{i3,8}(z)$  have not more than  $4.5^\circ$  maximum phase deviations from the ideal linear phase responses over wideband.

## 5. Comparisons

To verify the accuracies of finally obtained digital differentiators  $H_{d4,6,7}(z)$  and integrators  $H_{i1,2,3}(z)$ ; recently published the Gupta-Jain-Kumar (GJK) [22], the Al-Alaoui [24] and the Upadhyay-Singh (US) [25] recursive digital differentiators and integrators are considered; which are given in (25, 26), (27, 28) and (3, 29) respectively.

$$H_{gd}(z) = \frac{z^2(z-1)}{T(0.34)(2.541)(z+0.3935)(z^2-0.2081z+0.03858)}, \quad (25)$$

$$H_{gi}(z) = 0.34T \frac{(z+2.541)(z^2-0.2081z+0.03858)}{z^2(z-1)}, \quad (26)$$

$$H_{Al3d}(z) = \frac{0.01903 - 0.02905z^{-1} + 1.123z^{-2} - 1.181z^{-3}}{T(1 + 0.1846z^{-1} - 0.001748z^{-2} + 0.03484z^{-3})}, \quad (27)$$

$$H_{Al3i}(z) = \frac{3T(13 + 27z^{-1} + 27z^{-2} + 13z^{-3})}{80(1 - z^{-3})}, \quad (28)$$

$$H_{usi}(z) = \frac{T(z+0.571)(z+0.11)}{1.1552(z-1)(z+0.5025)}. \quad (29)$$

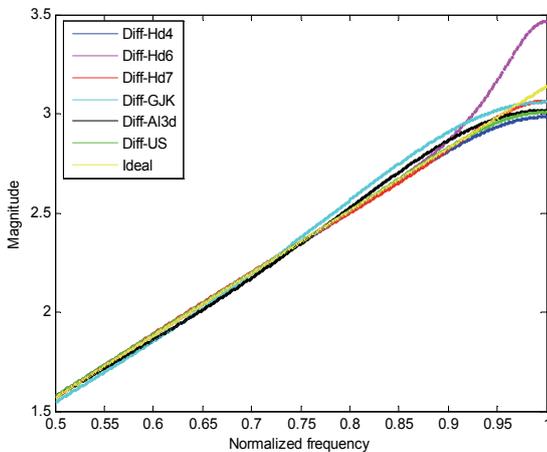


Fig. 4. Magnitude responses of the proposed and existing recursive digital differentiators with the ideal one.

It is observed that the magnitude responses of the proposed and the existing recursive digital differentiators with the ideal one are not differentiable up to the Nyquist frequency  $\omega = 0.5\pi$ . Therefore, Fig. 4 shows the magnitude responses of the proposed and the existing recursive digital differentiators with the ideal one only for the Nyquist frequency range of  $\omega = 0.5\pi$  to  $\omega = \pi$ . Further, Figs. 5 and 6 show the percentage relative errors (PREs) of the proposed and the existing recursive digital differentiators in magnitude responses with the ideal one for different Nyquist frequency ranges as  $\omega = 0$  to  $\omega = 0.84\pi$  and  $\omega = 0.84\pi$  to  $\omega = \pi$ .

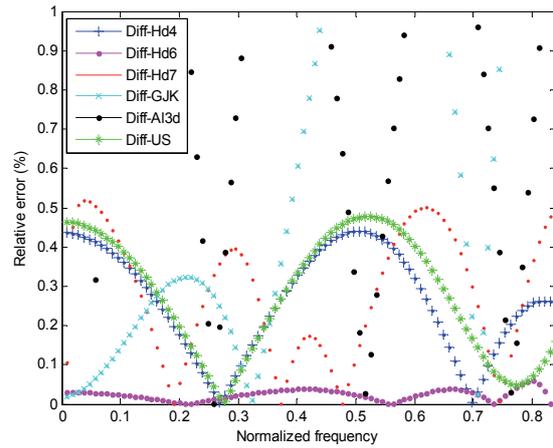


Fig. 5. PREs of the proposed and existing recursive digital differentiators for  $\omega = 0$  to  $\omega = 0.84\pi$ .

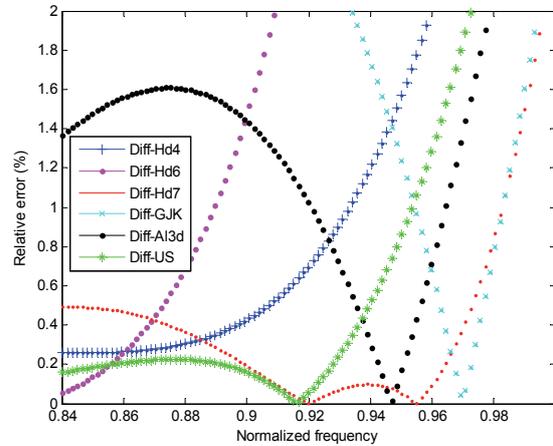


Fig. 6. PREs of the proposed and existing recursive digital differentiators for  $\omega = 0.84\pi$  to  $\omega = \pi$ .

Fig. 7 shows the magnitude responses of the proposed and the existing recursive digital integrators with the ideal one. Further, Figs. 8 and 9 show the PREs of the proposed and the existing recursive digital integrators in magnitude responses with the ideal one for different Nyquist frequency ranges as  $\omega = 0$  to  $\omega = 0.84\pi$  and  $\omega = 0.84\pi$  to  $\omega = \pi$ .

From Figs. 4-9, it is observed that the proposed recursive digital differentiator  $H_{d4}(z)$  and integrator  $H_{i1}(z)$  of second-order systems have MPREs of 0.44% up to the Nyquist frequency  $\omega = 0.9\pi$  and 1.5% up to  $\omega = 0.95\pi$  in

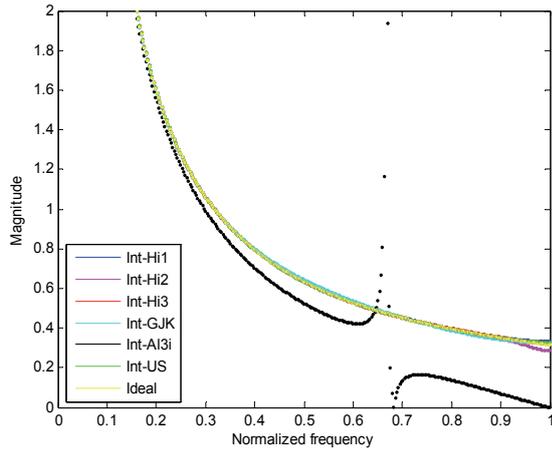


Fig. 7. Magnitude responses of the proposed and existing recursive digital integrators with the ideal one.

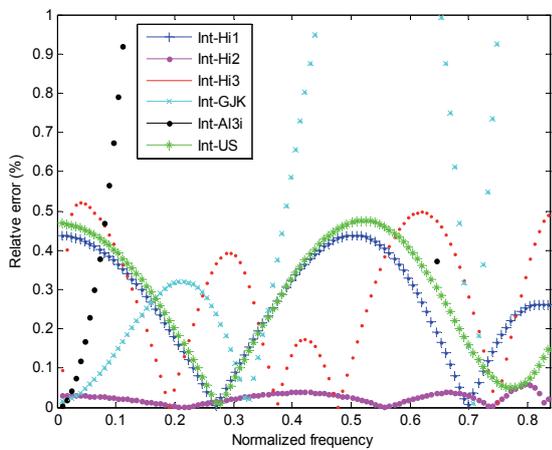


Fig. 8. PREs of the proposed and existing recursive digital integrators for  $\omega = 0$  to  $\omega = 0.84\pi$ .

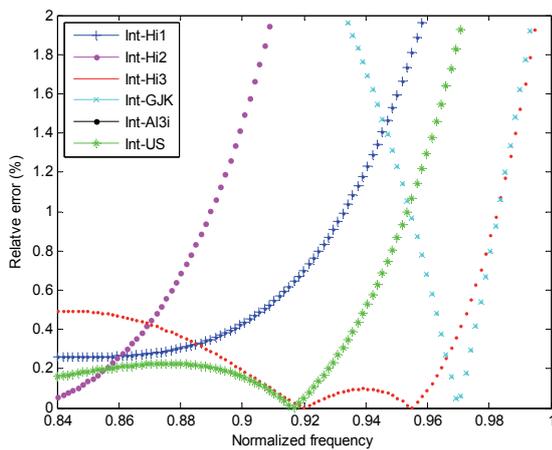


Fig. 9. PREs of the proposed and existing recursive digital integrators for  $\omega = 0.84\pi$  to  $\omega = \pi$ .

magnitude responses. Further, it is also observed that the proposed recursive digital differentiator  $H_{d6}(z)$  and integrator  $H_{i2}(z)$  of fourth-order systems have MPREs of 0.048% up to the Nyquist frequency  $\omega = 0.84\pi$  and 1.5% up to  $\omega = 0.9\pi$  in magnitude responses; while the proposed recursive digital differentiator  $H_{d7}(z)$  and integrator  $H_{i3}(z)$  of fourth-order systems have MPREs of 0.51% up to the

Nyquist frequency  $\omega = 0.975\pi$  and 1.5% up to  $\omega = 0.99\pi$  in magnitude responses. It is also observed that the existing US [25] recursive digital differentiator and integrator have MPREs of 0.48% up to the Nyquist frequency  $\omega = 0.94\pi$  and 1.5% up to  $\omega = 0.965\pi$  in magnitude responses. The magnitude responses of existing Gupta-Jain-Kumar [22] and the Al-Alaoui [24] recursive digital differentiators and integrators of third-order systems have more relative errors or almost un-comparable with the proposed and the existing US [25] recursive digital differentiators and integrators.

## 6. Conclusions

In this paper, new recursive digital differentiators and integrators are designed according to the importance of accuracy, bandwidth and the system simplicity. Further, the zero-reflection approach is applied to improve the phase responses of designed recursive digital differentiators and integrators. Results show that the proposed recursive digital differentiator  $H_{d6}(z)$  and integrator  $H_{i2}(z)$  of fourth-order systems have MPREs of 0.048% in magnitude responses up to the Nyquist frequency  $\omega = 0.84\pi$ . It is also shown that the proposed recursive digital differentiator  $H_{d4}(z)$  and integrator  $H_{i1}(z)$  of second-order systems have MPREs of 0.44% in magnitude responses up to the Nyquist frequency  $\omega = 0.9\pi$ . In addition, it is also shown that the proposed recursive digital differentiator  $H_{d7}(z)$  and integrator  $H_{i3}(z)$  of fourth-order systems have MPREs of 0.51% in magnitude responses up to the Nyquist frequency  $\omega = 0.975\pi$ . It is also observed that these proposed designs have maximum phase deviations of nearly  $6^\circ$  from the ideal linear phase responses over wideband. Therefore, finally proposed recursive digital differentiators and integrators may be better alternate of existing non-recursive digital differentiators and integrators.

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