MULTICOPTER ATTITUDE ESTIMATION USING DYNAMIC MODEL

Radek Baránek
Doctoral Degree Programme (4), FEEC BUT
E-mail: xbaran10@stud.feec.vutbr.cz

Supervised by: František Šolc
E-mail: solc@feec.vutbr.cz

Abstract: The paper is focused on attitude estimation of multicopters. Specifically the use of dynamic model of multicopter in the attitude estimation algorithm is investigated. The examined algorithm is compared to the standard algorithm GPS-INS, which do not need any information of the vehicle dynamics. Both algorithms are tested in simulations. The use of dynamic model provides bounded errors of attitude estimates even without GPS measurements. The drawback is the need for a good dynamic model of the multicopter and its parameters.

Keywords: Multicopter, Attitude estimation, Dynamic model

1 INTRODUCTION

Multicopters are well known aerial vehicles which consist of rigid frame with equidistantly placed motors with propellers. They are popular thanks to their mechanical simplicity since the only moving parts are the motors with propellers. Multicopters are usually used in autonomous regime where knowledge of the multicopter state is needed by controllers. The most crucial state component is attitude (orientation including heading).

The standard algorithm used for attitude estimation of multicopters equipped with different low cost sensors (gyroscope, accelerometer, magnetometer, GPS, barometer etc.) is GPS-INS [1]. The main drawback of this algorithm is the dependency on the GPS measurements. In the case of signal lost or GPS receiver failure the error of attitude estimates grows quickly and the estimate is usually useless after few seconds (depends on given sensor performance).

Including the vehicle dynamics into state estimation algorithm promises better performance even without GPS and is not entirely new idea. One of the first usages was on airplane attitude estimation [2]. Since that time some papers discuss even the use of multicopter dynamics for state estimation [3], [4]. The main expected advantages of the included vehicle dynamics are improved accuracy, bounded error even without GPS and slower position error growth when GPS is lost. The disadvantages are the required good knowledge of the dynamic model and its parameters for a given multicopter and required knowledge of propeller speeds or motor control signals.

2 ALGORITHM STRUCUTRE

Both algorithms mentioned in this paper are implemented as a extended Kalman filter which is a standard method for system state estimation. The Kalman filter is defined by state variables and then by state and measurement equations. In each iteration of the Kalman filter two steps are conducted. Firstly the new state is predicted based on the previous state using the state equations and selected inputs (sensors etc.). Second step is the correction step in which the predicted state is corrected based on the difference between the predicted (using measurements equations) and actually measured values. The various algorithm then differ in state and measurement equations and in the selection of inputs and measurements. The equations of extended Kalman filter can be found for example in [1].

3 MULTICOPTER MODEL

As both algorithms which will be presented in this paper partialy use equations from the full multicopter model, it is appropriate to briefly describe the simple multicopter model. The multicopter model can be divided into two parts. The dynamic part, which includes computation of acceleration and angular acceleration. The kinematic part then includes the standard 6 DoF kinematics which is valid for any free rigid body. Two coordinates frames are used throughout the description. The reference frame which is linked to the ground and the body frame which is linked with the multicopter axes. The vectors expressed in reference resp. body frame have subscript n reps. b. The vectors in reference and body frame are related by:

$$\vec{x}_b = \mathbf{R}_b^n(\vec{q})\vec{x}_n, \quad \vec{x}_n = \mathbf{R}_n^b(\vec{q})\vec{x}_b \tag{1}$$

where $\vec{q}$ is attitude quaternion and $\mathbf{R}$ is rotation matrix.

3.1 DYNAMIC PART

The dynamic part computes the total acceleration and angular acceleration driving the motion of the rigid body based on three main force sources:

1. Thrust of Propellers (Depends on propeller speeds or motor control signals):

$$\begin{bmatrix} F_b^z \\ M_b^x \\ M_b^y \\ M_b^z \end{bmatrix} = \begin{bmatrix} -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & -L\frac{\sqrt{3}}{2} & -L\frac{\sqrt{3}}{2} & 0 & L\frac{\sqrt{3}}{2} & L\frac{\sqrt{3}}{2} \\ L & \frac{L}{2} & -\frac{L}{2} & -L & -\frac{L}{2} & \frac{L}{2} \\ c_R & -c_R & c_R & -c_R & c_R & -c_R \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} \tag{2}$$

   where L is the length of the multicopter arm, c_R is the positive constant and T_i is thrust of i-th propeller. This particular example is valid for hexacopter (multicopter with six motors with propellers). The thrust of the propeller is a function of the propeller angular speed ω_i or control signal to motor controller s_i depending on which information is available:

$$T_i = f(\omega_i), f(s_i) \tag{3}$$

2. Aerodynamic Drag Force (Depends on multicopter body speed w.r.t. air mass):

$$\vec{F}_b^d = -(\vec{v}_b - \vec{w}_b) \cdot [u_x, u_y, u_z]^T \tag{4}$$

   where $\vec{v}_b$ is multicopter speed, $\vec{w}_b$ is wind speed and u_i are positive constants.

3. Gravity (Constant in reference frame):

$$\vec{F}_b^g = \mathbf{R}_b^n(\vec{q})m\vec{g}_n \tag{5}$$

The resulting acceleration is computed according to the second Newton law:

$$\vec{a}_b = \frac{\vec{F}_b}{m} \tag{6}$$

where $\vec{F}_b$ is sum of all forces and m is mass of the multicopter. The resulting angular acceleration is computed according to following equation:

$$\vec{\varepsilon}_b = \left[\vec{M}_b - (\vec{\omega}_b \times \mathbf{I}_b \vec{\omega}_b)\right] \cdot \mathbf{I}_b^{-1} \tag{7}$$

where $\vec{M}_b$ is sum of all moments (in this simple model only the thrusts of propellers produce moments), $\vec{\omega}_b$ is angular rate, and $\mathbf{I}_b$ is inertia matrix of multicopter.

3.2 Kinematic Part

Kinematic part include standard 6 DoF kinematics of free rigid body which is driven by acceleration and angular acceleration computed by dynamic part. The states of 6 DoF kinematics are velocity, angular rate, position and attitude quaternion. The differential equations for individual states are:

$$\dot{\vec{v}}_b = \vec{a}_b - \vec{\omega}_b \times \vec{v}_b \tag{8}$$

$$\dot{\vec{p}} = \mathbf{R}_n^b(\vec{q})\vec{v}_b \tag{9}$$

$$\dot{\vec{\omega}}_b = \vec{\varepsilon}_b \tag{10}$$

$$\dot{\vec{q}} = 0.5\vec{q} \cdot [0, \omega_x, \omega_y, \omega_z]^T \tag{11}$$

where $\vec{v}_b$ is velocity vector, $\vec{a}_b$ is acceleration vector including gravity, $\vec{\omega}_b$ is angular rate vector, $\vec{\varepsilon}_b$ is angular acceleration vector, $\vec{p}$ is the position vector and $\vec{q}$ is attitude quaternion. The equations (8), (9) form the translational section of 6 DoF kinematics while equations (10),(11) form the rotational part of 6 DoF kinematics.

4 GPS-INS

In this section the algorithm known as GPS-INS is briefly introduced. This algorithm is used for comparison of performance of algorithm with included multicopter dynamics. The GPS-INS algorithm process the data from gyroscopes, accelerometers, GPS receiver and optionally from magnetometer and barometer in order to estimates the states of rigid body to which the mentioned sensors are linked. As already mentioned the GPS-INS algorithm is implemented using extended Kalman Filter.

The GPS-INS has the following states: Attitude Quaternion, Velocity, Position, Gyroscope bias, Accelerometer bias. Except the states of sensor errors (gyroscope and accelerometer bias) the state equation are based on the 6 DoF kinematic equations. The accelerometer and gyroscopes are used as a inputs and they drive the time evolution of state equations. Accelerometers for translational part (equations (8), (9)) and gyroscope for rotational part, in this case consisting only of equation (11) since gyroscope measures the angular rate vector directly. The sensor biases are usually modeled as a random-walk or Gauss-Markov processes. The information about these processes can be found in [5].

GPS and optionally barometer and magnetometer measurements are selected as a measurements and are used to correct the predicted state. The measurement equations for GPS and barometer are directly mapping the position state. For magnetometer the measurement equation is:

$$\vec{m}_b = \mathbf{R}_b^n(\vec{q})\vec{m}_n \tag{12}$$

where $\vec{m}_b$ is predicted magnetometer measurements and $\vec{m}_n$ is magnetometer vector expressed in reference frame (constant - required parameter).

5 MODEL-BASED ALGORITHM

Contrary to GPS-INS model-based attitude estimation algorithm additionally process motor control signals or propeller speed. As a consequence the algorithm also includes multicopter dynamic equations. For simplicity of the algorithm only the translational part of multicopter dynamics is used, since the rotational part is well covered by gyroscope sensor. According to theory any additional information should improve the accuracy, but in the rotational part the improvement is not worth implementing. On top of that other hardly measurable parameters of multicopter would be required (inertia matrix etc.).

The state selection is the same as in the GPS-INS except the accelerometer bias, which is unobservable without GPS. The inputs are motor control signals (or propeller speeds) and gyroscope. The motor control signals are used to predict the translational part (instead of accelerometer) and gyroscope again predicts the rotational part. In this algorithm the accelerometer is used as a measurement in contrast to GPS-INS.

6 SIMULATIONS

The both algorithms were tested in simulations. The GPS-INS for comparison and model-based to prove its expecting performance. The used simulation scheme is depicted in figure 1. The reference model is used to generate testing trajectory. The true state is then used to generate simulated sensor data (adding noise etc.). Further the simulated sensor values are passed to both algorithms. The estimated state of both algorithms are compared to the true state (which is known exactly thanks to simulation). The results of simulations are summarized in figure 2 where the errors of Euler angles for both algorithms are plotted and in table 1 where the RMSe values and maximal instant errors for Euler angles are showed.

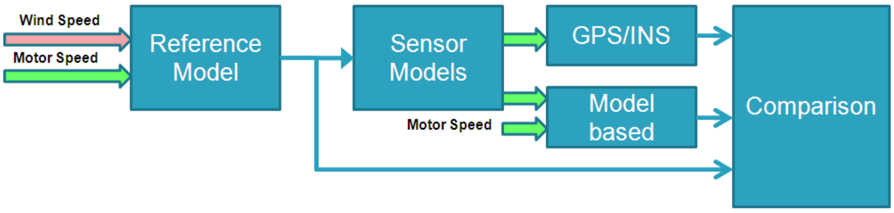

Figure 1: Simulation scheme

Algorithm	RMSe Roll [deg]	RMSe Pitch[deg]	RMSe Yaw [deg]	MAXe Roll [deg]	MAXe Pitch [deg]	MAXe Yaw [deg]
GPS-INS	1.0271	1.2056	1.3998	4.7947	4.6906	4.1645
Model-based	0.9666	0.6248	1.1820	3.4270	1.6125	3.4776

Table 1: Results of simulations - RMSe and maximal instant error values

7 CONCLUSION

The results of simulations showed outstanding performance of the model-base attitude estimation algorithm. Not only the error of attitude estimate is bounded without GPS measurements but is even smaller than in the GPS-INS algorithm. It has to be noted that the model and parameters used in the

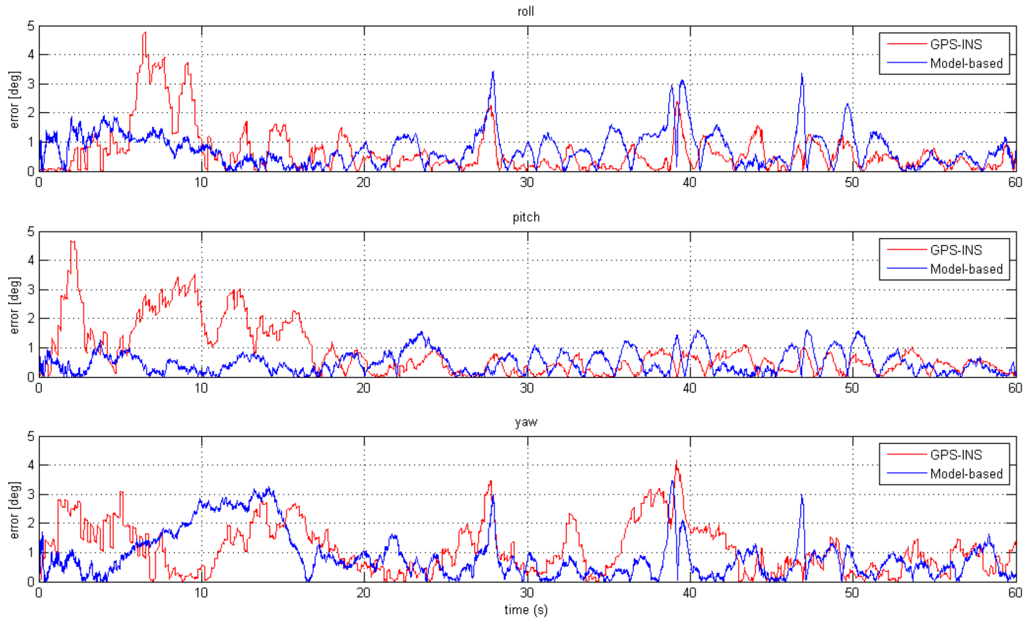

Figure 2: Results of simulations - Euler angles errors

estimation algorithm was exactly the same like in the reference model used for data generation. In real case this state is unachievable and both the model and the parameters of the model will be only approximation of reality. Thus the next step is the investigation of effects of inaccuracies in model and its parameters.

ACKNOWLEDGEMENT

The completion of this paper was made possible by grant No. FEKT-S-14-2429 - "The research of new control methods, measurement procedures and intelligent instruments in automation", and the related financial assistance was provided from the internal science fund of Brno University of Technology.

REFERENCES

[1] Shin, E. H., "Estimation Techniques for Low-cost Inertial Navigation," Ph.D. Thesis, Calgary, 2005.

[2] Koifman, M., and Bar-Itzhack, I., "Inertial Navigation System Aided by Aircraft Dynamics," IEEE Transactions on Control Systems Technology, Vol. 7, No. 4, 1999, pp. 487–493.

[3] Leishman, R., Macdonald, J., "Utilizing an Improved Rotorcraft Dynamic Model in State Estimation." Proceedings of 2011 IEEE/RSJ International Conference on Intelligent Robots and Systems, 2011 p. 5173-5178.

[4] Crocoll, P., Seibold, J., Scholz, G., Trommer, G. F., "Model-Aided Navigation for a Quadrotor Helicopter: A Novel Navigation System and First Experimental Results," NAVIGATION, Vol. 61, No. 4, Winter 2014, pp. 253–271.

[5] Rogers, M. R., "Applied Mathematics in Integrated Navigation Systems." American Institute of Aeronautics and Astronautics, Inc. Reston, Virginia. Second edition. 2003.