Heat storage efficiency and effective thermal output: Indicators of thermal response and output of radiant heating and cooling systems

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HEAT TRANSFER EFFICIENCY AND HEAT STORAGE EFFICIENCY: INDICATORS OF THERMAL RESPONSE OF RADIANT HEATING AND COOLING SYSTEMS

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Abstract

Alternative indicators of thermal response of radiant heating and cooling systems called heat transfer efficiency (*HTE*) and heat storage efficiency (*HSE*) have been proposed. The *HTE* and *HSE* were compared with established indicators represented by the time constant τ_{63}, response time τ_{95}, and thermal energy stored (*TES*). The comparison was performed for three wall cooling systems with various combinations of pipe location, configuration of material layers, and materials of the thermal core. Taking into account the whole response curve (*HTE*, *HSE*) instead of focusing on one specific point on the curve (τ_{63}, τ_{95}) allowed comparing the thermal response of systems with complex thermal behaviour by a single value. It also permitted predicting thermal response consistently regardless of the system and core material. *TES* predicted the thermal response of certain systems, but it may not be suitable for comparing the thermal response of radiant systems with different thermal admittance. Besides, a composite indicator called effective thermal output was proposed to consider both the steady-state and dynamic thermal performance. For thermally active building systems (TABS), using effective thermal output led to differences in thermal performance between the two core materials lower than indicated by the maximum (nominal) thermal output.

Keywords

Radiant heating; radiant cooling; thermal response; thermally activated building systems (TABS); thermal dynamics; response time

1. Introduction

In well-insulated modern and retrofitted existing buildings, installation of radiant heating and cooling systems can be preferable due to their benefits such as the suitability for combination with low-grade renewable energy sources [1,2,3], high sensible heating and cooling capacity [4], and comfortable thermal environment [5,6,7]. The response time of these systems is higher than that of convective systems and may be as high as tens of hours [8,9,10]. This, in combination with frequent changes in weather conditions and internal heat gains, requires using precise control strategies to fulfil the criteria on thermal comfort [11,12,13,14]. Knowing the thermal response of the radiant systems is therefore crucial to determine an efficient control strategy.

The existing body of research describes various methods to evaluate the thermal response of radiant heating and cooling systems (Table 1). The most common indicator is the time constant, which is defined as 63% of the final value of the variable observed (τ_{63}) such as the surface temperature of the radiant system, thermal output, or room temperature [15,16,17,18]. Alternatively, the thermal response can be expressed as response time defined as 80% (τ_{80}) [19], 95% (τ_{95}) [9] or other percentages of the final value. A single value of the time constant or response time sufficiently describes the thermal response in case that the temperature of the body is nearly uniform, i.e. that internal conduction can be neglected. For such systems, the response time τ_{95} equals three times the time constant τ_{63} [20]. Ning et al. [9] have shown that for radiant systems with pipes embedded in a layer of thermal mass, using a single value of the time constant or response time is not suitable because the ratio of τ_{95}/τ_{63} varies and the thermal behaviour of some of the systems may be too complex. They, therefore, recommended using several response times such as e.g., 25%, 50%, 63%, or 80% of the difference between the final and initial values.

The studies that use indicators other than time constant and response time include Price and Smith [21], who studied the thermal response of various structures of the building envelope to thermal load. The thermal response was expressed as the time in which the surface temperature reaches its peak value. Peak values of the surface temperature were also reported. Sourbron et al. [22] used thermal admittance (surface output/surface temperature) and thermal transmittance (pipe output/surface temperature). The difference between the transmittance and the admittance curve is proportional to the amount of energy stored in the thermal mass, which makes it a suitable indicator to quantify the controllability of radiant systems. Chikh [23] used room temperature overshoot and settling time after a step rise in the set point of the room temperature by 1°C in a room with radiant floor heating. The heat output was provided through a heat source located in various depths within the floor.

Kobayashi and Kohri [24] compared the thermal response of three types of floor heating systems to a step-change in pipe surface temperature from 20°C to 40°C using three indicators: (i) total heat transmission from pipe to the ambient space at upper and lower floor surfaces, (ii) heat absorption from pipe to the floor, and (iii) ratio of heat transmission to heat absorption. The cumulative difference between the heat transmission and absorption rates indicated the thermal storage. Other studies use step-up or step-down curves of surface temperature and compare the curves visually to observe the differences in the thermal response of various radiant systems [19,25,26] or various measurement points within the same system [27], or count the number of operation cycles needed to keep the cooling output between 90 and 63 % of its maximum value [8].

Several studies have tested the effect of design on the thermal response of radiant heating and cooling systems. Merabtine et al. [15] developed a simplified calculation model of the thermal behaviour of a heating slab considering various design parameters. The heat capacity of the slab and the water flowrate significantly affected the time constant τ_{63} as opposed to the insignificant effect of the thermal conductivity and the pipe diameter. Ning et al. [9] tested the response time τ_{95} of radiant heating and cooling, floor and ceiling systems as defined in ISO 11855 [28]. The concrete thickness, pipe spacing, and concrete type had a significant impact on the response time of Type E of thermally active building systems (TABS), whereas pipe diameter, room temperature, water temperature, and water flow regime did not. The small effect of some of the design parameters in combination with complex thermal behaviour may result in similar response times of radiant systems with different thermal dynamics. For example, Krajčík and Šikula [29] used τ_{95} to test the thermal response of TABS with pipes embedded in thermal insulation attached to the thermal core, TABS with pipes embedded in the core, and a system with a thermally activated plaster decoupled from the core. In some cases, the response times were similar despite the obvious differences in thermal dynamics.

The literature review has shown that (i) an indicator of thermal response may not reflect the differences in thermal dynamics between various radiant systems, (ii) a single value of response time is not enough to describe the thermal dynamics of systems with complex thermal behaviour, and (iii) studies that would directly compare the performance of several indicators are lacking. We, therefore, propose alternative indicators of thermal response called heat transfer efficiency (*HTE*) and heat storage efficiency (*HSE*). Unlike time constant and response time, *HTE* and *HSE* present single indicators of thermal response taking into account the evolution of surface temperature or thermal output from the beginning until it reaches steady-state. These indicators help describe the thermal response of radiant systems with complex thermal behaviour by a single indicator and thereby facilitate direct comparison of their thermal dynamics. The calculation principle of *HTE* using a decline curve of surface temperature was briefly outlined in Ref. [8]. This study provides a detailed explanation of the concept and calculation principle of *HTE* and *HSE*, compares them with established indicators of thermal response, and test their ability to overcome some of the limitations of the existing indicators. Besides, a composite indicator called effective thermal output is proposed that allows taking into account both the steady-state and dynamic thermal performance of radiant systems.

Table 1 Research studies pertaining to thermal response of radiant heating and cooling systems, and their comparison with the present study

Study	Radiant system	Determination method	Evaluation of thermal response
Athienitis [18] (1993)	Floor heating (not specified)	Step up curve of room temperature over time	Time constant, defined as the time to reach 63% of the steady state room temperature
Athienitis and Chen [30] (2000)	Floor heating (not specified)	Profiles of surface temperature over time	Delay time, defined as the time duration between the time that the room temperature falls below the set point and the time when it reaches its minimum value
Kobayashi and Kohri [24] (2003)	Floor heating (hydronic)	Step up or decay curve of: (1) heat transmission from pipe to room (2) heat absorption from pipe to floor (3) ratio of heat transmission to heat absorption	Visual evaluation and comparison of the curves for the various cases investigated. The heat transmission and heat absorption are not defined
Chikh [23] (2005)	Floor heating (not specified)	(1) Step up curve of room temperature over time (2) Response of room temperature to rise in set point	(1) Time constant, defined as the time to reach 63% of the steady state room temperature (2a) Overshoot of room temperature after a step change in set point of the room temperature (2b) Settling time after a step change in set point of room temperature
Sourbron et al. [22] (2009)	Combined floor and ceiling, heating and cooling (hydronic)	Profiles of thermal output and surface temperatures over time	(1) Admittance, defined as the ratio of thermal output from the surface to surface temperature (2) Transmittance, defined as the ratio of thermal output from pipes to surface temperature
Thomas et al. [19] (2011)	Floor heating (hydronic)	Step up curves of heat output over time	Response time, defined as the time to reach 80% of the nominal heat output
Zhao et al. [17] (2014)	Floor cooling (hydronic)	Step up curve of surface temperature. Decay curve of cooling output	Time constant, defined as the time to reach 63% of the steady state value
Ning et al. [10] (2015)	Floor and ceiling, heating and cooling (hydronic)	Step up curve of surface temperature over time	Time constant, defined as the time when the average surface temperature reaches 63.2% of the steady state value
Yu and Yao [16] (2015)	Floor heating (hydronic)	Step up curves of surface temp. and heating capacity	Time constant, defined as the time to reach 63% of the steady state value
Ning et al. [9] (2017)	Floor and ceiling, heating and cooling (hydronic)	Decay curve of surface temperature over time	Response time, defined as the time it takes for the surface temperature of a radiant system to reach 95% of the difference between its final and initial values when a step change in control of the system is applied as input
Shen et al. [27] (2017)	Generic heating and cooling panel	Step up curve of surface temperature over time	Visual evaluation and comparison of the curves for various measurement locations on the panel
Ferrarini et al. [31] (2018)	Generic heating panel (electric)	Step up curve of surface temperature over time	Time constant, calculated as $\tau = \frac{\rho c_p L_x}{h_{CONV}+h_{RAD}}$
Sun et al. [25] (2018)	Generic heating panel with flat heat pipes (electric)	Step up curve of surface temperature of the flat heat pipe over time	(1) Visual evaluation and comparison of the curves for various measurement locations on the flat heat pipe (2) Duration of rising stage of the surface temperature
Merabtine et al. [15] (2019)	Floor heating (hydronic)	Profiles of surface temperature over time	(1) Time constant, defined as the time to reach 63% of the steady state surface temperature. (2) Time delay, determined from the equation: $T_s(t) = \begin{cases} T_{s,0} & t < t_d \\ T_{s,\infty} + (T_{s,0} - T_{s,\infty})e^{-\frac{(t-t_d)}{\tau}} & t \geq t_d \end{cases}$
Krajčík and Šikula [8] (2020)	Wall cooling (hydronic)	(1) Step-down curve of surface temperature over time. (2) Curve of cooling output	(1) Heat transfer efficiency calculated from a step-down curve of surface temperature (2) Number of operation cycles to keep the cooling output between 90 and 63% its maximum
Present study	Wall cooling (hydronic)	Step-up and step-down curves of surface temperature over time	Heat transfer efficiency, heat storage efficiency, time constant τ_{63}, response time τ_{95}, energy stored

2. Indicators of thermal response used

Referring to Table 1, time constant and response time are the most frequent indicators to evaluate the thermal response of radiant systems and have been therefore selected as representatives of the established indicators. Thermal energy stored in the thermal mass has also been considered. Despite not being widely used as an indicator, it can provide useful insights into the thermal dynamics and is a suitable indicator to quantify the controllability of radiant systems of radiant systems [22]. Heat transfer efficiency (*HTE*) and heat storage efficiency (*HSE*) present novel indicators that may provide certain benefits but have not yet been properly described and compared with the established indicators.

2.1 *Time constant τ_{63} and response time τ_{95}*

Time constant τ_{63} is the most frequently used indicator of thermal response (Table 1). On the other hand, Ning et al. [9] found it preferable to use the response time τ_{95} to classify radiant heating and cooling systems according to their thermal response. Response time τ_{95} can be deduced from the time constant τ_{63} in case that the temperature of the body is nearly uniform and internal conduction can be neglected. The relationship between τ_{95} and τ_{63} may vary for various system types and designs, or it may even be impossible to deduce if the thermal behaviour is too complex [9,20]. Both τ_{63} *and* τ_{95} have been considered in the present study. They are defined as the time it takes for the surface temperature of a radiant system to reach 63% (τ_{63}) or 95% (τ_{95}) of the difference between its final and initial values when a step change in control of the system is applied as input.

2.2 *Thermal energy stored*

The rate of thermal energy stored in the heating/cooling structure is a function of its thermal admittance and transmittance. Changing the water temperature with frequencies at which the difference between the thermal energy admitted to the structure and transmitted to the interior is large means that energy is exchanged between the pipe and the structure without any meaningful effect on the indoor temperature. The amount of the thermal energy stored quantifies the thermal response and controllability of the thermally active structure [22,32,33]. To calculate the amount of thermal energy stored, the energy flows admitted to the structure and transmitted to the surfaces are integrated over time as follows:

$$TES = \sum_{i=1}^{i=n} \left(q_{stored,i} * \Delta t_i\right) = \sum_{i=1}^{i=n} \left[\left(q_{\mathrm{p}} - q_{\mathrm{int}} - q_{\mathrm{ext}}\right) * \Delta t_i\right] \qquad (\mathrm{Wh/m^2}) \tag{1}$$

where $q_{\mathrm{stored,i}}$ is the thermal energy stored in the time step Δt_{i} (W/m^2), Δt_{i} is the calculation time step (h), q_{p} is the heating or cooling power supplied to the structure through the pipe (W/m^2), q_{int} is the thermal output of the structure (W/m^2), and q_{ext} are thermal losses to the exterior (W/m^2).

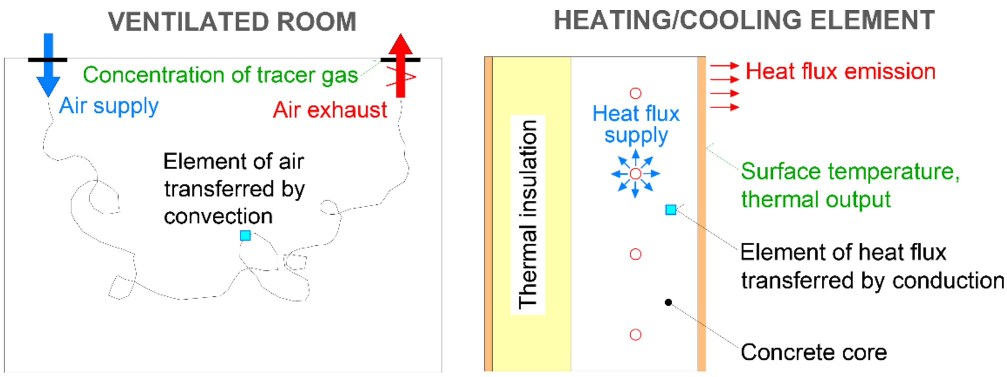
2.3 *Heat transfer efficiency (HTE) and heat storage efficiency (HSE)*

HTE shows how efficiently the heat is transferred from the pipes to the interior. Conversely, *HSE* indicates how efficiently the heat flux is distributed over the heating or cooling structure and thereby stored in the structure. A higher *HTE* (lower *HSE*) means a greater tendency of the heat to be transferred to the inner surface, whereas at a lower *HTE* (higher *HSE*) the thermal energy tends to be stored in the structure.

3 Concept and calculation principle of *HTE* and *HSE*

HTE and *HSE* are based on the mean age of heat flux concept. The mean age of heat flux is an analogy to the mean age of air used in buildings ventilation to characterize the ability of the ventilation system to efficiently distribute the fresh supply air in a ventilated room [6,34,35,36]. The analogy between the mean age of air and mean age of heat flux is illustrated in Fig. 1. In buildings ventilation, the age of air can be obtained by several methods using tracer gas, from which the most typical are the tracer gas step-up and step-down method. In the step-up method, the tracer gas is dosed at a constant rate throughout the whole measurement. The dosage starts at the time τ = 0 s, and its concentration starts to continuously increase until it reaches steady state. Conversely, in the step-down method, the tracer gas is dosed in the room before the measurement. The measurement starts when the supply of tracer gas stops and its concentration starts to decay due to ventilation.

In a ventilated room (Fig. 1a) the air containing tracer gas is transferred by convection from the supply, through the room, to the exhaust where the tracer gas concentration is measured. Thereby, the step-up or step-down curve is obtained. In the heating or cooling element (radiant wall, floor or ceiling, Fig. 1b), the heat is supplied through pipes and transferred by conduction to the surface where it is emitted to the room. Similarly to the tracer gas which spends a certain time in the room before it is exhausted, the heat or cool in the structure is not being transferred directly to the surface, but distributed and stored throughout the structure. In a step-up test, heat is supplied to the structure at constant water temperature and the surface temperature and thermal output start to increase until they reach steady state (Fig. 2). Conversely, in a step-down test, the heat is extracted from the structure at constant water temperature and the surface temperature and thermal output start plummeting until they reach equilibrium. Although in Fig. 2 and Eqs. 4 to 7 the calculation procedure is described using surface temperature, heat flux at the surface (thermal output) can be used for the calculation instead of the surface temperature providing that the heat transfer coefficient at the inner wall surface is constant.



a) Mean age of air **b) Mean age of heat flux**

Fig. 1 The analogy between mean age of air and mean age of heat flux

Mean age of air is a statistical concept based on the age distribution of the air components in a point [34,35,36]. The age of an element of air is the time elapsed since the element of air entered the room. The mean age of air is the mean age of all the elements at a certain point or in the whole room. As an analogy to buildings ventilation, the age of an element of heat flux is the time elapsed until the element of heat flux supplied to the structure through pipe reaches a point in the structure where it is stored. For a point within the structure, the local mean age of heat flux is the time it takes for the heat flux to reach and be stored in the actual point after entering the element. The structure mean age of heat flux is the mean age of all the heat flux in the structure. Using the mean age of heat flux concept, it is possible to determine the heat storage efficiency:

$$HSE = \frac{\delta_n}{2.\langle\bar{\delta}\rangle} * 100 \quad (\%) \tag{2}$$

where δ_n is the nominal time constant which is a measure of how fast the structure is charged by heat or cool (h); $\langle\bar{\delta}\rangle$ is the structure mean age of heat flux (h).

HSE is defined as the lowest possible mean age of heat flux $\delta_n/2$ and the actual structure mean age of heat flux $\langle\bar{\delta}\rangle$. A higher ratio of δ_n to $\langle\bar{\delta}\rangle$ means that the heat flux is being distributed more evenly over the structure and thereby the thermal energy is being accumulated at a higher rate. If $\delta_n/2$ is equal to $\langle\bar{\delta}\rangle$, the heat storage efficiency is 100% meaning that the thermal energy is being stored in the structure at the highest rate possible. Conversely, a lower ratio of δ_n to $\langle\bar{\delta}\rangle$, i.e. lower *HSE* indicates that the heat flux is being distributed over the structure less evenly. The rate of thermal storage is, therefore, lower and the heat transfer between pipes and surface is more efficient. The heat transfer efficiency is defined as:

$$HTE = 100 - HSE \quad (\%) \tag{3}$$

HTE is a measure of how efficiently the heat or cool supplied to the structure through pipes is transferred to the inner surface where it is emitted to the conditioned space.

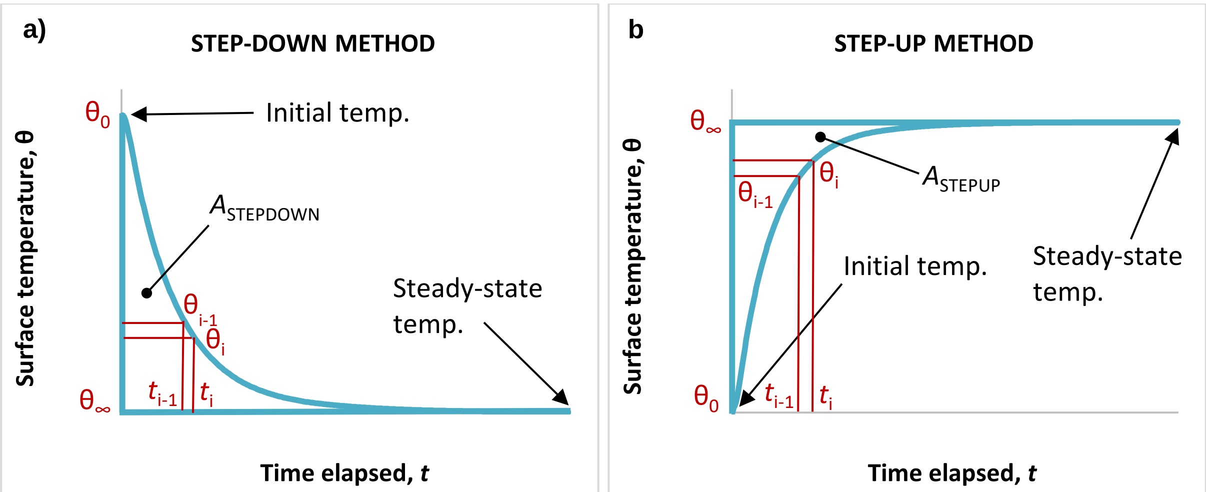


Fig. 2 Definitions of surface temperature, time, and area in the step-down and step-up method. Key: *θ* – surface temperature, *t* – time. *A* – area.

With the step-down method, the nominal time constant is obtained from the area under a decline curve, normalized by the initial temperature (Fig. 2a):

$$\delta_{\mathrm{n}} = \frac{A_{\mathrm{STEPDOWN}}}{(\theta_0 - \theta_\infty)} = \frac{\sum_{\mathrm{i=1}}^{\mathrm{i=n}} \left[\frac{(\theta_{\mathrm{i}} - \theta_\infty) + (\theta_{\mathrm{i-1}} - \theta_\infty)}{2} * (t_{\mathrm{i}} - t_{\mathrm{i-1}}) \right]}{(\theta_0 - \theta_\infty)} \quad (\mathrm{h}) \tag{4}$$

With the step-down method, the structure mean age of heat flux is calculated by the equation:

$$\langle \bar{\delta} \rangle = \frac{A_{\mathrm{w,STEPDOWN}}}{A_{\mathrm{STEPDOWN}}} = \frac{\sum_{\mathrm{i=1}}^{\mathrm{i=n}} \left[\frac{(\theta_{\mathrm{i}} - \theta_\infty) + (\theta_{\mathrm{i-1}} - \theta_\infty)}{2} * (t_{\mathrm{i}} - t_{\mathrm{i-1}}) * \frac{t_{\mathrm{i}} + t_{\mathrm{i-1}}}{2} \right]}{\sum_{\mathrm{i=1}}^{\mathrm{i=n}} \left[\frac{(\theta_{\mathrm{i}} - \theta_\infty) + (\theta_{\mathrm{i-1}} - \theta_\infty)}{2} * (t_{\mathrm{i}} - t_{\mathrm{i-1}}) \right]} \quad (\mathrm{h}) \tag{5}$$

Alternatively, the nominal time constant can be obtained by a step-up method from the area above an ascending curve (Fig. 2b). In such a case, it is calculated as follows:

$$\delta_{\mathrm{n}} = \frac{A_{\mathrm{STEPUP}}}{(\theta_\infty - \theta_0)} = \frac{\sum_{\mathrm{i=1}}^{\mathrm{i=n}} \left[\frac{(\theta_\infty - \theta_{\mathrm{i}}) + (\theta_\infty - \theta_{\mathrm{i-1}})}{2} * (t_{\mathrm{i}} - t_{\mathrm{i-1}}) \right]}{(\theta_\infty - \theta_0)} \quad (\mathrm{h}) \tag{6}$$

The mean age of heat flux is obtained by the step-up method as follows:

$$\langle \bar{\delta} \rangle = \frac{\sum_{\mathrm{i=1}}^{\mathrm{i=n}} \left[\frac{(\theta_\infty - \theta_{\mathrm{i}}) + (\theta_\infty - \theta_{\mathrm{i-1}})}{2} * (t_{\mathrm{i}} - t_{\mathrm{i-1}}) * \frac{t_{\mathrm{i}} + t_{\mathrm{i-1}}}{2} \right]}{\sum_{\mathrm{i=1}}^{\mathrm{i=n}} \left[\frac{(\theta_\infty - \theta_{\mathrm{i}}) + (\theta_\infty - \theta_{\mathrm{i-1}})}{2} * (t_{\mathrm{i}} - t_{\mathrm{i-1}}) \right]} \quad (\mathrm{h}) \tag{7}$$

The step-down method can be applied, e.g., using a decline curve of surface temperature after a cooling system is turned on. The step-up method can be used to calculate *HTE* and *HSE* from an ascending curve of surface temperature or thermal output. The two methods lead to identical outcomes regardless of the parameter used for the calculation (surface temperature or thermal output) in case that the heat transfer coefficient at the inner wall surface remains constant. This is illustrated in Fig. 3 for a wall TABS with pipes embedded in the thermal core made of reinforced concrete, concrete thickness of 40 cm, insulation thickness of 20 cm, and pipe spacing of 15 cm.

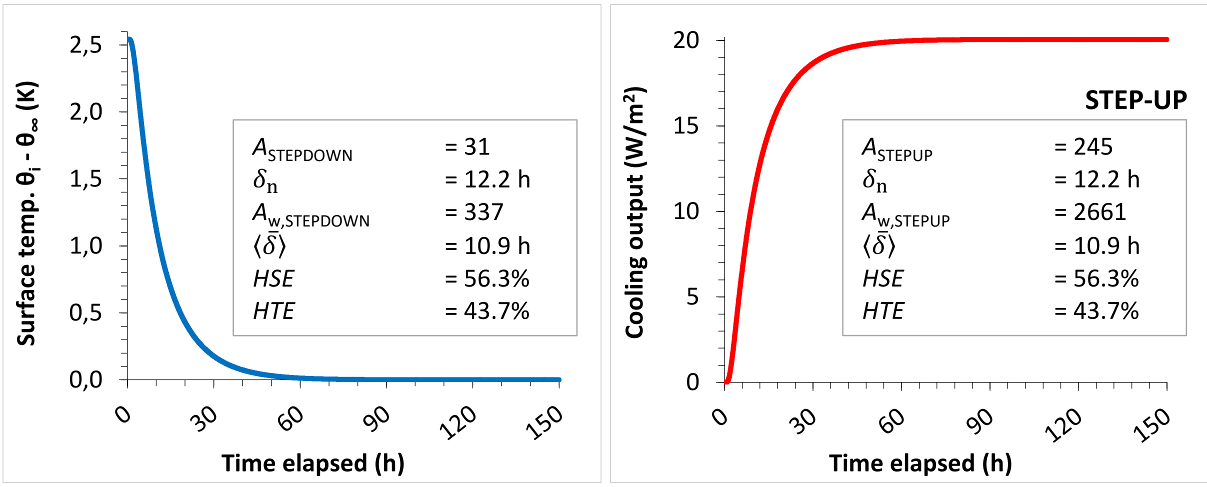


Fig. 3 *HTE* and *HSE* calculated by step-down and step-up method for a wall TABS operated in cooling mode.

4. Comparison of *HSE* and *HTE* with established indicators of thermal response

A case study is used to compare *HTE* and *HSE* with established indicators of thermal response. The case study involves three types of radiant systems differing from each other by the configuration of their material layers, location of pipes and thermal mass of the active layer (Fig. 4). The wall systems are operated as space cooling.

4.1 Description of the wall cooling systems

The three radiant wall systems are described as follows:

(a) **Wall W1** has the pipes embedded in the middle of the concrete core.

(b) **Wall W2** has the pipes located in the inner plaster underneath the wall surface. The plaster is thermally coupled to the concrete core.

(c) **Wall W3** has the pipes also located in inner plaster underneath the wall surface, but the plaster is thermally decoupled from the concrete core by thermal insulation.

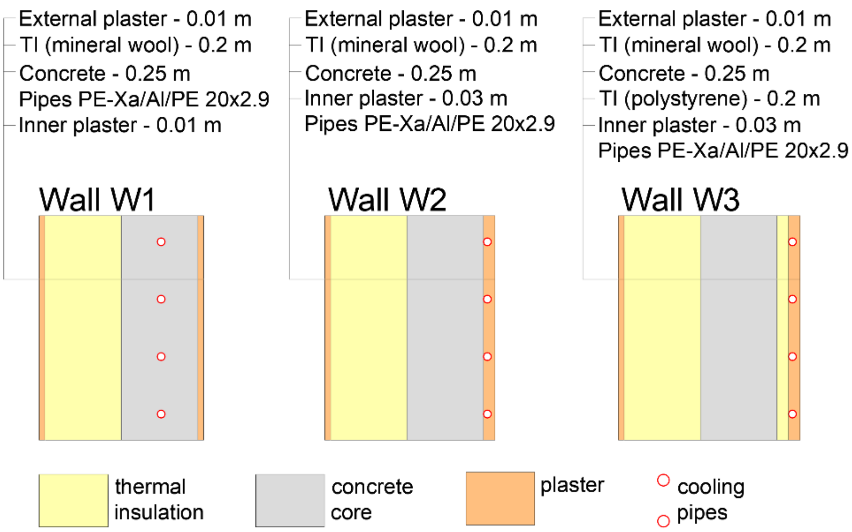


Fig. 4 Wall cooling systems used in the case study

The thermophysical properties of the individual material layers used in the calculation model are specified in Table 2. The thermal core is represented by two types of concrete with a substantially different thermal conductivity: aerated concrete with low conductivity and a thermally conductive reinforced concrete. In all the calculations the thermo-physical properties of materials were considered constant, isotropic, and temperature independent.

Table 2 Thermo-physical properties of material layers

No.	Material	Thickness *d* m	Volumetric weight *ρ* kg/m³	Thermal conductivity *λ* W/(m.K)	Specific heat capacity *c* J/(kg.K)
(1)	Inner plaster	0.01-0.03	1300	0.7	840
(2)	Insulation - EPS F *(only in Wall D)*	0.03	17	0.04	1020
(3)	Aerated concrete or reinforced concrete	0.25 0.25	600 2400	0.19 1.58	1000 1020
(4)	Insulation - mineral wool	0.2	20	0.04	940
(5)	Outer plaster Plastic pipe ø 20	0.01	1600 1200	0.8 0.35	840 1000

4.2 Boundary conditions and calculation principle of heat transfer

The heat transfer was computed using CalA software, which has been verified following ISO 10211 [37]. The governing equation describes the problem as 2D unsteady heat conduction:

$$\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)=\rho * c * \frac{\partial T}{\partial \tau} \quad (8)$$

where T is the temperature (K), τ is time (s), λ is thermal conductivity (W/(m.K)), ρ is bulk density (kg/m^3), and c is the specific heat capacity at constant pressure (J/(kg.K)).

The calculation principle has been described in detail in Refs. [8,38,39]. The Finite Volume Method was used for the heat transfer analysis. The computational mesh was structural, uniform and equidistant, and it was carefully refined so that it fulfilled the criteria on the cell size as defined in ISO 10211 [37]. The calculation was considered converged when it complied with the convergence criterion defined as:

$$\frac{\sum_{i=1}^{k} q_{\Gamma,i}}{\sum_{i=1}^{k}|q_{\Gamma,i}|} \leq 10^{-6} \quad (9)$$

where $q_{\text{enter,i}}$ is a sum of all the heat fluxes entering and exiting the wall through a boundary condition, and $|q_{\text{enter,i}}|$ is a sum of absolute values of these heat fluxes.

The computational model was previously validated for a wall system with pipes located in thermal insulation, and attached to the concrete core under summer [40] and winter climatic conditions [38]. The solver settings and climatic conditions used in the present study were identical to those used in Ref. [40]. In the present calculation model of the three wall systems (W1, W2, and W3), the calculation procedure and boundary conditions were identical to those of the validated model, whereas the thermophysical properties of the material layers were slightly adjusted to better suit practical applications. The calculation step was refined, and the simulation time was prolonged to increase the precision of the results.

The boundary conditions defining the specific heat flux on the surface of a computational domain were calculated according to Newton's law of cooling, assuming adiabatic wall boundaries (Fig. 5):

$$-\lambda.\left(\frac{\partial T}{\partial n}\right)_w = h.\left(T_w - T_f\right) \quad (10)$$

$$-\lambda.\left(\frac{\partial T}{\partial n}\right)_w = 0 \quad (11)$$

where *n* denotes a line perpendicular to surface; *w* denotes surface of an object; *h* is the overall heat transfer coefficient between radiant surface and environment (W/(m^2.K)); T_w is the temperature of wall surface (°C); T_f is the temperature of surrounding fluid (°C).

The heat transfer coefficient was 8 W/m^2 on the inner and 15 W/m^2 on the outer wall´s surface as defined in relevant standards [41,42]. The heat transfer coefficient for the water and pipe surface was 1218 W/(m^2.K). All the heat transfer coefficients were assumed constant over time.

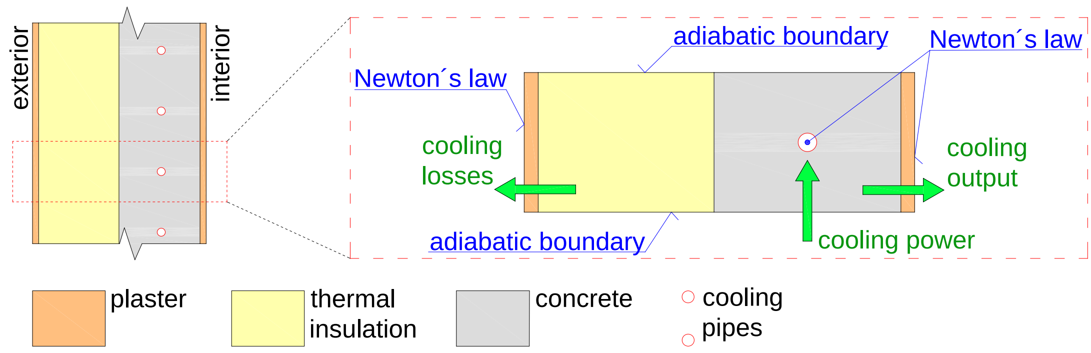


Fig. 5 Definition of boundary conditions and terminology

At the beginning of the observation period, the wall cooling system was turned off and in steady state. The temperature at the inner and outer surfaces, water temperature and temperature within the wall had a constant value of 26°C. Subsequently, a step-change was applied by turning the cooling system on, i.e. changing the water temperature in pipes from 26°C to 20°C, all other boundary conditions remaining the same.

Fig. 6a shows the relationship between simulation time step and root mean squared error (RMSE). The RMSE represents the difference between step-down curves of inner surface temperature over a 100-hour interval. The RMSE generally decreases with reduced time step and the difference between the time step of 1 and 2 min is small, about 2.10^{-3}. Fig. 6b shows the dependence of *HSE* on the simulation time. For the simulation time of 150 h, the *HSE* is always very close to the final value at 250 h. Based on these results, the simulation step of 2 min and simulation time of 150 h were used for all the subsequent calculations because they represent a suitable compromise between precision and simulation costs. At these conditions, the absolute uncertainty in *HSE* and *HTE* is estimated to be in the range ±1%. The absolute uncertainty in the time constant τ_{63} and response time τ_{95} is estimated to be within ±0.1 h and ±0.5 h, respectively.

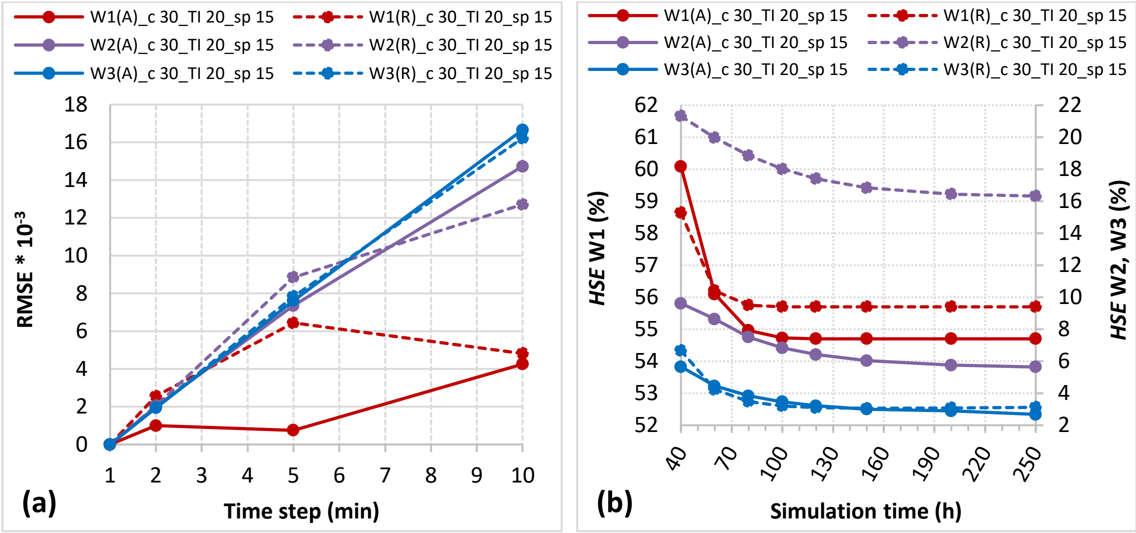


Fig. 6 The relationship between a) simulation time step and root mean squared error (RMSE), b) simulation time and Heat Storage Efficiency (*HSE*). Key: A – aerated concrete, R – reinforced concrete, *c* – core thickness in cm, *TI* – insulation thickness in cm, *sp* – pipe spacing in cm.

4.3 Thermal response curves

All the simulations have been done for nine realistic combinations of the concrete thickness (20, 30, and 40 cm) and pipe spacing (15, 20, and 25 cm). Insulation thickness was always 20 cm. The thermal response curves obtained for the three systems W1, W2, and W3 are shown in Fig. 7 for the two core materials, aerated concrete (A) and reinforced concrete (R).

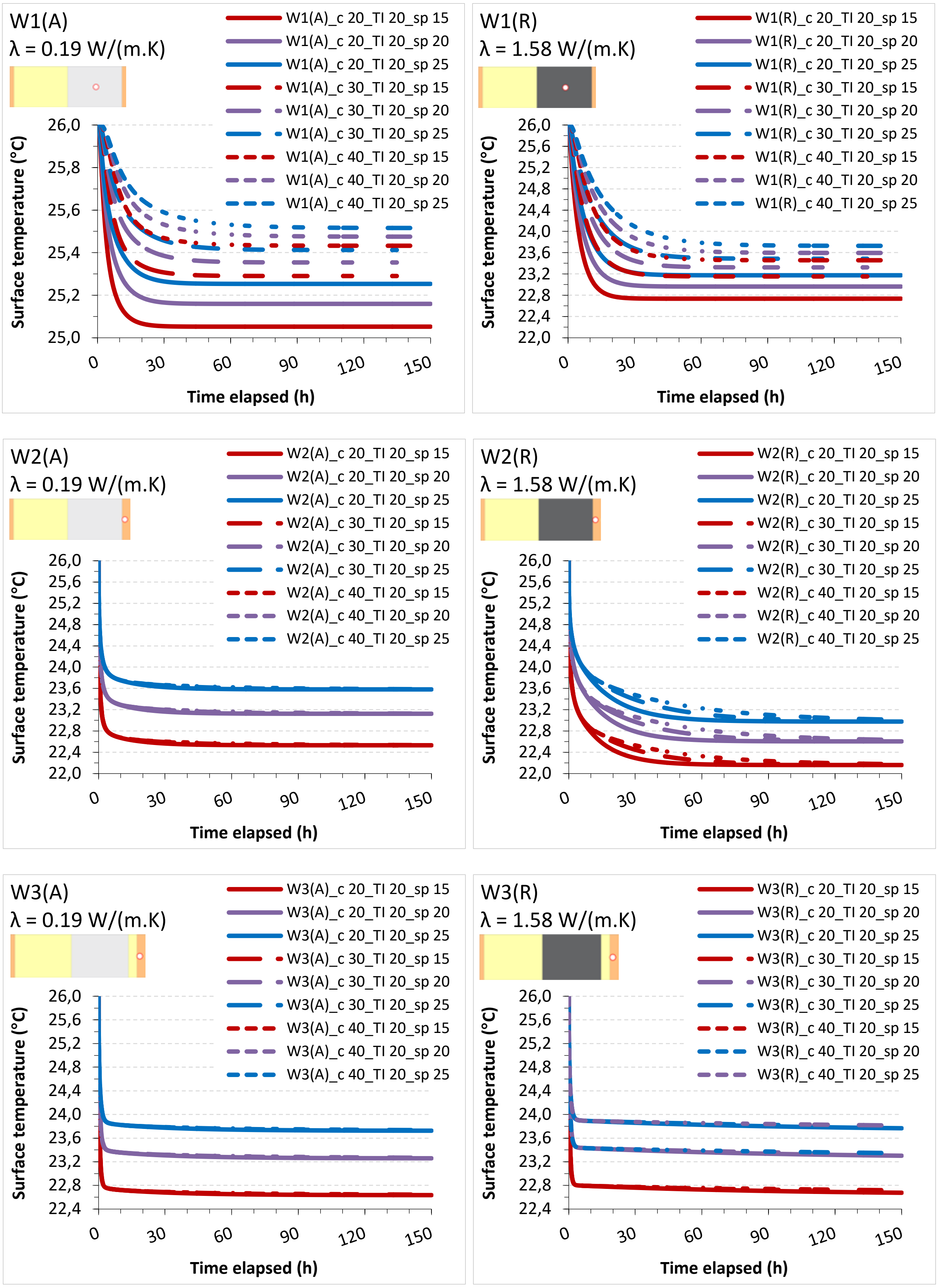


Fig. 7 Thermal response curves of the three wall cooling systems for two core materials: aerated (A) and reinforced (R) concrete. Key: *c* – core thickness in cm, *TI* – insulation thickness in cm, *sp* – pipe spacing in cm.

4.4 Time constant τ_{63} and response time τ_{95}

The time constant τ_{63} (Fig. 8a) indicates that the thermal response is slowest for TABS with pipes embedded in the thermal core W1(A) and W1(R), followed by TABS with a thermally conductive core and pipes underneath the surface W2(R). The response time is fast when pipes are attached to core with low thermal conductivity W2(A) and when the pipes are insulated from the thermal core (W3). This does not entirely match the trend when response time τ_{95} is used to evaluate the thermal response (Fig. 8b). For example, when using τ_{95} as an indicator, the response is slowest for the system with pipes attached to a thermally conductive core, W2(R). In general, τ_{63} and τ_{95} of the systems that accumulate thermal energy can vary over a wide interval depending on the thickness of the core and spacing of the pipes.

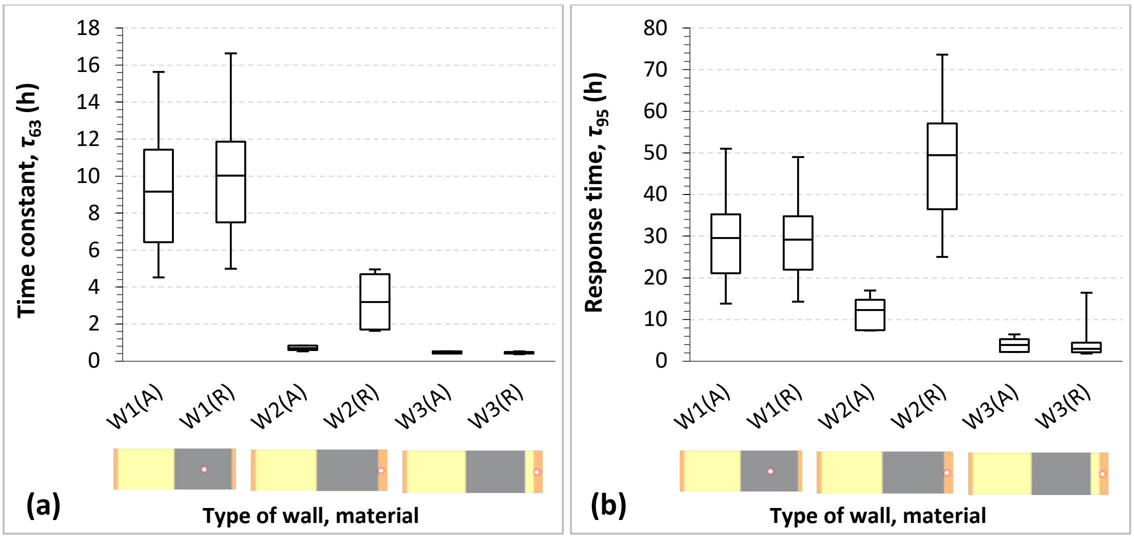


Fig. 8 Boxplots of the nine cases investigated for each cooling system: a) time constant τ_{63} and b) response time τ_{95}. Key: A – aerated concrete, R – reinforced concrete,

Fig. 9 shows the relationship between the time constant τ_{63} and response time τ_{95} for individual cases. The corresponding coefficients of determination are shown in the legend. A strong correlation and large variation in the values was found for the two TABS with pipes embedded in the thermal core (W1). For the systems with pipes insulated from the thermal core (W3) the correlation is weaker and the variation is much lower. The correlation is weakest for system W2(R). For systems W2(A) and W2(R), τ_{95} varies over a much wider interval than τ_{63} meaning that the difference in thermal response between the systems and individual cases depends on the definition of response time. This is also illustrated by the outlying value of τ_{95} in system W3(R) (see Fig. 8b where the maximum value is much higher than the rest). This is caused by the response curve of system W3 composed of three parts (Fig. 7): (i) the steep decline, (ii) the sharp bend, and (iii) the slow gradual decline. The surface temperature corresponding to τ_{63} lies on the steep part of the curve and τ_{63} is therefore always close to zero. The surface temperature corresponding to τ_{95} lies on the slowly declining part of the curve. The slow decline means that a small difference in surface temperature can lead to substantial differences in τ_{95}. These findings underline the fact that calculating several response times may be needed to describe the thermal dynamics of radiant systems with complex thermal behaviour.

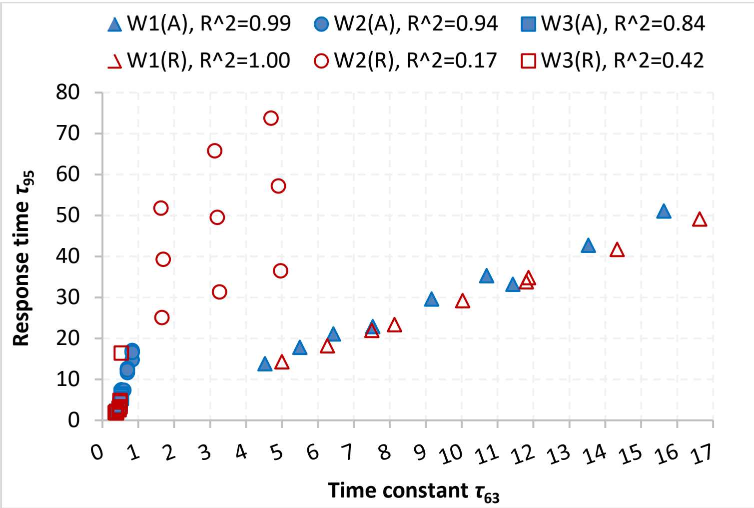


Fig. 9 Correlation between τ_{63} and τ_{95} for the three radiant systems and two core materials. Key: *A* – aerated concrete; *R* – reinforced concrete.

4.5 Thermal energy stored (TES)

The simulations have shown that for the system W3 with pipes separated from the thermal core, *TES* does not reach the peak even after the whole simulation period but keeps increasing. For W1 and W2, the increase is maximal during the first hours. As expected, *TES* increased with the core thickness of and density of pipe spacing. The values of *TES* differ substantially for various combinations of the cooling system and core material, as illustrated in Fig. 10a for the concrete thickness of 30 cm, insulation thickness of 20 cm, and pipe spacing of 15 cm. A smaller amount of *TES* indicates a smaller difference between energy admitted to the structure and energy transmitted to the interior. This means faster transmission of thermal energy from pipe to the interior and consequently more rapid thermal response. Conversely, higher *TES* indicates a slower thermal response.

Fig. 10b shows boxplots of the nine combinations of concrete thickness and pipe spacing for each cooling system. The two systems with pipes attached to a thermally conductive core, W1(R) and W2(R), store significantly more energy than the other systems. *TES* is low for the two systems with pipes attached to a core with low thermal conductivity, W1(A) and W2(A), because of their low thermal admittance resulting from the low thermal conductivity of the core. *TES* is also low for the two systems with pipes insulated from the thermal core (W3) due to the low thermal mass of the thermally active plaster, which indicates a fast thermal response.

TES logically predicts a slow thermal response for TABS with thermally conductive core, W1(R) and W2(R), and fast thermal response for systems with pipes insulated from the core (W3). The discrepancy between *TES* and the other indicators is largest for system W1(A). In this case, *TES* indicates a fast thermal response similar to W2(A) and W3(A). This contradicts the visual comparison of response curves in Fig. 7, and a slow thermal response predicted by τ_{63} and τ_{95} (Fig. 8) and *HTE* (see Fig. 11, Section 4.7). This is because *TES* reflects the low thermal admittance of structure W1(A) rather than the shape of the response curve. It means that *TES* can be used to compare the thermal response of systems with similar thermal admittance. Thus, TES has limited use as an indicator of thermal response.

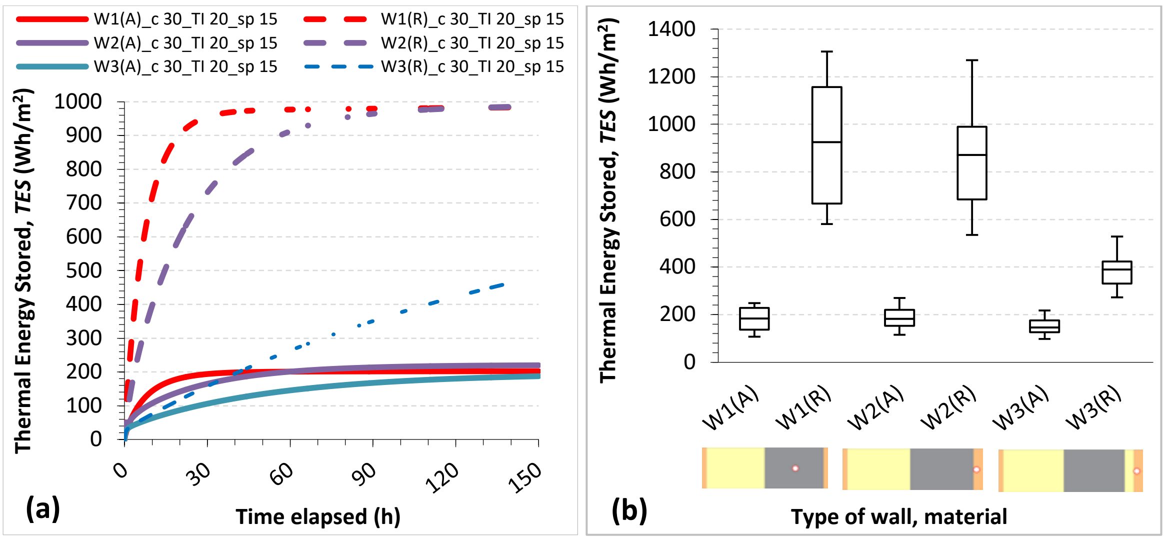


Fig. 10 Thermal energy stored (*TES*) over 24 h: a) Representative curves, b) Boxplots of the nine cases investigated for each cooling system. Key: *A* – aerated concrete; *R* – reinforced concrete.

4.6 Heat transfer efficiency (HTE) and heat storage efficiency (HSE)

HTE and *HSE* are complementary indicators meaning that their sum is always 100%. The boxplots in Fig. 11 visualize the relationship between *HTE* and *HSE*. The results show the differences in the thermal response of the cooling systems. *HTE* and *HSE* are ranging from 40 to 60% for the systems with pipes embedded in the thermal core (W1). *HTE* is substantially lower for W1 with pipes in the core than for W2 and W3 with pipes underneath the surface because locating the pipes in the core causes a more even distribution of heat flux in the structure. Thereby, the energy is stored at a higher rate and consequently, its transfer to the surface is less efficient.

HTE is about 90% for the system with pipes attached to a core with low thermal conductivity, W2(A), because the high thermal resistance of the core results in less efficient thermal storage. The *HTE* is lower for W2(R) with a thermally conductive core because the low thermal resistance of the core allows more efficient heat flux distribution and thermal storage. *HTE* is highest for system W3 regardless of the core material. Locating the pipes close to the surface and insulating them from the thermal core causes inefficient thermal storage.

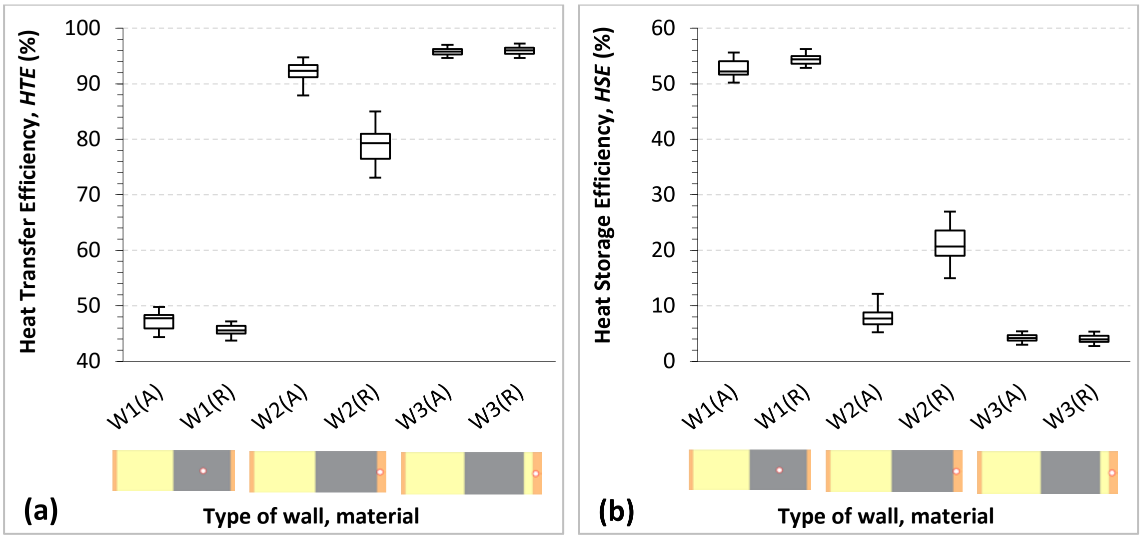


Fig. 11 Boxplots of the nine cases investigated for each cooling system: a) Heat transfer efficiency (*HTE*), b) Heat storage efficiency (*HSE*). Key: *A* – aerated concrete, λ = 0.19 W/(m.K); *R* – reinforced concrete, λ = 1.58 W/(m.K).

The age-of-heat-flux concept considers the whole response curve whereas the response-time concept focuses on one point on the curve. The information provided about thermal response can, therefore, differ depending on the method as illustrated in Figs. 12 and 13 where *HSE* is compared with τ_{63} and τ_{95} for the individual cases. Time constant τ_{63} shows a fast thermal response of W2(A) and W3, but does not show any differences in thermal response between the systems and the individual cases. Response time τ_{95} may lead to differences between the individual cases much wider than τ_{63}, e.g. in case of W2(R) and W3(R). This confirms the need for calculating several response times (e.g. τ_{25}, τ_{50}, τ_{63}, τ_{95}) as pointed out by Ning et al. [9]. On the other hand, *HSE* (*HTE*) reflects even subtle differences between the individual cases and systems but the values are more consistent as compared to τ_{95} whereby excessive scatter of the values is prevented. Thus, the potential advantage of using HSE (*HTE*) as compared to the response-time concept is that it allows comparing radiant systems with complex thermal behaviour using a single value.

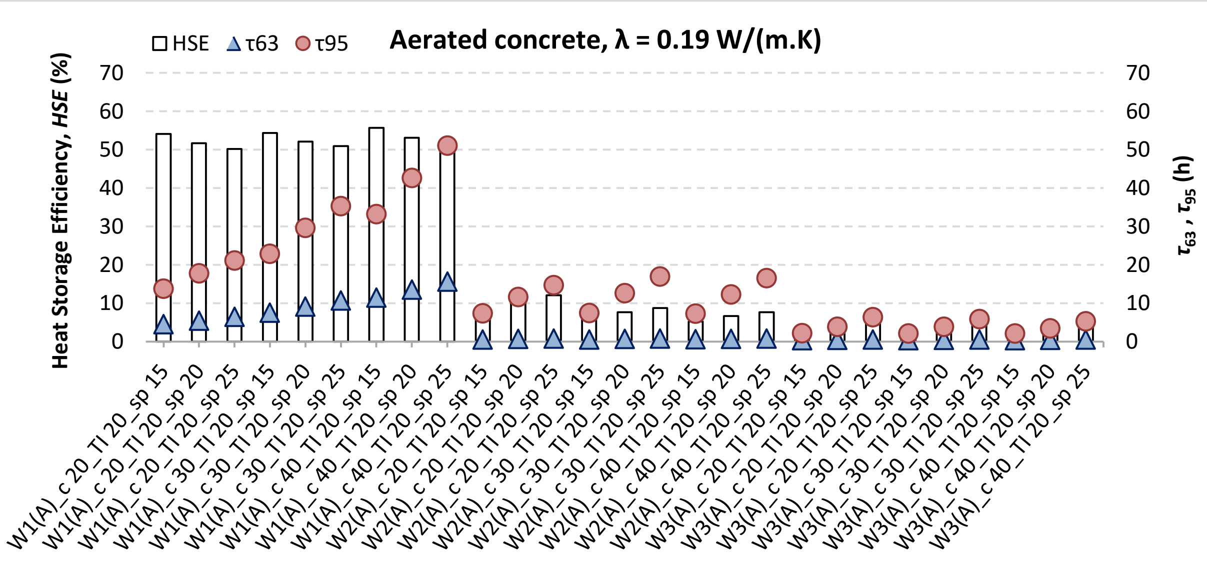


Fig. 12 Comparison of HSE, τ_{63}, and τ_{95} for thermal core made of aerated concrete. Key: *c* – core thickness in cm, *TI* – insulation thickness in cm, *sp* – pipe spacing in cm.

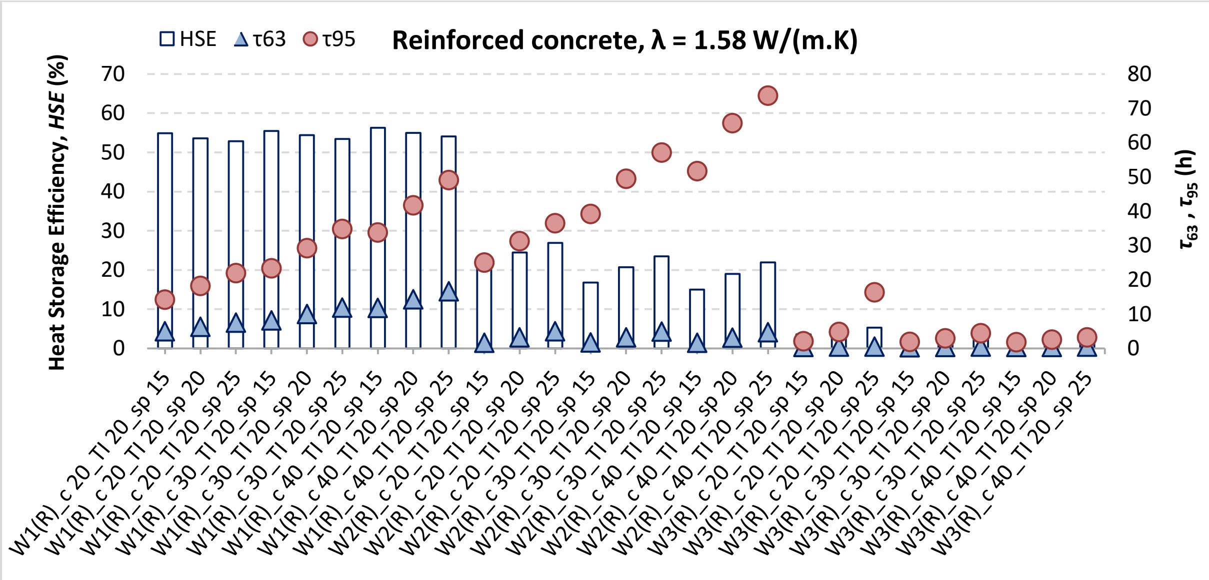


Fig. 13 Comparison of *HSE*, τ_{63}, and τ_{95} for thermal core made of reinforced concrete. Key: *c* – core thickness in cm, *TI* – insulation thickness in cm, *sp* – pipe spacing in cm.

4.7 Effective thermal output

In addition to *HTE* and *HSE*, we propose a composite indicator of steady-state and dynamic thermal performance called effective thermal output ($q_{int,eff}$). The effective thermal output reflects both the steady-state thermal performance represented by the heating/cooling capacity and the dynamic thermal performance represented by *HTE*. In other words, it considers the magnitude of thermal output as well as the speed with which the output is attained. It is defined as:

$$q_{int,eff} = (q_{int} * HTE) / 100 \qquad (W/(m^2.K)) \tag{12}$$

where q_{int} is the maximum value of thermal output which corresponds to the nominal heating/cooling capacity, expressed in W/m^2 per temperature difference between water and room temperature (W/(m^2.K)). In the present study, the maximum (nominal) cooling output was used, and the temperature difference between water and room temperature was 6 K in all the cases. *HTE* is heat transfer efficiency (%).

Figs. 14 and 15 show the differences in effective thermal output ($q_{int,eff}$) between some of the systems and cases. For system W1, *HTE* is similar for aerated and reinforced concrete but the nominal cooling output is substantially higher for reinforced concrete because of its high thermal conductivity. The effective thermal output shows that the difference between the two core materials is considerably lower than indicated by the nominal cooling output. Nevertheless, the thermally conductive core is still preferable.

Effective thermal output also equalizes the performance of the two systems with pipes attached to the thermal core, W2(A) and W2(R). The nominal cooling output is lower, and *HTE* is higher for the core made of aerated concrete, W2(A). However, looking at the effective thermal output, the performance is similar regardless of the core material. For systems W3(A) and W3(R) the nominal and effective thermal output are always similar because the *HTE* is very close to 100% in all cases. These findings suggest that using effective thermal output can be useful especially when comparing TABS (W1, W2).

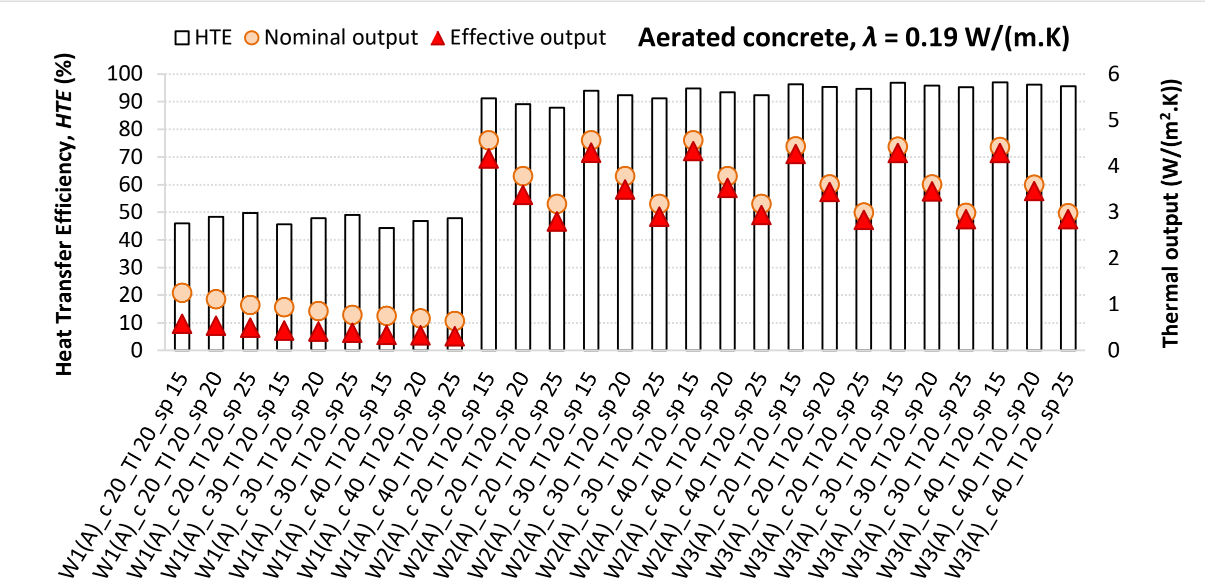


Fig. 14 Comparison of nominal and effective thermal output for thermal core made of aerated concrete. Key: *c* – core thickness in cm, *TI* – insulation thickness in cm, *sp* – pipe spacing in cm.

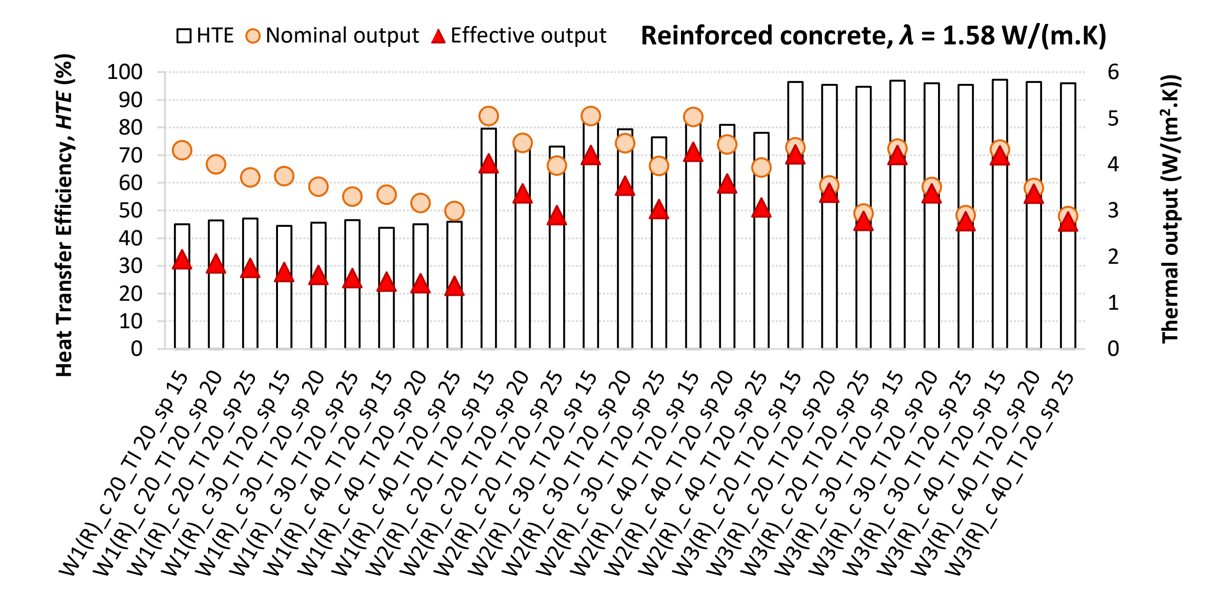


Fig. 15 Comparison of nominal and effective thermal output for thermal core made of reinforced concrete. Key: *c* – core thickness in cm, *TI* – insulation thickness in cm, *sp* – pipe spacing in cm.

5 Conclusion

Novel indicators of thermal response of radiant heating and cooling systems called heat transfer efficiency (*HTE*) and heat storage efficiency (*HSE*) have been proposed. Unlike the commonly used time constant and response time that focus on a single point on the response curve, *HTE* and *HSE* take into account the evolution of the response curve from the beginning until it reaches steady-state. The *HTE* and *HSE* have been compared with established indicators of thermal response represented by the time constant τ_{63}, response time τ_{95}, and thermal energy stored (*TES*). Besides, a composite indicator called effective thermal output has been proposed for a combined evaluation of the steady-state and dynamic thermal performance of radiant systems. The novel indicators can serve to evaluate the thermal performance of various types of radiant systems as well as to develop a control strategy in buildings with radiant systems. The conclusions that may be drawn from this study are:

- Using the whole response curve (*HTE*, *HSE*) rather than one specific point on the curve (τ_{63}, τ_{95}) helps predict the thermal response consistently regardless of the system and core material. It clearly showed the differences between individual systems and materials of the thermal core while preventing excessive scatter of the values.

- For systems with complex thermal behaviour, the results depend on the definition of response time. Calculating *HTE* and *HSE* using the whole response curve allowed comparing such systems with a single value, i.e. without the need to produce several values of response time.
- *TES* predicted the slow thermal response of TABS with thermally conductive core and the fast thermal response of systems with pipes insulated from the core. However, *TES* can be used to compare the thermal response of systems with similar thermal admittance (or similar thermal conductivity if the difference in water and room temperature is constant). Thus, it has limited use as an indicator of thermal response.
- For TABS, using the composite indicator called effective thermal output led to differences in thermal performance between the two materials of thermal core considerably lower than indicated by the maximum (nominal) thermal output.

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