DEPENDENCY OF THE CONVERGENCE RATE MEAN EXTENT OF VARIATION ON THE REPETITIONS NUMBER IN WEAKLY CONNECTED TOPOLOGIES

Martin Kenyeres

Doctoral Degree Programme (2), FEEC BUT

E-mail: kenyeres@phd.feec.vutbr.cz

Bohumil Novotný

Doctoral Degree Programme (3), FEEC BUT

E-mail: xnovot59@stud.feec.vutbr.cz

Supervised by: Vladislav Škorpil

E-mail: skorpil@feec.vutbr.cz

Abstract: This paper deals with the stochastic distributed algorithm – the push-sum protocol. We examine the effect of experiments repetitions on the mean of the convergence rates quantities. The main goal of the executed experiments is to show how many repetitions of the push-sum protocol are necessary to achieve a statistically credible representative of the obtained set of data. Within this paper, we have focused on weakly connected structures.

Keywords: Distributed computing, The push-sum protocol, The convergence rate mean extent of variation

1. INTRODUCTION

The centralized way of computation, which was frequently used in the past, is substituted by the novel solution – distributed computing [1]. The systems whose functionality is executed in this way are labelled as the distributed systems [1]. The agents forming these systems are characterizable by a limited awareness of the other elements within the same system as well as the whole system.

This paper continues the paper entitled as the dependency of the convergence rate mean extent of variation on the repetitions number in strongly connected topologies, where the convergence rate mean extent of variation was examined in a fully-connected mesh topology executing the push-sum protocol. Within this paper, our attention is focused on a line topology, which is a representative of weakly connected structures. We examine the effect of experiment repetitions number on the mean of the convergence rates quantities. Although the experiment executed more times ensures more statistically credible data, too extensive execution presents a demanding computing process. Therefore, the main motivation of this paper is to show how the number of the repetitions of the same experiment affects the credibility of the obtained results.

The experiment within this paper is executed in a line topology with the size of eight agents. The number of repetitions is 10, 100, 1000, 10 000 and 100 000. The experiment is repeated five times for each number of repetitions. At the end, the obtained results are discussed and compared with the ones obtained in the strongly connected topology.

2. THE PUSH-SUM PROTOCOL

The push-sum protocol is a multi-purpose gossip-based aggregation algorithm, which fulfills its functionality by an iterative pair-wise distribution of the aggregated values among particular agents. It is a stochastic distributed protocol, which means that the outputs are supposed to differ when the experiment is repeated despite the same inputs. As mentioned, the push-sum protocol is supposed to solve more than one problem. In this paper, we have focused on the calculation of the average determined by all the initial values. Below is described the execution of the protocol [1]:

- The initial inner states of all the nodes represent the values from which the average is calculated. The initial weights are set to 1 for each node.
- Each node sends a half of its current state and a half of its weight to one of its neighbors chosen uniformly at random. The same value is stored in the inner memory and participates in further computation. This is repeated at each iteration.
- The sum of the inner state and the inner states sent by the adjacent nodes is the inner state for the next iteration. The sum of the weight stored in the inner memory and the weights sent by the adjacent nodes is the weight for the next iteration.
- The ratio of these two sums (the sum of the inner states is the numerator) poses the estimation of the average.
- The previously described procedure is repeated until the consensus is reached

3. THE EXAMINED TOPOLOGY

Within this paper, we focus our attention on a line topology. This topology was chosen as a representative of weakly connected structures, in which the protocol is supposed to reach slower convergence rates. The mentioned topology consists of eight agents and is shown in Fig. 2.

Figure 1: The examined line topology

4. EXPERIMENTS

The results obtained from the numerical experiments are shown in this section.The experiments were executed in order to show the behavior of the convergence rate mean extent of variation [2] of a data set varies when the number of repetitions changes. The number of the repetitions were changed and the experiment was repeated five times for each number.

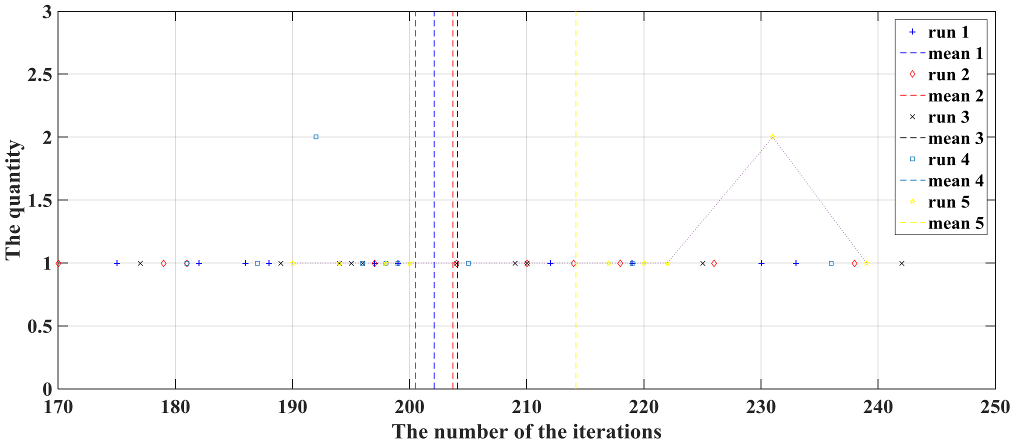

Figure 2: The results for 10 repetitions

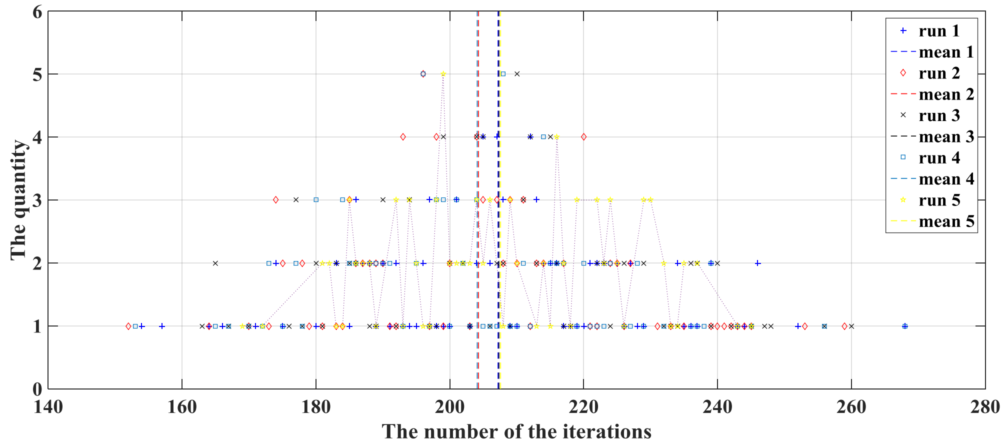

Figure 3: The results for 100 repetitions

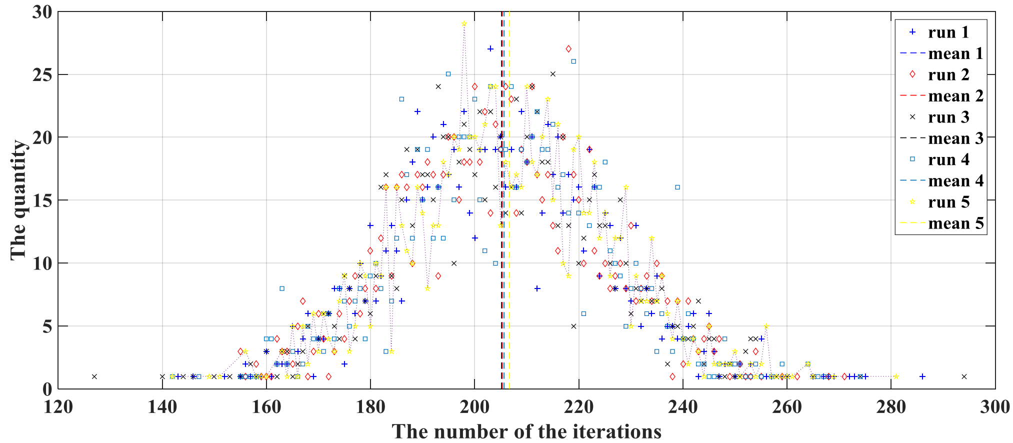

Figure 4: The results for 1 000 repetitions

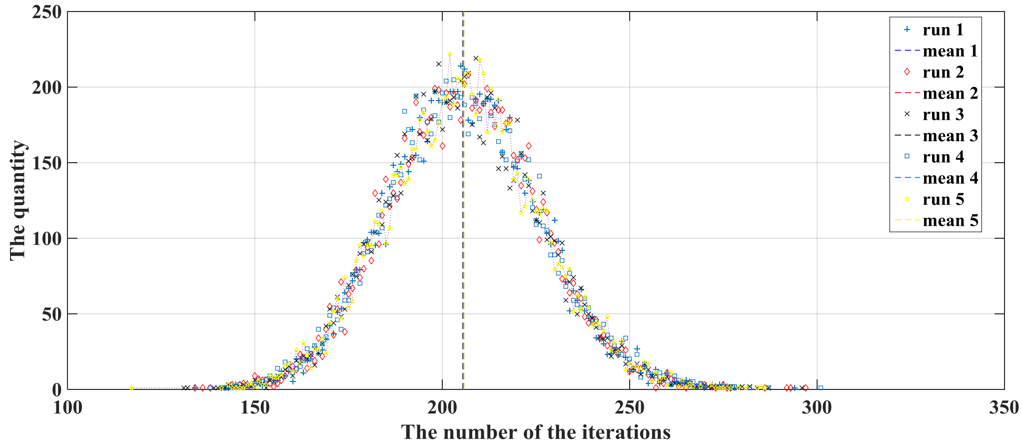

Figure 5: The results for 10 000 repetitions

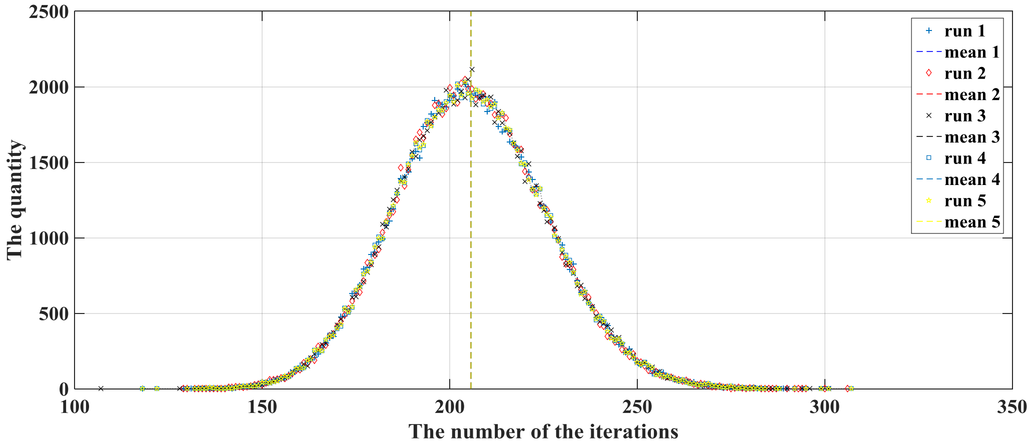

Figure 6: The results for 100 000 repetitions

	10	100	1 000	10 000	100 000
Run 1	202.1	207.2	205.298	205.7752	205.6805
Run 2	203.7	204.29	205.263	205.7373	205.6190
Run 3	204.1	207.34	205.165	205.5086	205.6778
Run 4	200.5	204.1	205.646	205.7110	205.6724
Run 5	214.2	207.59	206.677	205.7225	205.6736
Extent of variation	13.7	3.49	1.512	0.2666	0.0615

Table 1: The convergence rate mean extent of variations for the particular numbers of the repetitions

As we can see from the results, the behavior is same as in the strongly connected topology. The more time the experiment is repeated, the smaller the convergence rate mean extent of variation is for a particular number of the repetitions. Therefore, like in the previous experiment, a higher number of the repetitions results in more credible results.

5. COMPARISON WITH STRONGLY CONNECTED TOPOLOGY

In the following part, we have focused on a comparison of the convergence rate mean extent of variation in strongly (fully-connected mesh) and weakly connected (line) topology. Both structures are of the same size and had the same initial conditions. In Fig. 8, the convergence rate mean extent of variation for both topologies and for 10, 100, 1000, 10 000 and 100 000 repetitions has been shown. As we can see from the graph, the values are decreasing as the number of the repetitions is increasing. Furthermore, the weakly connected structure achieves worse results than the better-connected one.

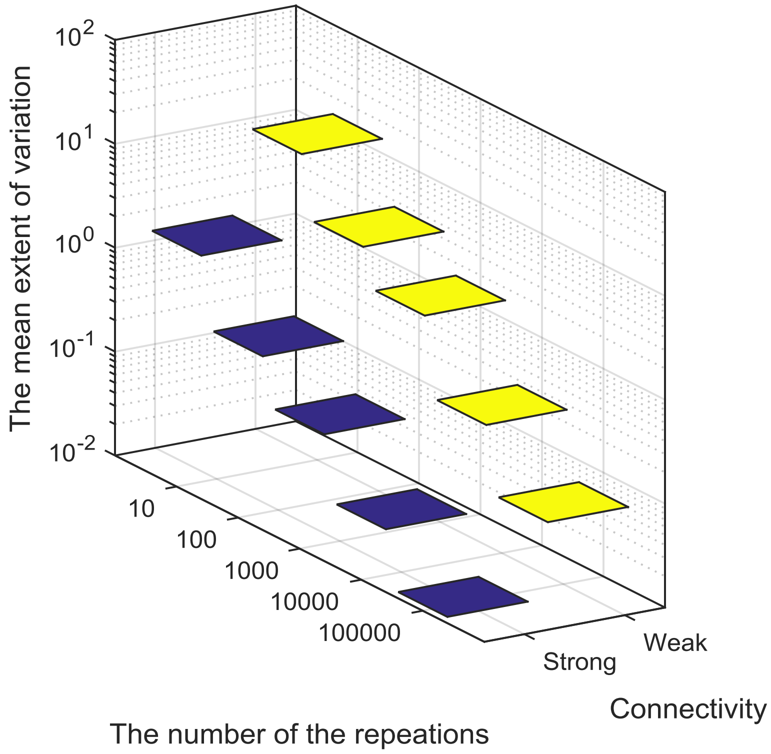

Figure 7: The results for 100 000 repetitions

6. CONCLUSION

In this paper, the impact of the numbers of the push-sum protocol repetitions on the convergence rate mean extent of variation is examined. The experiments were executed on a line topologies with the size of eight agents. The obtained results prove that increasing number of repetitions enhances statistical credibility of the gained results. The behavior of the examined phenomena is same as in the strongly connected topologies with the difference that the mentioned parameter achieves higher value in the weakly connected structure.

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