Robust Power and Subcarrier Allocation for OFDM-Based Cognitive Radio Networks Considering Spectrum Sensing Uncertainties

Houshang FATHI, Seyed Mohammad-Sajad SADOUGH

Cognitive Telecommunication Research Group,
Department of Electrical Engineering, Faculty of Electrical and Computer Engineering,
Shahid Beheshti University G. C., 1983963113, Tehran, Iran.

Email: s_sadough@sbu.ac.ir

Abstract. *In this paper, we address power and subcarrier allocation for cooperative cognitive radio (CR) networks in the presence of spectrum sensing errors. First, we derive the mutual interference of primary and secondary networks affecting each other by taking into account spectrum sensing errors. Then, taking into account the interference constraint imposed by the cognitive network to the primary user and the power budget constraint of cognitive network, we maximize the achievable data rates of secondary users. Besides, in a multi secondary user scenario, we propose a suboptimal but low complexity power and subcarrier allocation algorithm to solve the formulated optimization problem. Our numerical results indicate that the proposed power loading scheme increases the cognitive achievable data rates compared to classical power loading algorithms that do not consider spectrum sensing errors.*

Keywords

Cognitive radio (CR), imperfect spectrum sensing, robust algorithm, subcarrier allocation, power allocation.

1. Introduction

The radio spectrum is inherently a scarce resource especially in wireless communication networks. Moreover, recent studies have shown that the spectrum is not used optimally and spectrum scarcity is more due to ineffective policies in assigning the spectrum that restricts its use solely to licensed users. A promising approach to solve the spectrum scarcity is cognitive radio (CR) technology that proposes to dynamically allocate the spectrum to users. In CR, secondary users should constantly monitor a predefined frequency band allocated to licensed primary in order to detect vacant frequency opportunities, commonly referred to as spectrum holes, where this operation is called spectrum sensing [1], [2]. Obviously, in practice, during the spectrum sensing process, it is essential for secondary users to reliably detect the primary user's signal in order to avoid interference from the secondary transmission to the primary network. However, due to environmental conditions and transmission impairments, the spectrum sensing process is an imperfect process, i.e., its results have some uncertainties.

The Federal Communication Commission (FCC) has suggested geo-location and database access as an alternative to conventional spectrum sensing for TV band devices (TVBD) to access the available channels. However, conventional spectrum sensing is still needed for an optimal usage of the radio spectrum in future applications as suggested by the FCC [3].

Optimal power allocation in practical CR deployment, increases the transmission capacity of the network and optimizes the power consumption. More precisely, traditional methods proposed for power allocation, do not consider the coexistence of secondary and primary networks, and hence, these methods impose an intense interference to primary users [4], [5]. The interference from secondary user imposed on the primary user depends from one side on the spectral interval between the primary and secondary systems, and from another side on the power allocated to the secondary users. Moreover, in practice, the interference imposed from secondary users on primary users should not exceed a predefined threshold. In [6], a CR system based on orthogonal frequency division multiplexing (OFDM) is considered where in order to reduce interference of the cognitive network on the primary network, the authors have suggested to eliminate dynamically adjacent subbands. Obviously, this technique reduces the bandwidth efficiency. In [7], a power allocation problem is formulated and solved at the CR. The power allocation strategy proposed in [8], aimed at maximizing the ergodic capacity or minimizing the outage probability of the secondary users. In [9], the authors propose a CR power allocation algorithm that maximizes the downlink capacity of secondary users so that the interference to the primary user remains in a tolerable range. However, in this work, spectrum sensing is assumed ideal and the error caused by spectrum sensing is not taken into account. Similar CR power allocation schemes are provided in [10], [11], [12], [13], based on different theoretical assumptions, without however, considering the practical scenario where the spectrum sensing block is imperfect and has some sensing errors. In [14], the authors propose to consider spectrum sensing errors in their power allocation scheme. However, the system model in [14] is very simplistic since it assumes only a single secondary user (i.e., not a secondary network composed of multiple CR terminals) without considering any form of subcarrier allocation.

In this work, we assume a CR *network* composed of *multiple* cooperative secondary users. Then, we extend and generalize initial results in [14] by proposing both *power* and *OFDM subcarrier* allocation in a multi secondary user scenario where the solution of the problem is much different from a single secondary user scenario. More precisely, we first derive probabilistic parameters corresponding to spectrum sensing errors based on *detection probability* and *false alarm probability*, which characterize the spectrum sensing block. Second, we formulate the effect of spectrum sensing errors and its impact on the mutual interference between the secondary network and the primary network. The derived interference parameters are used for optimal power allocation problem at the CR network. More precisely, we express the power allocation optimization problem so as to maximize the data rate of the secondary network, provided that the interference to the primary user subbands does not exceed a predefined amount. We also propose a suboptimal but low complexity algorithm for solving the optimization problem.

The rest of this paper is organized as follows. The system model and our main assumptions about the model are presented in Section 2. Section 3 describes the spectrum sensing methodology. Section 4 characterizes and derives different kinds of interferences that can occur in our considered transmission scenario. Section 5 presents the main part of the contribution, i.e., the proposed CR power loading algorithm that takes into account spectrum sensing errors. Section 6 provides numerical results and discussions about the performance obtained by the proposed power loading algorithm and finally, Section 7 draws our conclusions.

2. System Model and Main Assumptions

We consider a downlink transmission scenario composed of one pair of primary transmitter/receiver and multiple secondary (CR) terminals, as depicted in Fig. 1. The primary and cognitive transmitters use an OFDM signaling scheme with N subcarriers. As shown in Fig. 2, the total available bandwidth licensed to primary transmission is equal to B Hz where $B = N\Delta f$, and the CRs try to have an opportunistic access to each of the primary subcarriers via spectrum sensing.

Let us denote by $h^{SS}_{k,i}$ ($k \in \{1,...,K\}$ and $i \in \{1,...,N\}$) the fading channel gain between the CR transmitter and the k-th CR receiver on the i-th subcarrier, by h^{SP}_{i} ($i \in \{1,...,N\}$)

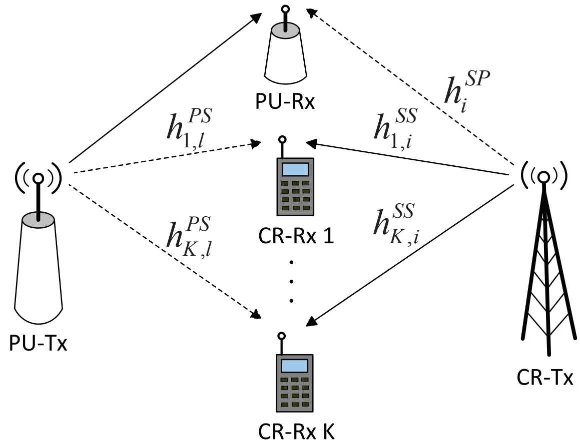

Fig. 1. Topology of the considered transmission scenario.

the channel gain between the CR transmitter and the primary receiver on the i-th subcarrier, and by $h^{PS}_{k,l}$ ($k \in \{1,...,K\}$ and $l \in \{1,...,N\}$) the channel gain between the primary transmitter and the k-th CR receiver on the i-th subcarrier. These instantaneous fading gains are assumed to follow a Rayleigh distribution and assumed perfectly known at the CR transmitter.

3. Spectrum Sensing: An Imperfect Process

In CR systems, one of the factors that increases the interference level at the primary transmitter is errors induced by spectrum sensing, which is inherently an imperfect process. In what follows, we establish the main relations that let us to take into account errors occurring during the spectrum sensing process for preventing harmful interference from the secondary network and increasing the data rates achieved by the CR network. Sensing the presence of a primary transmitter inside the i-th frequency subband can usually be viewed as a binary hypothesis testing problem with hypothesis $\mathcal{H}^i_0$ and $\mathcal{H}^i_1$ defined as:

$$\begin{cases} \mathcal{H}^i_0: & \text{the primary is not transmitting in the } i\text{-th subband,} \\ \mathcal{H}^i_1: & \text{the primary is transmitting in the } i\text{-th subband.} \end{cases}$$

Notice that the above two hypotheses denote the *actual* presence or absence of the primary in the i-th subband. However, in practice, the CR network can only have access to an *imperfect* estimate of the above hypothesis as a result of spectrum sensing. We denote these estimated hypothesis $\widehat{\mathcal{H}}^i_0$ (for the absence of the primary signal in i-th subband) and $\widehat{\mathcal{H}}^i_1$ (for the presence of the primary signal in i-th subband).

In the sequel, we assume that the cooperative CR network has made its final decision about the presence or the absence of the primary signal using the spectrum sensing unit. As shown in Fig. 2, the total available bandwidth is divided to N subbands with a bandwidth equal to Δf. Due

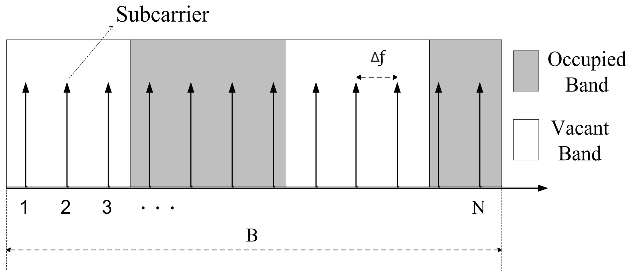

Fig. 2. Total primary bandwidth available for an opportunistic access to the CR network.

to imperfect spectrum sensing, an exact information about the presence of the primary signal in each subband is not available and so the CR user can potentially transmit its data through each of the subbands. We start by defining the probability $P\left(\widehat{\mathcal{H}}_0^i\right)$ as:

$$\begin{aligned} P\left(\widehat{\mathcal{H}}_0^i\right) &= P\left(\widehat{\mathcal{H}}_0^i, \mathcal{H}_0^i\right) + P\left(\widehat{\mathcal{H}}_0^i, \mathcal{H}_1^i\right) \\ &= P\left(\widehat{\mathcal{H}}_0^i | \mathcal{H}_0^i\right) P\left(\mathcal{H}_0^i\right) + P\left(\widehat{\mathcal{H}}_0^i | \mathcal{H}_1^i\right) P\left(\mathcal{H}_1^i\right) \\ &= (1 - P_{fa}^i) P\left(\mathcal{H}_0^i\right) + (1 - P_d^i) P\left(\mathcal{H}_1^i\right) \end{aligned} \quad (1)$$

where $P_{fa}^i = P\left(\widehat{\mathcal{H}}_1^i | \mathcal{H}_0^i\right)$ is referred to as false-alarm probability in i-th subband, $P_d^i = P\left(\widehat{\mathcal{H}}_1^i | \mathcal{H}_1^i\right)$ is referred to as detection probability in i-th subband. Considering (1), the conditional probability α_i can be defined and derived as:

$$\begin{aligned} \alpha_i \triangleq P\left(\mathcal{H}_0^i | \widehat{\mathcal{H}}_0^i\right) &= \frac{P\left(\mathcal{H}_0^i, \widehat{\mathcal{H}}_0^i\right)}{P\left(\widehat{\mathcal{H}}_0^i\right)} \\ &= \frac{P\left(\widehat{\mathcal{H}}_0^i | \mathcal{H}_0^i\right) P\left(\mathcal{H}_0^i\right)}{P\left(\widehat{\mathcal{H}}_0^i\right)} = \frac{(1 - P_{fa}^i) P\left(\mathcal{H}_0^i\right)}{P\left(\widehat{\mathcal{H}}_0^i\right)}. \end{aligned} \quad (2)$$

The probabilistic framework defined above for characterizing the spectrum sensing process, will be used in the sequel for taking into account sensing inaccuracies in the proposed CR power loading scheme.

4. Interference Analysis

4.1 Evaluation of the Interference Introduced by the Primary's Signal to the CR's Signal

The power density spectrum of the primary signal after M-point fast Fourier transform (FFT), can be expressed by the expected value of the periodogram [6] as:

$$E\{I_M(w)\} = \frac{1}{2\pi M} \int_{-\pi}^{\pi} \phi_{PU}(e^{jw}) \left(\frac{\sin(w - \psi)M/2}{\sin(w - \psi)/2} \right)^2 d\psi \quad (3)$$

where w represents the frequency normalized to the sampling frequency and $\phi_{PU}(e^{jw})$ is the power density spectrum of the primary signal. The interference introduced by the l-th primary subcarrier to the i-th CR subcarrier that is allocated to k-th secondary user, denoted as $J_{k,li}$, can be written as:

$$J_{k,li} = |h_{k,l}^{PS}|^2 \int_{(n-\frac{1}{2})\Delta f}^{(n+\frac{1}{2})\Delta f} E\{I_M(w)\}\, dw \quad (4)$$

where $n = |l - i|$, i.e., $n\Delta f$ is the distance in frequency between the i-th CR subcarrier and the l-th primary subcarrier. In what follows, we aim at deriving the interference introduced by the l-th primary subcarrier to the i-th CR subcarrier under the assumption that the spectrum sensing process is imperfect. In other words, the primary and the CR can potentially transmit simultaneously over the same subcarrier, i.e., when the spectrum sensing process makes an incorrect decision (we may thus have interference scenarios where $i = l$). We assume that the spectrum sensing process has made its decision in favor of one of the two hypothesis $\widehat{\mathcal{H}}_0^i$ (i.e., the primary is absent in i-th subband) or $\widehat{\mathcal{H}}_1^i$ (i.e., the primary is operation in i-th subband). However, depending on the spectrum sensing decision ($\widehat{\mathcal{H}}_0^i$ or $\widehat{\mathcal{H}}_1^i$), the interference level is different. The aim of this part is to derive and characterize these interference levels under an imperfect spectrum sensing. More precisely, we derive the primary network interference on the secondary network by taking into account spectrum sensing errors. When the decision of spectrum sensing block is $\widehat{\mathcal{H}}_0^i$, the secondary network can transmit data in i-th subband, but if the decision is $\widehat{\mathcal{H}}_1^i$, secondary network will not transmit data in this subband. When the decision of spectrum sensing block is $\widehat{\mathcal{H}}_0^i$, one of the following two cases occurs: **i)** The primary is not present in the i-th subband ($\mathcal{H}_1^i$). Thus, both the primary and the secondary will simultaneously send data on this subband and the wrong decision of spectrum sensing block causes intense interference. Moreover, we should also consider the primary interference in other subbands on the secondary in i-th subband. **ii)** The primary is not present the i-th subband ($\mathcal{H}_0^i$). In this case, the spectrum sensing block has taken the right decision and there is only the primary interference in other subband on the secondary in the i-th subband. As a result, the average total interference imposed by the primary network on k-th secondary operating in i-th subband under imperfect spectrum sensing, denoted $\widetilde{J}_{k,i}$, writes:

$$\begin{aligned} \widetilde{J}_{k,i} &= \sum_{l=1}^{N} P(\mathcal{H}_1^l, \widehat{\mathcal{H}}_0^i) J_{k,li} \\ &= \sum_{l=1, l\neq i}^{N} P(\mathcal{H}_1^l, \widehat{\mathcal{H}}_0^i) J_{k,li} + P(\mathcal{H}_1^i, \widehat{\mathcal{H}}_0^i) J_{k,ii} \\ &= \sum_{l=1, l\neq i}^{N} P(\mathcal{H}_1^l) P(\widehat{\mathcal{H}}_0^i) J_{k,li} + P(\mathcal{H}_1^i | \widehat{\mathcal{H}}_0^i) P(\widehat{\mathcal{H}}_0^i) J_{k,ii} \\ &= P(\widehat{\mathcal{H}}_0^i) \left[\sum_{l=1, l\neq i}^{N} P(\mathcal{H}_1^l) J_{k,li} + (1 - \alpha_i) J_{k,ii} \right] \end{aligned} \quad (5)$$

where $J_{k,ii}$ is the interference caused by simultaneous transmission of primary and k-th secondary over the i-th subcarrier.

4.2 Evaluation of the Interference Introduced by the CR's Signal to the Primary's Signal

We assume the transmission of CR is performed by ideal Nyquist pulse and with power of $q_{k,i}$ in i-th subband allocated to the k-th secondary, so the power density spectrum related to the k-th CR transmission over the i-th subband can be written as [6]:

$$\phi_{k,i}(f) = q_{k,i}T_s(\frac{\sin\pi f T_s}{\pi f T_s})^2$$

where T_s is the symbol duration, equal to $1/\Delta f$. The interference introduced by the i-th subband of CR to the l-th subband of primary can be written as:

$$\begin{aligned} I_{k,il} &= |h_l^{SP}|^2 \int_{(n-\frac{1}{2})\Delta f}^{(n+\frac{1}{2})\Delta f} \phi_{k,i}(f)df \\ &= q_{k,i}T_s|h_l^{SP}|^2 \int_{(n-\frac{1}{2})\Delta f}^{(n+\frac{1}{2})\Delta f} (\frac{\sin\pi f T_s}{\pi f T_s})^2 df \\ &= q_{k,i}\Theta_{il} \end{aligned} \tag{6}$$

where Θ_{il} is defined as:

$$\Theta_{il} \triangleq T_s|h_l^{SP}|^2 \int_{(n-\frac{1}{2})\Delta f}^{(n+\frac{1}{2})\Delta f} (\frac{\sin\pi f T_s}{\pi f T_s})^2 df. \tag{7}$$

By following a similar approach to the computation of $\widetilde{J}_{k,i}$, we can compute $\widetilde{I}_{k,i}$ as:

$$\begin{aligned} \widetilde{I}_{k,i} &= P(\widehat{\mathcal{H}}_0^i)\left[\sum_{l=1,l\neq i}^{N} P(\mathcal{H}_1^l)I_{il} + (1-\alpha_i)I_{ii}\right] \\ &= q_{k,i}P(\widehat{\mathcal{H}}_0^i)\left[\sum_{l=1,l\neq i}^{N} P(\mathcal{H}_1^l)\Theta_{il} + (1-\alpha_i)\Theta_{ii}\right] \\ &= q_{k,i}\bar{I}_i \end{aligned} \tag{8}$$

where $\widetilde{I}_{k,i}$ is the average total interference imposed under imperfect spectrum sensing by the CR transmission over the i-th subcarrier on the primary network. Finally, the total averaged interference level imposed for secondary transmission on the primary transmission in the i-th subcarrier, denoted $\bar{I}_i$, is derived as:

$$\bar{I}_i \triangleq P(\widehat{\mathcal{H}}_0^i)\left[\sum_{l=1,l\neq i}^{N} P(\mathcal{H}_1^l)\Theta_{il} + (1-\alpha_i)\Theta_{ii}\right]. \tag{9}$$

5. Proposed Power and Subcarrier Allocation Scheme

Here, we aim at allocating the power to each subcarrier of the OFDM-based CR so as to maximize the achievable information rate for the CR network while keeping the instantaneous interference introduced to the primary below a predefined threshold. Considering an ideal coding scheme and using the Shannon capacity formula, the cognitive achievable data rate at the i-th subcarrier that is allocated to the k-th secondary, denoted $R_{k,i}$ for $i \in \{1,\ldots,N\}$ and $k \in \{1,\ldots,K\}$, is given by:

$$R_{k,i}(q_{k,i}) = \alpha_i \Delta f \log_2\left[1 + \frac{|h_{k,i}^{SS}|q_{k,i}}{\sigma_{k,i}^2 + \widetilde{J}_{k,i}}\right] \tag{10}$$

where $q_{k,i}$ is the total transmit power in the i-th subcarrier that is allocated to the k-th secondary, and $\sigma_{k,i}^2$ denotes the additive white Gaussian noise (AWGN) variance.

Using (10), the proposed cognitive power allocation under imperfect spectrum sensing is formulated as the following optimization problem:

$$C = \max_{q_{k,i},x_{k,i}} \sum_{k=1}^{K}\sum_{i=1}^{N} x_{k,i}R_{k,i}(q_{k,i})$$

subject to:

$$\begin{aligned} &\sum_{k=1}^{K}\sum_{i=1}^{N} x_{k,i}q_{k,i}\bar{I}_i \le I_{th}, \\ &\sum_{k=1}^{K}\sum_{i=1}^{N} x_{k,i}q_{k,i} \le Q, \\ &\sum_{k=1}^{K} x_{k,i} \le 1, \\ &q_{k,i} \ge 0 \quad \forall k\,\forall i, \\ &x_{k,i} \in \{0,1\} \quad \forall k\,\forall i \end{aligned} \tag{11}$$

where C indicates the transmission capacity of the CR user, I_{th} is the total tolerable interference at the primary and Q is the total maximum power constraint at the CR network; $x_{k,i}$ is a binary variable indicating that subcarrier i is allocated to secondary user k or not, whereas each subcarrier can be allocated to only one secondary user. The optimization problem (11) for $K = 1$ is a standard convex problem that can be solved in a straightforward manner by using the Lagrange multiplier method [15].

Proposition: In a single CR user case ($K = 1$), the optimal solution is derived as:

$$q_i^* = \max\left\{0, \frac{1}{\lambda + \mu\bar{I}_i} - \frac{\sigma_i^2 + \widetilde{J}_i}{|h_i^{SS}|^2}\right\} \quad , i = 1,\ldots,N \tag{12}$$

where $\lambda \ge 0$ and $\mu \ge 0$ are the Lagrange multipliers.

Proof: The proof is provided in the appendix.

The optimization problem (11) for the case of multiple secondary users is a mixed integer non-linear programming (MINLP) problem , for which the optimal solution is NP-hard [16]. However, in the sequel, we propose a suboptimal algorithm with low complexity to solve it. First, we define $\eta_{k,i}$ as follows:

$$\eta_{k,i} = \frac{|h_{k,i}^{SS}|^2}{\sigma_{k,i}^2 + \widetilde{J}_{k,i}} \quad , \forall i,k. \tag{13}$$

Then, for each subcarrier i and CR user k, we calculate $\eta_{k,i}$ and we follow a greedy scheme [17], [18], i.e., for the i-th subcarrier, the secondary having the maximum value $\eta_{k,i}$ is allowed to transmit over this subcarrier. The index of this user is denoted $\widetilde{k}_i$, i.e., we have:

$$\widetilde{k}_i = \arg\max_{k} \eta_{k,i}, \quad i = 1, \ldots, N. \tag{14}$$

Second, we allocate the power to subcarriers as follows:

$$q^*_{\widetilde{k}_i,i} = \widehat{Q} \frac{\eta_{\widetilde{k}_i,i}}{\sum_{i=1}^{N} \eta_{\widetilde{k}_i,i}} \quad , i = 1, \ldots, N \tag{15}$$

where $\widehat{Q}$ is defined in Tab. 1, that is the power level that satisfies the constraints of power budget and interference in (11). Obviously, in our algorithm, greater power is allocated to subcarriers having greater parameter $\eta_{\widetilde{k}_i,i}$.

Complexity issues of the proposed algorithm: We can find the optimal solution of the problem (11) by first finding the optimal subband allocation to the secondary users by an exhaustive search and then the power allocation can be optimized by another exhaustive search over subbands. Hence, the complexity of the optimum solution is $O(NK^N)$. The complexity of the optimal algorithm proposed in [19] is $O(KN^2)$. Our proposed suboptimal algorithm has a reduced complexity of $O(KN)$.

Thus, although suboptimal, the proposed algorithm is less complex in terms of practical implementation. The proposed suboptimal and low complexity algorithm for CR power and subcarrier allocation is provided below.

Proposed power and subcarrier allocation algorithm

Set $x_{k,i} = 0, \quad \forall i \forall k$
Set $q^*_{k,i} = 0, \quad \forall i \forall k$
Calculate $\eta_{k,i} = \frac{|h^{SS}_{k,i}|^2}{\sigma^2_{k,i} + \bar{J}_{k,i}}, \quad \forall i \forall k$
$\widetilde{k}_i = \arg\max_k \eta_{k,i}, \quad i = 1, \ldots, N$
$x_{\widetilde{k}_i,i} = 1, \quad i = 1, \ldots, N$
$\bar{q} = Q/N$
Calculate $\bar{I}_i, \quad i = 1, \ldots, N$ from (9)
Calculate $\widehat{I} = \bar{q} \sum_{i=1}^{N} \bar{I}_i$
if $\widehat{I} > I_{th}$
Set $\quad \bar{q} = \frac{Ith}{\sum_{i=1}^{N} \bar{I}_i}$
end
Set $\widehat{Q} = N\bar{q}$
Set $q^*_{\widetilde{k}_i,i} = \widehat{Q} \frac{\eta_{\widetilde{k}_i,i}}{\sum_{i=1}^{N} \eta_{\widetilde{k}_i,i}}, \quad i = 1, \ldots, N$
if $q^*_{\widetilde{k}_i,i} = 0, \quad i = 1, \ldots, N$
Set $\quad x_{\widetilde{k}_i,i} = 0$
end
Calculate $\alpha_i, \quad i = 1, \ldots, N$ from (2)
Calculate $C = \sum_{k=1}^{K} \sum_{i=1}^{N} \alpha_i x_{k,i} \Delta f \log_2 \left[1 + q_{k,i} \eta_{k,i}\right]$

Tab. 1. Pseudo code of the proposed power and subcarrier allocation algorithm.

6. Numerical Results and Discussion

Throughout this section, the total available bandwidth (B) is assumed equal to 12 MHz. We also assume that 12 OFDM subbands are available for cognitive transmission with a bandwidth equal to 1 MHz for each subcarrier. It is assumed that a long-enough cyclic prefix (CP) is employed at the OFDM transmitter. The *a priori* probabilities about the presence of the primary in different subbands are gathered in vector $P(\mathcal{H}_1) = [0.75, 0.6, 0.7, 0.2, 0.15, 0.25, 0.1, 0.55, 0.7, 0.6, 0.2, 0.3]$. We also assume that the probability of false alarm is equal to 0.08 in all subbands and the detection probability of subbands is equal to $P_d = [0.97, 0.94, 0.96, 0.98, 0.95, 0.99, 0.98, 0.97, 0.96, 0.95, 0.98, 0.99]$. Moreover, the noise variance is set to 0 dBm and the fading coefficients for all channels are assumed to follow a Rayleigh distribution with an average channel power gain of 0 dBm. The CR network has 5 secondery users ($K = 5$).

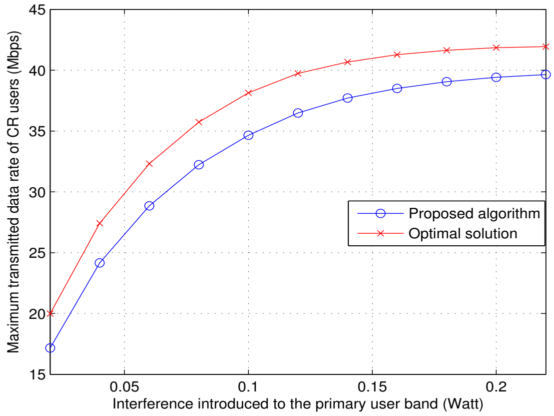

Fig. 3. Maximum achieved data rate of CR user versus the average interference imposed to the primary user band.

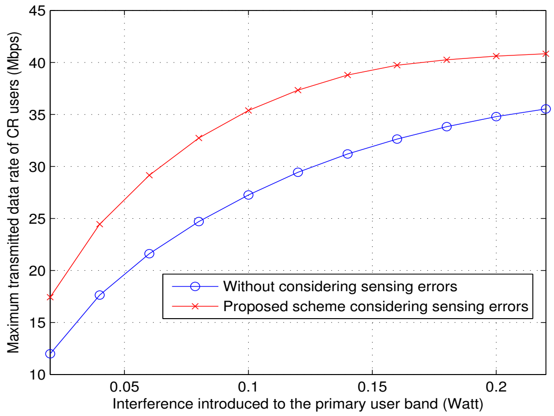

Fig. 4. Maximum achieved data rate of CR user versus the average interference imposed to the primary user band.

Figure 3 depicts the maximum transmitted data rate of CR user versus the average interference imposed to the primary band where the transmit power budget (Q) is fixed and set to 3 Watt. In this figure, for the sake of comparison, we have provided the data rates achieved by the optimal algorithm proposed in [19] to obtain the optimal power allocation of (11). This figure shows that the proposed algorithm provides data rates which are not very far from those achieved by the optimal solution in [19].

Figure 4 depicts the maximum transmitted data rate of CR user versus the average interference imposed to the primary user band in which the transmit power budget is fixed and set to 3 Watt. This figure shows that the proposed scheme leads to higher CR data rates for a given interference level imposed to the primary user band.

Similar plots are provided in Fig. 5 showing the maximum transmitted data rate of CR user versus this time the total maximum power constraint of CR network and a fixed predefined interference threshold is $I_{th} = 0.12$ Watt. Again, we observe the superiority of our proposed method compared to the conventional method that does not consider errors induced by spectrum sensing.

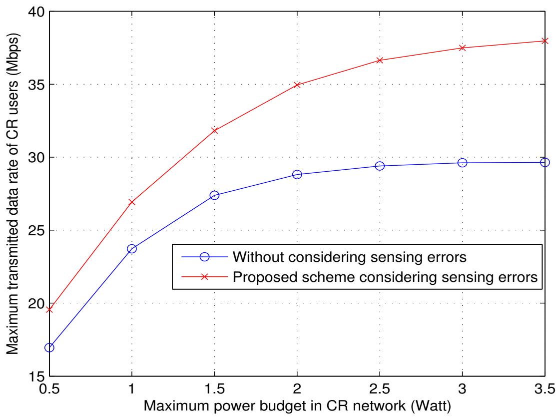

Fig. 5. Maximum achieved data rate of CR user versus the total maximum power constraint at the CR network.

Figure 6 depicts the power allocation (q_i) and interference factor ($\bar{I}_i$) in CR subbands at a fixed interference level ($I_{th} = 0.12$ Watt) and a fixed total power budget ($Q = 3$ Watt). We observe that by using our proposed algorithm, a larger amount of power is allocated to subcarriers with a lower interference level, and vice versa.

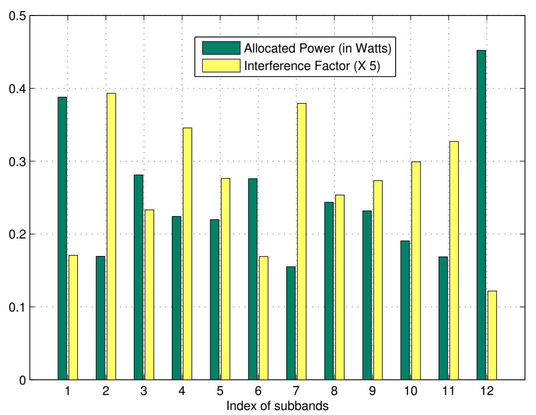

Fig. 6. CR power allocation among OFDM subcarriers, $I_{th} = 0.12$ Watt and $Q = 3$ Watt.

7. Conclusion

We proposed power and subcarrier allocation scheme for OFDM-based CR system while taking spectrum sensing errors into account. We provided appropriate relations for modeling the mutual interference between the CR network and the primary network. We have satisfied the required QoS of the primary network by considering the interference imposed from the CR to the primary. Similarly, we have taken into account the interference imposed from the primary network on the CR network to satisfy the secondary QoS, in terms of achievable rates. Besides, we propose a suboptimal power allocation algorithm with lower complexity to solve the optimization problem. It was shown that the rates achieved by using the proposed algorithm are not very far from rates achieved by the optimal (but not practical) solution. Moreover, simulation results confirmed that the proposed scheme maximizes the data rate achieved by the CR network, while keeping the interference imposed on primary network within a tolerable limit.

Appendix

Derivation of q_i^* in (12)

We start by converting the optimization problem in (11) to the standard form [15]. To this end, we write the problem as:

$$\min_{q_i} -\sum_{i=1}^{N} \alpha_i \Delta f \log_2 \left[1 + \frac{|h_i^{SS}|^2 q_i}{\sigma_i^2 + \widetilde{J}_i}\right] \tag{16}$$

subject to:

$$\sum_{i=1}^{N} q_i \bar{I}_i - I_{th} \leq 0, \tag{17}$$

$$\sum_{i=1}^{N} q_i - Q \leq 0, \tag{18}$$

$$-q_i \leq 0 \quad \forall i. \tag{19}$$

The problem in (16) is a nonlinear convex problem, in other words, the objective function is nonlinear and concave with respect to q_i, so, the optimal solution can be obtained by the Karush-Kuhn-Tucker (KKT) conditions [15].

Considering Lagrange multipliers λ', μ' and ν_i' for the inequality constraints in (17), (18) and (19), respectively, we can define the Lagrangian L associated with (16), (17), (18) and (19) as:

$$\begin{aligned} L(q_i, \lambda', \mu', \nu_i') = &- R_i(q_i) + \lambda' (\sum_{i=1}^{N} q_i \bar{I}_i - I_{th}) \\ &+ \mu' (\sum_{i=1}^{N} q_i - Q) - \nu_i' q_i \end{aligned} \tag{20}$$

where

$$R_i(q_i) = \sum_{i=1}^{N} \alpha_i \Delta f \log_2 \left[1 + \frac{|h_i^{SS}|^2 q_i}{\sigma_i^2 + \widetilde{J}_i} \right].$$

Now we express the gradient of $L(q_i, \lambda', \mu', \nu_i')$ with respect to q_i as:

$$\nabla_{q_i} L(q_i, \lambda', \mu', \nu_i') = -\frac{\Delta f}{\ln 2} \cdot \frac{1}{q_i + 1/\eta_i} + \lambda' \bar{I}_i + \mu' - \nu_i' \quad (21)$$

where

$$\eta_i \triangleq \frac{|h_i^{\mathrm{SS}}|^2}{\sigma_i^2 + \widetilde{J}_i}.$$

To solve the problem, we can write the KKT conditions as:

$$\begin{aligned}
\sum_{i=1}^{N} q_i^* \bar{I}_i - I_{th} &\le 0,\\
\sum_{i=1}^{N} q_i^* - Q &\le 0,\\
-q_i^* &\le 0 \quad \forall i,\\
\lambda' &\ge 0,\\
\mu' &\ge 0,\\
\nu_i' &\ge 0,\\
\lambda' (\sum_{i \in V} q_i^* \bar{I}_i - I_{th}) &= 0,\\
\mu' (\sum_{i \in V} q_i^* - Q) &= 0,\\
-\nu_i' q_i^* &= 0 \quad \forall i,\\
-\frac{1}{q_i^* + 1/\eta_i} + \lambda \bar{I}_i + \mu - \nu_i &= 0 \qquad (22)
\end{aligned}$$

where $\lambda = \frac{\ln 2}{\Delta f}\lambda'$, $\mu = \frac{\ln 2}{\Delta f}\mu'$ and $\nu_i = \frac{\ln 2}{\Delta f}\nu_i'$. Considering the condition $\nu_i' q_i^* = 0$, we can remove ν_i from (22) and rewrite it as:

$$q_i^* \ge 0 \quad \forall i, \qquad (23)$$

$$\sum_{i=1}^{N} q_i^* \bar{I}_i = I_{th}, \qquad (24)$$

$$\sum_{i=1}^{N} q_i^* = Q, \qquad (25)$$

$$\frac{1}{q_i^* + 1/\eta_i} = \lambda \bar{I}_i + \mu. \qquad (26)$$

Substituting q_i^* from (26) into (24) and (25), we get the Lagrange multipliers, μ and λ.

Considering that (23) should be equal or greater than zero, we derive q_i^* as:

$$q_i^* = \max\left\{0, \frac{1}{(\lambda + \mu \bar{I}_i)} - \frac{1}{\eta_i}\right\} \quad \forall i, \qquad (27)$$

and this ends the proof.

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