

MATHEMATICAL MODEL OF HEAT PUMP

Jiri Pitron

Doctoral Degree Programme (1.) FEEC BUT

E-mail: xpitro00@stud.feec.vutbr.cz

Supervised by: Petr Mastny

E-mail: mastny@feec.vutbr.cz

Abstract: In this paper different energy states of a heat pump are described. Equations used for mathematical description of the heat pump in the Matlab Simulink are presented. Created model is used to calculate the energy flows in the system according to different input parameters. The simulation involves an accumulation tank, which is controlled by individual input and output parameters. Simulation results have been compared with the experimental measured values on a heat pump in a laboratory.

Keywords: Heat pump, heating, mathematical modeling, Simulink.

1. NOMENCLATURE

- q_{ref}, q_{air} mass flow of refrigerant, air entering into an evaporator ($\text{kg}\cdot\text{s}^{-1}$)
- q_s mass flow of water input to the accumulation tank ($\text{kg}\cdot\text{s}^{-1}$)
- s_{ev}, s_c entropy of refrigerant behind the expansion valve, the compressor ($\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$)
- s_{evap}, s_{co} entropy of refrigerant behind the evaporator, the condenser ($\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$)
- T_{ev}, T_{evap} temperature of refrigerant behind the expansion valve, the evaporator (K)
- T_c, T_{co} temperature of refrigerant behind the compressor, the condenser (K)
- T_1, T_2 input and output temperatures of the air (K)
- c_a, c specific heat capacity of air, of water ($\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$)
- η_c the efficiency of the compressor (-)

2. INTRODUCTION

The heat pump (HP) is device which offers energy-efficient way to provide heating and cooling in many applications. HP can be installed in residential and office buildings to reduce energy consumption. The main function of the HP is to pump heat from low temperature heat sources to high temperature heat sinks, thus providing both comforts heating and cooling. Conventional HP includes basic items such as evaporator, compressor, condenser and expansion (throttling) valve. Those components have different parameters and possess different operational characteristics, particularly under transient conditions. It is conceivable, that the transient behavior of the integrated HP system is quite different from those of the single components. However, it is possible to force the heat from a source at a lower temperature to a sink at a higher temperature using a relatively small quantity of mechanical work. In heating mode, the heat pump transfers heat from the ground or the outdoor air to a building or an industrial application. Theoretically, the total heat delivered by the heat pump is equal to the heat extracted from the heat source including the amount of mechanical work. It is important to notice that the heat extracted from the source is renewable energy in the form of low-temperature heat [1], [2].

2.1. MATHEMATICAL EQUATIONS

This part describes the equations for thermal changes in the heat pump. These equations are used in designs of simulation program Simulink.

Suggestion of this simulation has been created with validation on a specific heat pump. However, a description of all the changes which take place in the heat pump circuit specially describing the refrigerant is considerably more difficult. Therefore simulation considers some simplifications.

- In the entire heat pump system is considered a constant mass flow of refrigerant q_{ref} .
- The simulation calculates the energy value of the sub-components. When some value is changed, the impact on the other calculations in compliance with precision of the calculation has to be considered.

In this simulation it is necessary to know temperatures between individual sub-components and then enthalpy of the refrigerant can be determined.

The energy of refrigerant entering into an evaporator:

$$Q_{ev} = q_{ref} \cdot s_{ev} \cdot T_{ev} \quad (\text{W}) \quad (1)$$

The heat flow received from the environment through the evaporator can be expressed:

$$Q_{evap} = q_{air} \cdot c_a \cdot (T_1 - T_2) \quad (\text{W}) \quad (2)$$

Thermal power delivered by the compressor into a refrigerant:

$$Q_{comp} = q_{ref} \cdot (s_c - s_{evap}) \cdot (T_c - T_{evap}) \quad (\text{W}) \quad (3)$$

The energy input of refrigerant into the expansion valve:

$$Q_{cond} = q_{ref} \cdot s_{co} \cdot T_{co} \quad (\text{W}) \quad (4)$$

Subsequently it can be expressed, that the heat flux input to the storage tank is reduced of the efficiency the heat exchanger, which is around 70-80 %:

Equation (4) is used for the calculation as a simplification, because the s_{ev} and T_{ev} are not known. It presumes $Q_{cond} = Q_{evap}$.

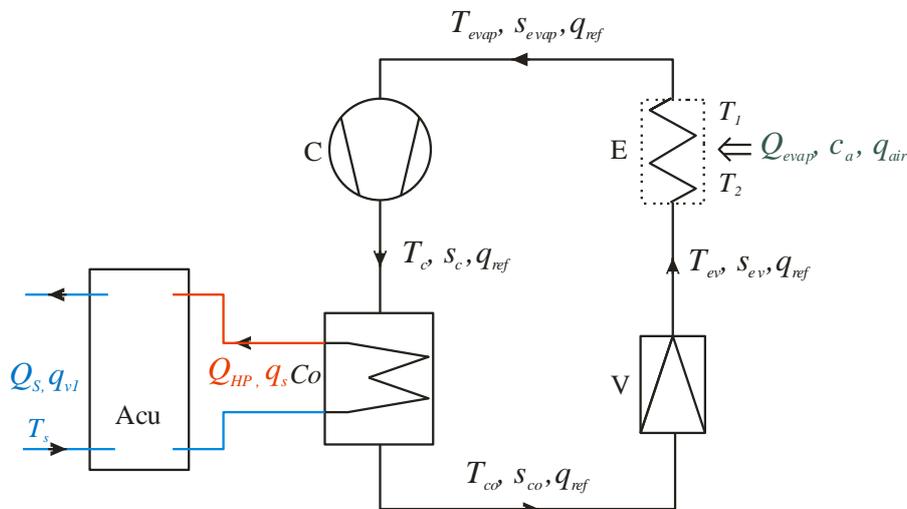


Figure 1: Scheme of heat pump

$$Q_{HP} = \left(\frac{Q_{exp} + Q_{evap} + Q_{comp} - Q_{cond} \cdot c \cdot q_s}{q_{ref} \cdot \frac{s_{co} - s_c}{2}} \right) \cdot Q_z \quad (\text{W}) \quad (5)$$

The simulation uses increments of energy obtained by the heat pump. In this simulation the energy inputted to an accumulation tank means that this energy is not dependent on the temperature of water output from the accumulation tank into the condenser. The calculation of mass flow of hot water into the accumulation tank is determined:

$$q_s = \left(\frac{q_{ref} \cdot \frac{s_{co} - s_c}{2}}{c} \right) \quad (\text{kg} \cdot \text{s}^{-1}) \quad (6)$$

Coefficient of performance (*COP*) is related to the heating mode and it is determined from the energy output of the heat pump and the electrical energy [3], [4]:

$$COP = \frac{Q_{HP}}{W_c} = \frac{Q_{HP}}{\frac{Q_c}{\eta_c}} \quad (-) \quad (7)$$

2.2. SIMULATION IN SIMULINK

The above-described equations have been implemented into the program Simulink. There is the accumulation tank, which can be monitored and controlled. For example it can be used for opening and closing of the individual valves of hot water from the heat pump or the cold water which should be heated.

$$\begin{aligned} c &= 4180 \text{ (J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}) & c_a &= 1010 \text{ (J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}) & \rho &= 998 \text{ (kg} \cdot \text{m}^{-3}) \\ T_o &= 293.15 \text{ (K) (20 }^\circ\text{C)} & T_{o_an} &= 295.15 \text{ (K) (22 }^\circ\text{C)} & & \text{initial temperature in the Acu} \\ V_{an} &= 0.201 \text{ (m}^3) & S_{an} &= 1.11 \text{ (m}^2) & & \text{volume, surface of Acu} \\ k_{an} &= 26 \text{ (W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}) & & & & \text{heat losses an Acu} \end{aligned}$$

3. COMPARATION OF SIMULATED AND MEASURED VALUES

The following part shows the results of the measurement performed on the heat pump. The time of the entire measurement is 90 min (5400 s). During this time, the heat pump has been working continuously and providing the heat energy to the accumulation tank.

In the next step the measured values are compared with the results of the simulation model described above. In Figure 2 the measured temperatures in the laboratory heat pump system are shown. The measurements correspond with the values in Figure 1.

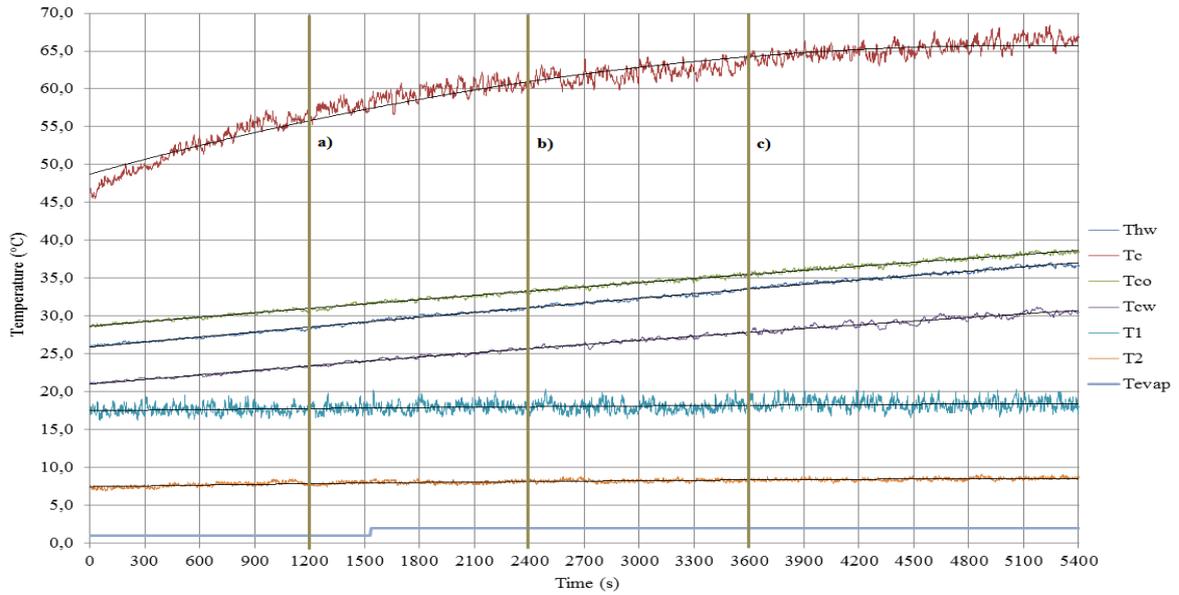


Figure 2: Measured temperature in the heat pump system

Because of evaluation the simulation results have been compared with temperatures in the Figure 3. Temperatures have been used to calculate the entropies. These values have been used to validate the created model.

	T_c (K)	s_c (J·kg ⁻¹ ·K ⁻¹)	T_{evap} (K)	s_{ev} (J·kg ⁻¹ ·K ⁻¹)	T_{co} (K)	s_{co} (J·kg ⁻¹ ·K ⁻¹)	Q_{HP} (W)	COP (-)
a)	329.15	1742.00	274.15	1725.0	304.15	1148.00	622.0	2.96
b)	335.15	1744.00	275.15	1726.0	306.15	1157.10	624.9	2.64
c)	337.15	1745.10	275.15	1726.0	308.65	1169.10	633.5	2.49

Table 1: Measured values

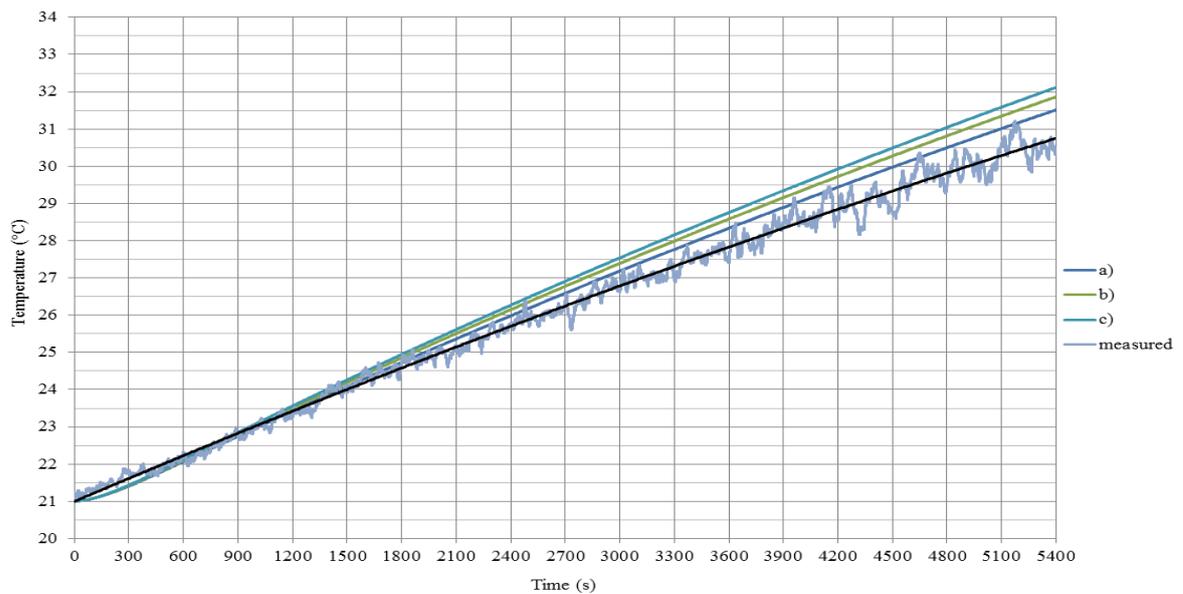


Figure 3: Temperatures in accumulation tank

In Figure 3 the temperatures of the accumulation tank water are demonstrated. The temperatures increase due to the heat received by the water. Each curve a), b), c) represents simulation parameters in Table. The highest output temperature has been achieved with the parameters of the simulation in the line c).

4. CONCLUSION

In this paper the equations regarding the refrigerant energy states of the heat pump's individual components have been described. The equations have been used to create a Simulink model of the heat pump.

Next part shows the measurements performed on the laboratory heat pump. Measured values have been plotted and they are in Figure 2. The adequate entropies were determined based on temperatures from Figure 2. These values have been used as the parameters for the creating of model and the results are in Figure 3.

In Figure 3 the temperatures are affected by the measured (calculated) energy taken from the condenser to the accumulation tank. For the line a) measured energy is $1.640 \text{ kJ}\cdot\text{s}^{-1}$, calculated energy is $1.560 \text{ kJ}\cdot\text{s}^{-1}$, for the line b) measured energy is $1.706 \text{ kJ}\cdot\text{s}^{-1}$, calculated energy is $1.615 \text{ kJ}\cdot\text{s}^{-1}$ and for the line c) measured energy is $1.773 \text{ kJ}\cdot\text{s}^{-1}$ and calculated energy is $1.655 \text{ kJ}\cdot\text{s}^{-1}$. After comparing these results, it can be stated, that the output temperature after 90 minutes can be higher than the calculated, but Figure 3 shows the opposite trend. Explanation of this behavior is that in the different energy loss of the accumulation tank has been used in the simulation. The heat loss of the accumulation tank is higher than loss considered in the modeled process.

Other values that can be compared are the coefficient of performance. Values of simulated COP are in Table 1. Based on the measuring there is $\text{COP}=2.8$ for the same condition with line a) of Table 1, $\text{COP}=2.73$ for line b) and $\text{COP}=2.63$ for line c), respectively.

In case that produce thermal energy is constant and water temperature increases then input energy for compressor is getting higher and value of COP is getting lower in the same time.

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