OFDM Signal Detector Based on Cyclic Autocorrelation Function and its Properties

Vladimír ŠEBESTA, Roman MARŠÁLEK, Zbyněk FEDRA

Dept. of Radio Electronics, Brno University of Technology, Purkyňova 118, 612 00 Brno, Czech Republic

sebesta@ feec.vutbr.cz, marsaler@feec.vutbr.cz, fedraz@feec.vutbr.cz

Abstract. *This paper is devoted to research of the general and particular properties of the OFDM signal detector based on the cyclic autocorrelation function. The cyclic autocorrelation function is estimated using DFT. The parameters of the testing signal have been chosen according to 802.11g WLAN. Some properties are described analytically; all events are examined via computer simulations. It is shown that the detector is able to detect an OFDM signal in the case of multipath propagation, inexact frequency synchronization and without time synchronization. The sensitivity of the detector could be decreased in the above cases. An important condition for proper value of the detector sampling interval was derived. Three types of the channels were studied and compared. Detection threshold SNR = -9 dB was found for the signal under consideration and for two-way propagation.*

Keywords

Spectrum sensing, cyclic autocorrelation function, OFDM, fading channel, frequency offset.

1. Introduction

Cognitive radio systems have to reliably detect the existence of a primary user in the given frequency band [1]. In the future, many wireless communication systems will use OFDM [2]. That is why detecting OFDM signals is very important.

Many of the proposed and described detectors of OFDM signals are the cyclostationary detectors. They have a great advantage of robustness against noise uncertainty. Feature detectors detect cyclostationarity caused by cyclic prefix [3] or cyclostationarity artificially embedded in OFDM signal [4].

The basic theory of cyclostationary detectors is given in [5]. There are the two main tools of cyclostationary OFDM detectors: spectral correlation function (SCF) and cyclic autocorrelation function (CAF). This paper is focused on detectors based on the cyclic autocorrelation function.

In [6] the authors compare the advantages and deficiencies of energy detection and cyclostationary detection, and then put forward an improved solution.

A cyclostationary statistical test based on the spectrum sensing algorithm (CST method) is proposed in [7]. Simulations are carried out in the AWGN channel. Part of the theory is cognate with that in [8].

An effort to improve performance of cyclostationary detectors led to the research into more sophisticated and complex algorithms. Cyclostationarity detectors utilizing multiple cyclic frequencies of OFDM signals are introduced and analyzed in [9] and [10]. The paper [11] is devoted to the detection of weak signals if multiple signals with different received-power levels are captured simultaneously. The proposed detection method suppresses the effects of previously-detected signals in the cyclic autocorrelation domain, and so increases the detection probability of weak signals. Paper [12] is focused on cyclostationary classifying different OFDM signals.

In this paper, an attempt is made to analyze and describe important properties of the detector published in [8]. The detector uses cyclostationarity established by the cyclic prefix. It will be shown that an optimal detector function requires a certain relation to hold between the OFDM signal parameters and the detector parameters. A significant violation of this relation can lead to the failure of the detection.

This paper is organized as follows. The fundamental description of the chosen detector is given in section 2. The basic behavior of the statistic T is analyzed in section 3. The impact of multipath propagation, frequency offset, time offset and sampling frequency is investigated in section 4. Finally, the overall performance of the detector is shown in section 5. The conclusion is given in section 6.

2. Algorithm

The conjugate cyclic autocorrelation function at cyclic frequency α can be estimated as [8]

$$\hat{R}_x^{\alpha} = \frac{1}{N}\sum_{n=0}^{N-1} x(n)x^*(n-\tau)\exp(-j2\pi\alpha n/N) \qquad (1)$$

where $x(n)$ is the signal being examined, N is the number of signal elements in the detector FFT, n is normalised time, and τ is the normalized lag parameter in the autocorrelation.

The detection statistic T is defined via a covariance matrix. For a vector of zero mean random variables, an estimate of the covariance matrix can be computed as [8]

$$\hat{\mathbf{\Sigma}}_{2c} = \begin{bmatrix} A & B \\ B & C \end{bmatrix} \quad (2)$$

where the matrix elements are computed as

$$A = \frac{1}{N}\sum_{k=0}^{N-1} \mathrm{Re}\left(\hat{R}_x^{\alpha_k}\right)^2 , \quad (3)$$

$$B = \frac{1}{N}\sum_{k=0}^{N-1} \mathrm{Re}\left(\hat{R}_x^{\alpha_k}\right)\mathrm{Im}\left(\hat{R}_x^{\alpha_k}\right) \quad (4)$$

and

$$C = \frac{1}{N}\sum_{k=0}^{N-1} \mathrm{Im}\left(\hat{R}_x^{\alpha_k}\right)^2 \quad (5)$$

where α_k is the normalized frequency, $\alpha_k = k$.

A test statistic T is expressed as

$$T = \mathbf{R}_\alpha \mathbf{\Sigma}_{2c}^{-1} (\mathbf{R}_\alpha)^T \quad (6)$$

where [5]

$$\mathbf{R}_\alpha = [\mathrm{Re}(\hat{R}_{\mathrm{x}}^{\alpha}), \mathrm{Im}(\hat{R}_{\mathrm{x}}^{\alpha})] \quad (7)$$

and α is the supposed cyclic frequency.

The parameters of the testing signal were chosen according to 802.11g WLAN. The number of subcarriers was 64, the length of the cyclic prefix was 16 samples, and the subcarrier modulation employed was 16-QAM. The sampling interval in the OFDM modulator was normalized to $T_{\mathrm{sOFDM}}= 1$ Cyclic frequency $\alpha = 1/(T_{\mathrm{sym}}+ T_{\mathrm{CP}})$. T_{sym} is the length of the OFDM symbol, and T_{CP} is the length of the cyclic prefix.

The length 2048 of the FFT register and the decimation ratio $D = 8$ are the same as in [8], but sampling interval T_{sDET} in the detector is not equal to that in the modulator. The basic value of the sampling interval in detector is $T_{\mathrm{sDET}} \cong 1.001$ for reasons which will be stated in section 3.4.

3. Basic Behavior of the Statistic

The value of the statistic T depends on both the signal parameters and the detector parameters.

If the received signal is pure noise, then the value of the statistic T is not dependent on the power of the noise. But the value of the statistic T is slightly dependent on the waveform of the noise.

The statistic T of the signal and noise is dependent on the SNR. An example of the dependence is shown in Fig. 1. For all values of SNR the same shapes of the signal and of the noise were used. T was equal to 1.5097 for pure noise.

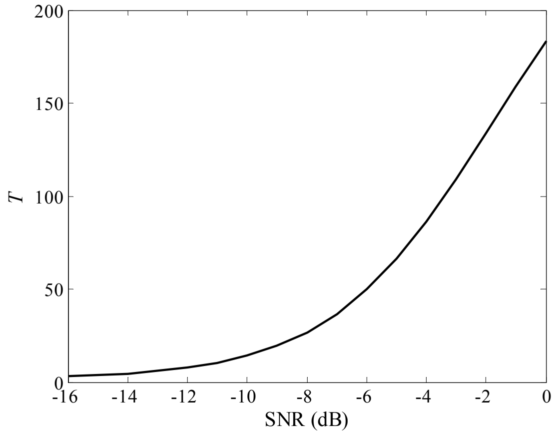

Fig. 1. T versus SNR.

The inversion matrix in (6) is given by the known formula

$$\mathbf{\Sigma}_{2c}^{-1} = \begin{bmatrix} A & B \\ B & C \end{bmatrix}^{-1} = \frac{1}{AC - B^2}\begin{bmatrix} C & -B \\ -B & A \end{bmatrix} = \begin{bmatrix} D & E \\ E & F \end{bmatrix} \quad (8)$$

where D, E and F are the real constants.

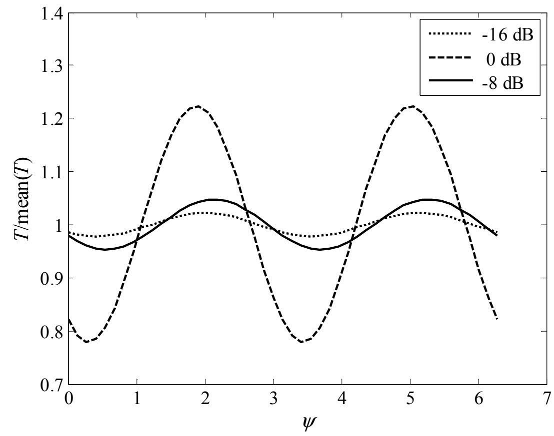

Fig. 2. T versus argument of $\hat{R}_x^{\alpha}$.

If the received signal is noise or signal and noise with negative SNR, then the relations $B^2 << \mathrm{AC}$ and $E \approx 0$ are valid and equation (8) can be rewritten as

$$\begin{bmatrix} A & B \\ B & C \end{bmatrix}^{-1} \approx \begin{bmatrix} \frac{1}{A} & 0 \\ 0 & \frac{1}{C} \end{bmatrix}. \quad (9)$$

In the case of pure noise, it can be observed that $A \approx C$ and therefore $D \approx F \approx \frac{1}{A}$. The statistic T for pure noise is then given by

$$T \approx \left[\mathrm{Re}(\hat{R}_x^\alpha)\ \ \mathrm{Im}(\hat{R}_x^\alpha)\right]\begin{bmatrix} D & 0 \\ 0 & D \end{bmatrix}\begin{bmatrix} \mathrm{Re}(\hat{R}_x^\alpha) \\ \mathrm{Im}(\hat{R}_x^\alpha) \end{bmatrix} = \\ = D\left[\mathrm{Re}(\hat{R}_x^\alpha)^2 + \mathrm{Im}(\hat{R}_x^\alpha)^2\right] = D\left|\hat{R}_x^\alpha\right|^2 = \frac{\left|\hat{R}_x^\alpha\right|^2}{A}. \tag{10}$$

Similarly, the statistic T for signal and noise is given by

$$T \approx \left[\mathrm{Re}(\hat{R}_x^\alpha)\ \ \mathrm{Im}(\hat{R}_x^\alpha)\right]\begin{bmatrix} \frac{1}{A} & 0 \\ 0 & \frac{1}{C} \end{bmatrix}\begin{bmatrix} \mathrm{Re}(\hat{R}_x^\alpha) \\ \mathrm{Im}(\hat{R}_x^\alpha) \end{bmatrix} = \\ = \left[\frac{1}{A}\mathrm{Re}(\hat{R}_x^\alpha)\ \ \frac{1}{C}\mathrm{Im}(\hat{R}_x^\alpha)\right]\begin{bmatrix} \mathrm{Re}(\hat{R}_x^\alpha) \\ \mathrm{Im}(\hat{R}_x^\alpha) \end{bmatrix} = \\ = \left[\frac{1}{A}\mathrm{Re}(\hat{R}_x^\alpha)^2 + \frac{1}{C}\mathrm{Im}(\hat{R}_x^\alpha)^2\right]. \tag{11}$$

This result shows that the statistic T could be slightly dependent on the phase ψ of complex number $\hat{R}_x^\alpha$, when negative SNR approaches zero. Fig. 2 shows an example of the relative value of T as a function of the phase ψ for three values of SNR. Fixed shapes of the signal and of the noise have been used. The oscillation amplitude increases when the power of the signal increases.

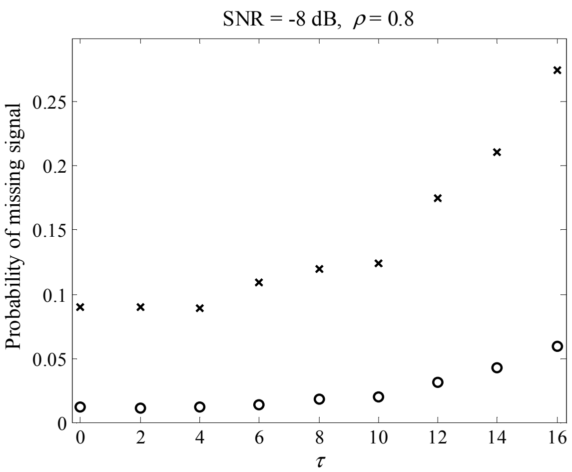

Fig. 3. Probability of missing detection versus τ. False alarm probability: x-mark 0.01, circle 0.1.

4. Partial Effects

4.1 Multipath Propagation

A consequence of multipath propagation is the violation of the cyclic prefix. This represents partial damage to periodicity and hence it also decreases the detection capability of the cyclostationary detector.

A simple two-way model was chosen to examine the above phenomenon. The received multipath signal $s_{\mathrm{MP}}(n)$ is the sum of the signal $s(n)$ and it's delayed and attenuated version $\rho s(n-\tau)$

$$s_{\mathrm{MP}}(n) = s(n) + \rho s(n-\tau)\,. \tag{12}$$

The effect of a gain ρ and a normalized delay τ on the probability of the missing detection was studied and estimated using 3.10^4 simulations. Two false alarm probabilities, 0.01 and 0.1, were used. SNR was set to -8 dB. Results of computer experiments are shown in Fig. 3 and Fig. 4.

Fig. 3 shows that the probability of the missing detection grows when the delay τ approaches the length of the cyclic prefix. It is a consequence of the cyclic prefix violation.

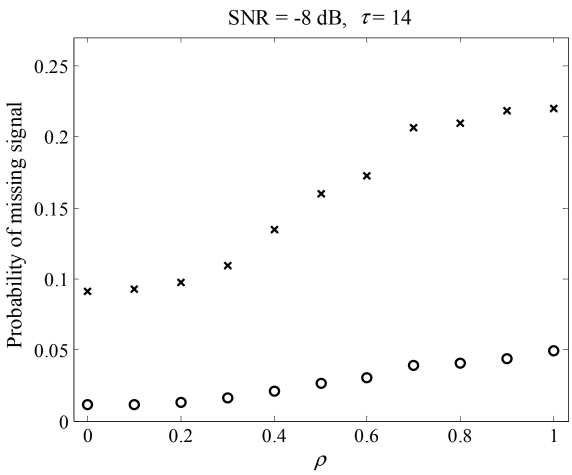

Fig. 4. Probability of missing signal versus ρ. False alarm probability: x-mark 0.01, circle 0.1.

Similarly, Fig. 4 shows the probability of the missing signal increasing as the gain ρ increases towards 1. This is a consequence of the cyclic prefix violation again.

Multipath propagation of the signal sometimes causes an increase of the probability of the missing signal. It will be shown using the exponentially decaying Rayleigh fading channel model in section 5.

4.2 Frequency Offset

The description of the frequency components of the function $y(t,\tau) = x(t)x^*(t-\tau)$, $t \in [T_{\mathrm{sym}},\ T_{\mathrm{sym}}+T_{\mathrm{CP}}]$, $\tau = T_{\mathrm{sym}}$ can serve to clarify the effect of the frequency offset. The expression $y(t,\tau)$ is present in the discrete time version as a part of the right side of (1).

For a noiseless signal, the frequency component for discrete angular frequency

$$k\omega_1 = k\frac{2\pi}{T_{sym}} \tag{13}$$

is given by

$$y_k(t,\tau) = c_k \exp[j(k\omega_1 + 2\pi\delta_f)t]. \\ .c_k^* \exp[-j(k\omega_1 + 2\pi\delta_f)(t - T_{\mathrm{sym}})] = \\ = \left|c_k\right|^2 \exp(j2\pi\delta_f T_{\mathrm{sym}}) \tag{14}$$

where δ_f is the frequency offset.

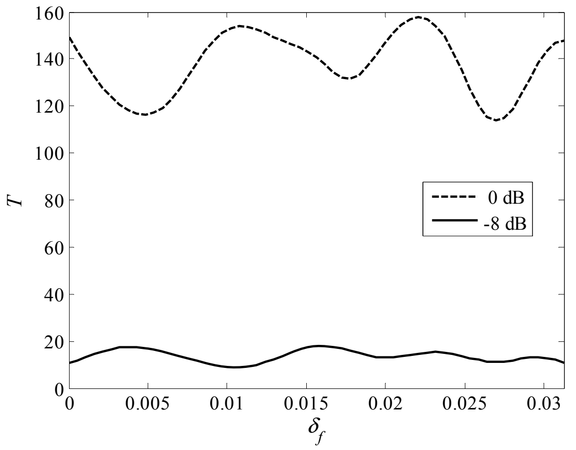

Fig. 5. The statistic T as a function of the frequency offset δ_f .

Generally, the result is a complex constant, but it is a real constant for zero frequency offset. The complexity of the function $y(t,\tau) = x(t)x^*(t-\tau)$ leads to the complexity of $\hat{R}_x^\alpha$. It yields the consequences mentioned in section 3, Fig. 2.

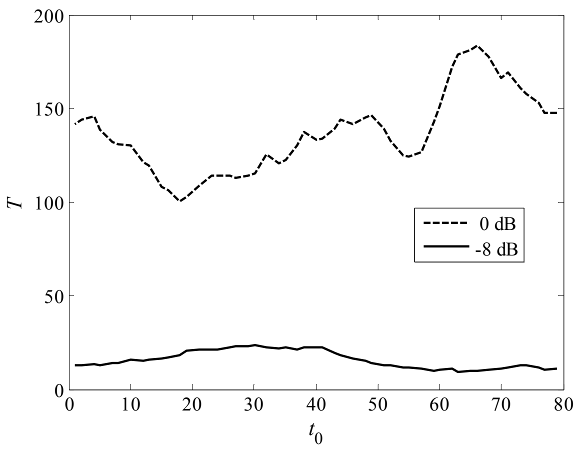

Fig. 6. The statistic T versus origin of sampling.

The statistic T of the signal and noise is dependent on the frequency offset. An example of the dependence is shown in Fig. 5. An irregularity of the curves is caused by the fact that their shape is affected by several frequency components $y_k(t,\tau)$ of the function $y(t,\tau)$. For both values of SNR the same shapes of the signal and of the noise were used. The sensitivity of the detector is not constant, but changes in the sensitivity are not too significant. This is important, because thanks to this feature the detector does not require carrier synchronization.

4.3 Timing Offset

The statistic T of the signal and noise is dependent on the origin t_0 of the sampling. An example of the dependence is shown in Fig. 6. For both values of SNR the same shapes of the signal and of the noise were used. The sensitivity of the detector is not constant, but the changes are not substantial. This is important, because the detector does not require time synchronization.

It can be shown that the development of the statistic T is dependent on the SNR, data, and sampling frequency in the detector.

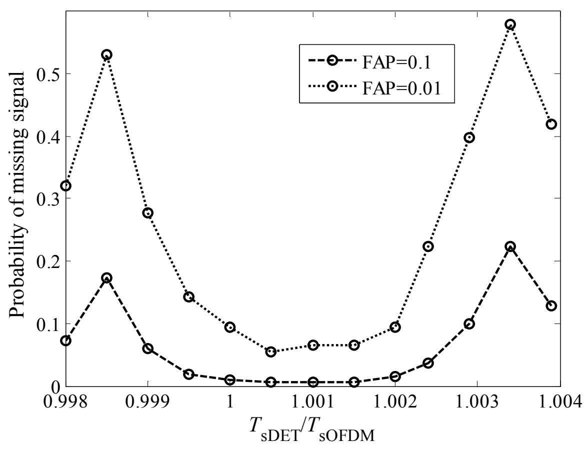

Fig. 7. Dependence of probability of missing signal on sampling interval.

4.4 Sampling Frequency

The determination of $\hat{R}_x^\alpha$ using DFT is analogous to the frequency analysis of harmonic signals via DFT [13], [14]. The estimate $\hat{R}_x^\alpha$ is given as the k_α-th element in the frequency domain,

$$k_\alpha = \mathrm{Round}\left[\frac{L_{\mathrm{FFT}}.T_{\mathrm{sDET}}}{(T_{\mathrm{sym}} + T_{\mathrm{CP}})} d\right] \tag{15}$$

where L_{FFT} is the number of elements in the FFT register, and d is the decimation ratio. A frequency error ε_α is given by the equation

$$\varepsilon_\alpha = \alpha - \frac{\mathrm{Round}(\alpha L_{\mathrm{FFT}} T_{\mathrm{sDET}} d)}{L_{\mathrm{FFT}} T_{\mathrm{sDET}} d}. \tag{16}$$

The nonzero error yields an undesirable decrease of the estimate $\hat{R}_x^\alpha$. It is desirable to achieve zero frequency error. In other words, the expression $\alpha L_{\mathrm{FFT}} T_{\mathrm{sDET}} d$ should be an integer or close to an integer. In our case L_{FFT}= 2048, d = 8 and the optimal value of the T_{sDET}= 205/204.8. These simplified considerations are confirmed by the results of computer simulations given in Fig. 7. The dotted line with circles denotes false alarm probability (FAP) 0.01, the dashed line with circles indicates false alarm probability 0.1. The signal-to-noise ratio was -8 dB. The number of experiments was 10 000, the data and noise were pseudo-random. It is shown that an improper value of the detector sampling interval T_{sDET} could lead to impaired detection ability.

Changes of the sampling frequency are connected with quantization errors of the lag parameter τ. This could bring some secondary effects.

5. Detection Performance

Overall detection performance has been tested using computer simulations. The data and noise were pseudorandom, the frequency offset δ_f was uniformly distributed in the normalized interval from -0.012 to 0.012, and the origin of sampling was uniformly distributed in the interval from 0 to 80 samples. The sampling period was set to the optimal value.

The result for one-way propagation (OW) is shown in Fig. 8. The threshold for the false alarm probability (FAP) 0.1 and the probability of missed detection 0.1 is about -9.8 dB. The number of experiments was 20 000.

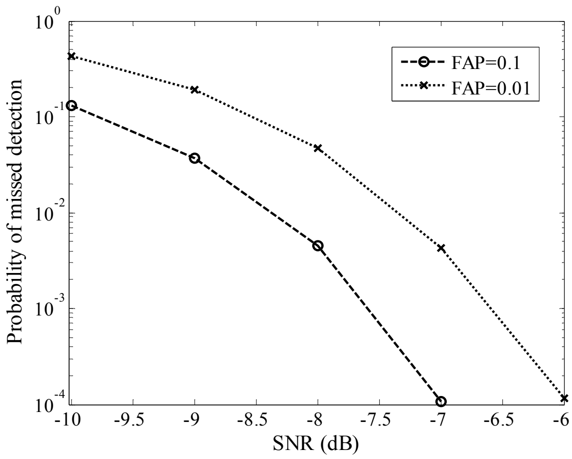

Fig. 8. One-way propagation case.

The behavior of the detector for the two-way (TW) propagation case is shown in Fig. 9. The parameters of the two-way channel were $\rho = 0.8$ and $\tau = 14$. Here, the threshold for the false alarm probability 0.1 and the probability of missed detection 0.1 is found to be about -9 dB. Parameter T_{rms} of this two-way channel is equal to 6.8293.

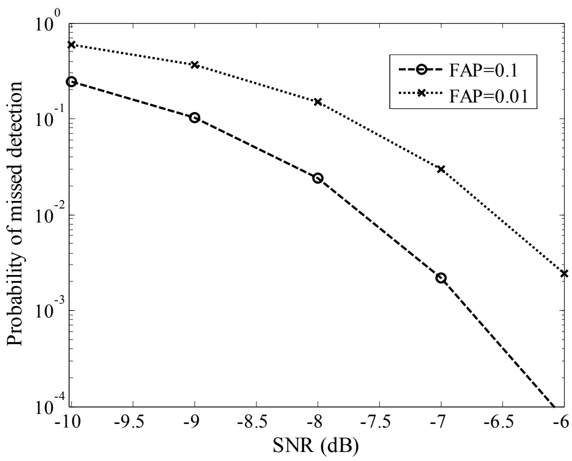

Fig. 9. Two-way propagation case.

The exponentially decaying Rayleigh fading channel model was used for other experiments. This model is suitable for evaluation of IEEE 802.11g [16]. The T_{RMS} value in a typical office environment is 50 ns [16]. It is equivalent to value 1 of normalized time T_{rms}. Such a channel cannot significantly impair the cyclic prefix. The performance of the channel is almost the same as the performance of the one-way channel. However, growing T_{rms} results in a deterioration of the detector sensitivity.

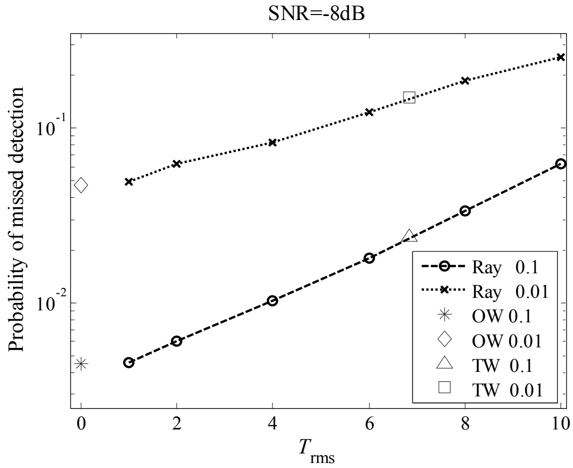

Fig. 10. Comparison of the channel models.

When the parameter T_{rms} of the exponentially decaying Rayleigh channel was set to 6.8293, the behavior of the detector was almost the same as the behavior of the detector in the case of the above two-way channel.

A comparison of the results for all three types of the propagation is given in Fig. 10. The abbreviation Ray 0.1 denotes the exponentially decaying Rayleigh fading channel model, and FAP=0.1. OW is the one-way propagation model, and TW is the two-way propagation model, $\rho = 0.8$ and $\tau = 14$. The figure shows that the behavior of the exponentially decaying Rayleigh fading channel with $T_{\mathrm{rms}}= 1$ is not different from the one-way model and that the behavior of the exponentially decaying Rayleigh fading channel with $T_{\mathrm{rms}}= 6.8293$ is not significantly different from the two-way model with the same T_{rms}.

6. Conclusions

In this paper, it is shown that the detector based on the cyclic autocorrelation function is able to detect OFDM signals in cases of multipath propagation, inaccurate frequency synchronization, and inaccurate time synchronization. The frequency offset and the timing offset imply complex changes of the statistic T, but do not jeopardize the ability to detect OFDM signals. Multipath propagation causes a violation of the cyclic prefix and, consequently, a decrease in the detector sensitivity for larger values of T_{rms}. This is shown in Fig. 10.

Important is the choice of the detector sampling interval. The expression $\alpha L_{\mathrm{FFT}} T_{\mathrm{sDET}} d$ should be an integer or close to an integer.

Acknowledgements

This research has been supported by the research grant of the Czech Science Foundation No. 102/09/0776, by the research project Advanced Electronic Communication Systems and Technologies MSM 0021630513 and by the research project FEKT-S-11-12 MOBYS. The research leading to these results has received funding from the European Community's Seventh Framework Programme (FP7/2007-2013) under grant agreement no. 230126.

References

[1] QUAN, Z., et al. Collaborative wideband sensing for cognitive radios. *IEEE Signal Processing Magazine*, 2008, vol. 25, no. 6, p. 60-73.

[2] ERGEN, M. *Mobile Broadband. Including WiMAX and LTE*. New York: Springer, 2009.

[3] NOGUET, D., BIARD, L., LAUGEIOS, M. Cyclostationarity detectors for cognitive radio: Architectural tradeoffs [online]. Hindawi Publishing Corporation, *EURASIP Journal on Wireless Communications and Networking*, 2010 - [cit. 23-01-2011]. Available on www: http://www.hindawi.com/journals/wcn/2010/526429.html.

[4] ZHANG, H, LE RUYET, D., TERRE, M. Signal detection for OFDM/OQAM system using cyclostationary signatures. In *IEEE 19th International Symposium on Personal, Indoor and Mobile Radio Communications*. Cannes (France), 2008, p. 1-5.

[5] DANWATÉ, A. V., GIANNAKIS, G. B. Statistical tests for presence of cyclostationarity. *IEEE Transactions on Signal Processing*. 1994, vol. 42, no. 9, p. 2355-2369.

[6] SUN, Z., WANG, Q., CHE, CH. Study of cognitive radio spectrum detection in OFDM system. In *Asia-Pacific Conference on Wearable Computing Systems*. Shenzhen (China), 2010, p. 235-238.

[7] AN, J., YANG, M., BU, X. Spectrum sensing for OFDM systems based on cyclostationary statistical test. In *6th International Conference on Wireless Communications Networking and Mobile Computing (WiCOM).* Chengdu (China), 2010, p. 1 – 4.

[8] TURUNEN, V. et al. Implementation of cyclostationary feature detector for cognitive radios. In *Proceedings of the 4th International Conference on CROWNCOM '09*. Hannover (Germany), 2009, p. 1-4.

[9] KIM, M., KIMTHO, P., TAKADA, J.-i. Performance enhancement of cyclostationarity detector by utilizing multiple cyclic frequencies of OFDM signals. In *IEEE Symposium on New Frontiers in Dynamic Spectrum*, Singapore, 2010, p. 1-8.

[10] LUNDEN, J. *et al.* Spectrum sensing in cognitive radios based on multiple cyclic frequencies. In *Proceedings of the 2nd Int. Conf. on Cognitive Radio Oriented Wireless Networks and Communications*. Orlando (USA), 2007, p. 37-43.

[11] HARADA, H., FUJII, H., FURUNO, T., MIURA, S., OHYA, T. Iterative cyclostationarity-based feature detection of multiple primary signals for spectrum sharing scenarios. In *IEEE Symposium on New Frontiers in Dynamic Spectrum*. Singapore, 2010, p. 1-8.

[12] HAN, N., SOHN, S. H., KIM, J. M. Cyclic autocorrelation based blind OFDM detection and identification for cognitive radio. *Journal of Communication and Computer* [online]. May 2009, vol. 6, no. 5, [cit. March 17, 2011]. Available in www: <www.informatics.org.cn/doc/ucit200905/ucit20090507.pdf>. ISSN 1548-7709.

[13] MITRA, S. K. *Digital Signal Processing. A Computer-Based Approach*. The McGraw-Hill Companies, Inc., 1998.

[14] ŠEBESTA, V. Estimating a spectral correlation function under the conditions of imperfect relation between signal frequencies and a sampling frequency. *Radioengineering*, 2010, vol. 19, no. 1, p. 1-5.

[15] ŠEBESTA, V. Estimating a spectral correlation function and its errors owing to inexact frequencies. In *Proceedings of 20th International Conference RADIOELEKTRONIKA 2010*. Brno (Czech Rep.), 2010. p. 129-132.

[16] LIEW, C. H., et al. *Fixed and Mobile Channel Models Identifications* [online]. SUIT_64, 2006. [cit. May 5, 2011]. Available on: http://suit.av.it.pt/dissemination/deliverables/SUIT_64_D2.1_channelmodel.pdf

About Authors ...

Vladimír ŠEBESTA was born in Předín, Czech Republic. He received the M.Sc. degree in electrical engineering from the Czech Technical University, Prague, in 1961 and the Ph.D. degree from Brno University of Technology in 1974. His research interests include the general areas of statistical signal processing and digital communications. Currently, he is a Professor Emeritus, Brno University of Technology, Czech Republic.

Roman MARŠÁLEK was born in Brno, Czech Republic. He received the M.Sc. degree from Brno University of Technology, Czech Republic, in 1999 and the Ph.D. degree from Université de Marne la Vallée, France, in 2003. Currently he is an associate professor at Brno University of Technology, Department of Radio Electronics. His research interests include digital communication theory and signal processing applied to radio communications.

Zbyněk FEDRA was born in Dačice, Czech Republic in 1981. He graduated from Brno University of Technology in 2004 and received the Ph.D. degree in 2008. He is currently an assistant professor at the Department of Radio Electronics, Brno University of Technology. His research interests are multicarrier communication and signal processing.