

# Calculation and Comparison of Turbulence Attenuation by Different Methods

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**Abstract.** This paper is focused on the effect of turbulence in atmospheric transmission media. A short introduction of the origin of turbulent atmosphere is given. Two classical methods for the calculation of turbulence attenuation are mentioned and briefly described in the paper – the Rytov approximation and Andrews’s method, and a new technique designed by the authors of this paper – the method of available power – is presented in short as well.

We have run simulations in Matlab to compare the results of turbulence attenuation given by the classical methods and by the new technique - the method of available power. These calculations were performed for communication wavelengths of 850 nm and 1550 nm. The distance between the optical transmitter and receiver of horizontal links was set to values ranging from 0 m to 2500 m. We have taken into account the homogenous turbulence with low ( $10^{-16} \text{ m}^{-2/3}$ ), medium ( $10^{-15} \text{ m}^{-2/3}$ ) and high ( $10^{-14} \text{ m}^{-2/3}$ ) structure parameter of refractive index  $C_n^2$ .

## Keywords

Free space optical links, turbulence attenuation, method of available power, the Rytov approximation.

## 1. Introduction

Atmospheric transmission medium is the environment for free space optical links operation. Typical effects occur during optical wireless transmission processes. We need a stable atmosphere to avoid optical signal distortions. Diffraction, scintillation and atmospheric attenuation are the most significant effects influencing optical signals in free space links. Only scintillation due to atmospheric turbulence will be discussed in the paper and the methods for determining turbulence attenuation due to scintillation will be compared.

## 2. Atmospheric Turbulence Theory

Atmospheric turbulence, generated by the temperature difference between the Earth’s surface and the atmosphere,

produces effects on optical waves that have been of great interest to scientists for many years.

Atmospheric turbulence disrupts the coherence of both laser radiation and optical wave during the whole day – most significantly about midday or midnight due to the highest temperature difference between the Earth’s surface and the atmosphere. Wave front distortion in the optical wave induced by atmospheric turbulence results in a broadening of the beam, random variations of the position of the beam centroid, called beam wander, and a redistribution of the beam energy within a cross section of the beam, leading to irradiance fluctuation [1].

Atmospheric temperature and wind variations, including boundary conditions, create local unstable air masses, causing them eventually to break up into turbulent eddies or cells of many different scale sizes with constant  $C_n^2$ . These inhomogeneities range in size from macroscale to microscale, and hence, in effect, form a continuum of decreasing “eddy” size [1].

The reason for optical signal scintillation is simply illustrated in Fig. 1. The scintillation is caused by the turbulent cells, which are areas with variant refractive indexes.

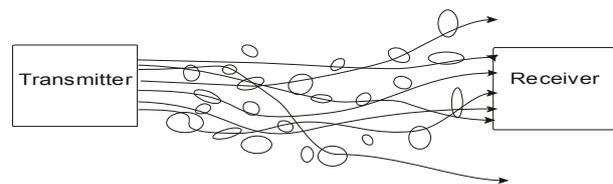


Fig. 1. Atmospheric transmission media with turbulent cells.

There exist a few physical descriptions of turbulence attenuation. The Rytov approximation and Andrews’s method are currently used in technical practice, but the method of available power is a new way of determining turbulence attenuation, designed by the authors of this paper.

The purpose of this paper is not a detailed description of the particular methods. We compare numerical results of currently used methods for the determination of turbulence attenuation (the Rytov approximation, Andrews’s method) with the new method of available power.

### 2.1 The Rytov Approximation

The Rytov approximation starts from the premise that an air mass behaves as a fluid. On the basis of Rytov's analysis [3], the relationship between refractive index structure parameter  $C_n^2$ , which characterizes the volume of atmospheric turbulence (we assume constant  $C_n^2$  and low turbulence), and relative variance of optical intensity  $\sigma_{I,rel}^2$  was set by Rytov as

$$\sigma_{I,rel}^2 = K \cdot C_n^2 \cdot k^{7/6} \cdot L^{11/6} \tag{1}$$

where  $K$  is the constant 1.23 for the plane wave, and 0.5 for the spherical wave,  $C_n^2$  is the refractive index structure parameter,  $k$  presents the wave number, and  $L$  signifies the distance between the transmitter and receiver of the optical wireless link. Equation (1) holds true for

$$\sigma_{I,rel}^2 < 1. \tag{2}$$

The intensity and the speed of the fluctuations (scintillation frequency) increase with wave frequency. For a low turbulence, the scintillation variance  $\sigma_\chi^2$  [dB<sup>2</sup>] can be expressed by the relation [2]

$$\sigma_\chi^2 = 23.17 \cdot C_n^2 \cdot k^{7/6} \cdot L^{11/6}. \tag{3}$$

Scintillation peak-to-peak amplitude is equal to  $4\sigma_\chi$  and turbulence attenuation related to scintillation is equal to  $2\sigma_\chi$  [dB] and thus the relation for turbulence attenuation  $\alpha_{RYTOV}$  can be written as [2]

$$\alpha_{RYTOV} = 2 \cdot \sqrt{23.17 \cdot k^{7/6} \cdot C_n^2 \cdot L^{11/6}}. \tag{4}$$

This method is not concerned with the effect of aperture averaging.

### 2.2 Andrews's Method

This method is derived on the basis of a detailed mathematical analysis of the turbulence in atmospheric transmission media presented by Larry C. Andrews [3]. The resultant expression for the theoretical mean variance of optical intensity is given as

$$\bar{\sigma}_I^2(D_{RXA}) \cong \exp \left[ \frac{0.49 \cdot \beta_0^2}{(1 + 0.18 \cdot d^2 + 0.56 \beta_0^{12/5})^{7/5}} + \frac{0.51 \cdot \beta_0^2 (1 + 0.69 \cdot \beta_0^{12/5})^{-5/6}}{1 + 0.90 \cdot d^2 + 0.62 \cdot d^2 \cdot \beta_0^{12/5}} \right] - 1 \tag{5}$$

In relation (5), two parameters are used. The first one is parameter  $d$ , which contains information about the wavelength  $\lambda$  of optical source, distance between transmitter and receiver  $L$ , and receiving optical lens diameter  $D_{RXA}$

$$d = \sqrt{\frac{2 \cdot \pi}{4 \cdot \lambda \cdot L}} \cdot D_{RXA} \tag{6}$$

while the second parameter is  $\beta_0^2$  given by the relation

$$\beta_0^2 = 0.5 \cdot C_n^2 \cdot k^{7/6} \cdot L^{11/6}, \tag{7}$$

which is formally the same as the Rytov relative variance of optical intensity for spherical waves.

This method takes into account the size of receiving optical lens  $D_{RXA}$ . Aperture averaging is the most valuable asset of Andrews's method. Due to this fact we expect the lowest resultant turbulence attenuation when Andrews's method is applied.

In this case the turbulence attenuation in dB is given by the relation [4]

$$\alpha_{ANDREWS} = 10 \cdot \log \left| 1 - \sqrt{\bar{\sigma}_I^2(D_{RXA})} \right|. \tag{8}$$

### 2.3 Method of Available Power

The two methods mentioned above work with the idea of homogenous atmospheric transmission media and do not take into account the profile of laser beam intensity while the theory of available power designed by the authors of this paper does take into account optical intensity distribution in the laser beam.

The new method for determining turbulence attenuation is based on an analysis of the optical intensity profile and works with the redistribution of optical intensity within the laser beam. The name of the method is derived from the fact that we integrate optical intensity, and available optical power is the result.

In non-turbulent atmospheric transmission media we can observe the standard intensity profile of the laser beam (Fig. 2) but in the case of turbulent atmosphere the optical intensity fluctuations are different in dependence on the position in the beam (Fig. 3).

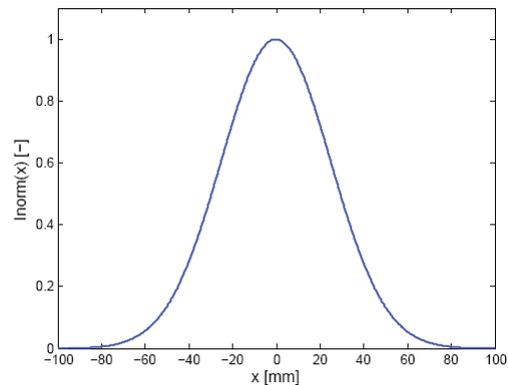


Fig. 2. Gaussian beam optical intensity distribution in non turbulent atmosphere – illustration.

The determination of turbulence attenuation by the method of available power is based on the assessment of the volume of available power  $V_{AP}$  in [5]. This quantity is evaluated using the function of available power  $f_{AP}(x,y)$ , which we obtain by finding the local minimum in each of

the scanned pixels of the laser beam intensity profile during a sufficiently long time.

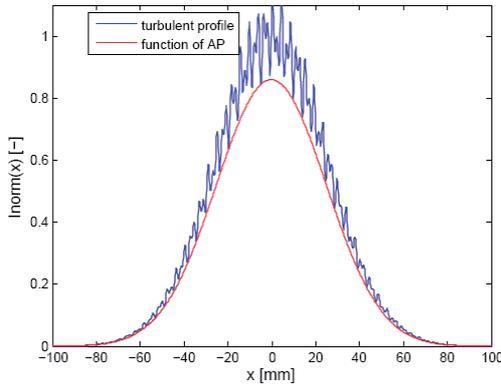


Fig. 3. Gaussian beam optical intensity distribution in turbulent atmosphere – illustration.

This method is valid only when the function of available power  $f_{AP}(x,y)$  has a value higher than zero. Otherwise we are beyond the validity limit.

To define the volume of available power the following equation is essential

$$V_{AP} = \int_{D_{RXA}} f_{AP}(x,y) dx dy \quad (9)$$

where the limits of integration are defined by the size of the receiving optical lens  $D_{RXA}$ .

We introduced the relative volume of available power  $V_{AP,r}$ , which is given as a ratio of the volume of available power  $V_{AP}$  to the volume of available power in non-turbulent atmosphere  $V_{AP,0}$  [5]

$$V_{AP,r} = \frac{V_{AP}}{V_{AP,0}} \quad (10)$$

Finally the turbulence attenuation given by the method of available power can be evaluated using the following expression [4]

$$\alpha_{AP} = 10 \cdot \log V_{AP,r} \quad (11)$$

The resultant value is given in dB.

### 3. Assessment of Turbulence Attenuations

We calculated turbulence attenuation using the Rytov approximation, Andrews’s method and the method of available power. Atmospheric windows with wavelengths of 850 nm and 1550 nm are used in the computations. Refractive index structure parameters  $C_n^2$  were set to  $10^{-16} \text{ m}^{-2/3}$ ,  $10^{-15} \text{ m}^{-2/3}$  and  $10^{-14} \text{ m}^{-2/3}$ . All simulations were run in Matlab and the resultant values of the turbulence attenuation were within the validity limits of the respective methods.

The first calculation (Fig. 4) proceeded with the following parameters:  $\lambda = 850 \text{ nm}$ ,  $C_n^2 = 10^{-16} \text{ m}^{-2/3}$ ,  $D_{RXA} = 10 \text{ cm}$ , and  $L = (0-2500) \text{ m}$ . According to the results obtained the method of available power and the Rytov approximation give similar resultant turbulence attenuations. It is evident that the results obtained by Andrews’s method are significantly different. We have to take into account that this method indicates the lowest possible turbulence attenuation for the defined conditions.

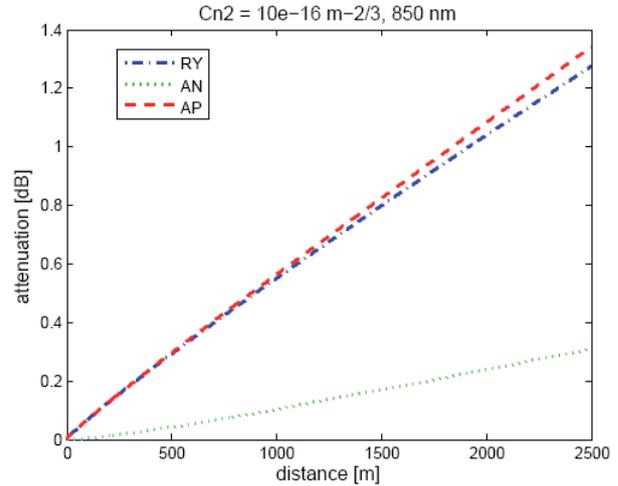


Fig. 4. Comparison of attenuation given by the Rytov approximation (RY), Andrews’s method (AN) and the method of available power (AP) for  $C_n^2 = 10^{-16} \text{ m}^{-2/3}$  and wavelength of 850 nm.

In comparison with the first calculation, the second one (Fig. 5) has a different value of the parameter  $C_n^2$ , in this case set to the value  $10^{-15} \text{ m}^{-2/3}$ , which means a higher volume of atmospheric turbulence. The trend of turbulence attenuation is analogous to the previous case.

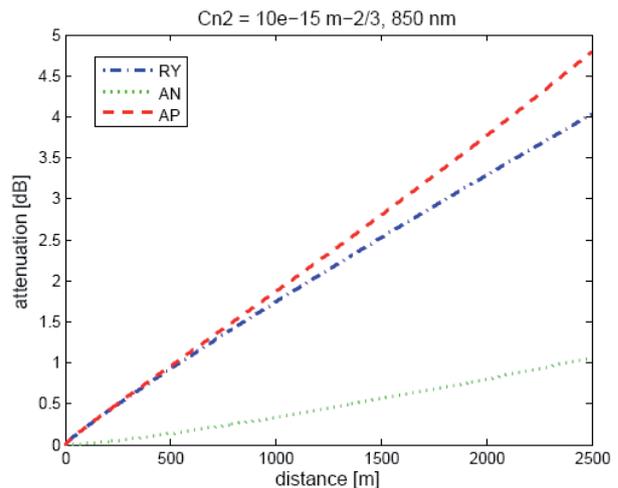
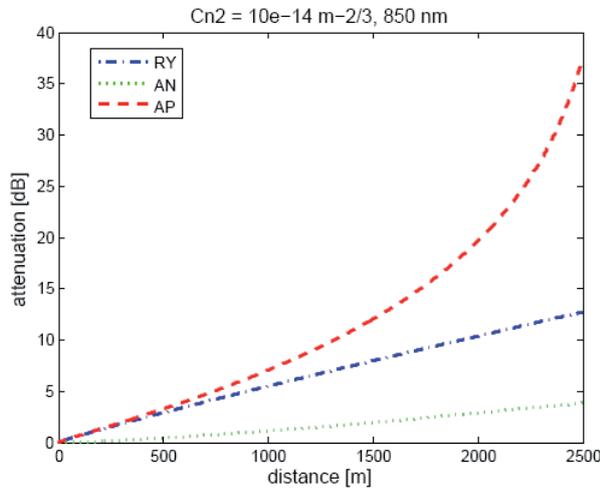


Fig. 5. Comparison of attenuation given by the Rytov approximation (RY), Andrews’s method (AN) and the method of available power (AP) for  $C_n^2 = 10^{-15} \text{ m}^{-2/3}$  and a wavelength of 850 nm.

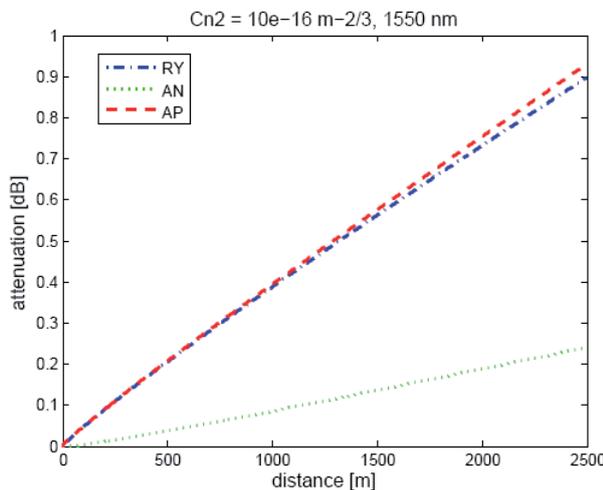
When we calculate with  $C_n^2 = 10^{-14} \text{ m}^{-2/3}$  (higher atmospheric turbulence), we obtain characteristics as

shown in Fig. 6. We observe different curves of turbulence attenuation given by the Rytov approximation and the method of available power, in particular for longer distances. This is because the method of available power determined the worst case of the turbulence attenuation for defined conditions.



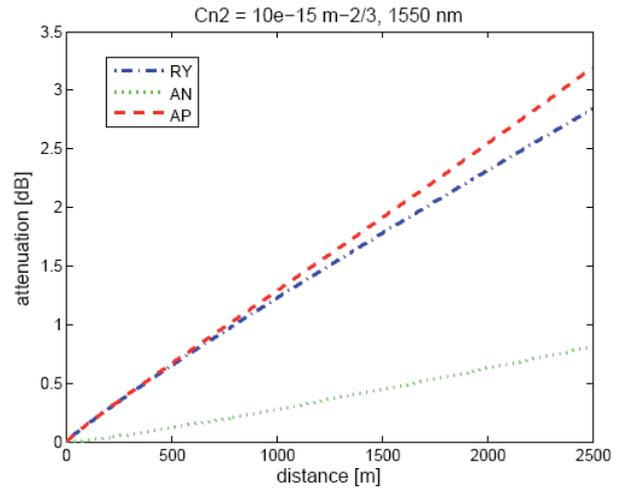
**Fig. 6.** Comparison of attenuation given by the Rytov approximation (RY), Andrews's method (AN) and the method of available power (AP) for  $C_n^2 = 10^{-14} \text{ m}^{-2/3}$  and wavelength of 850 nm.

We performed the same calculations also for the 1550 nm wavelength. There is almost no difference in the characteristics determined by the Rytov approximation and by the method of available power (Fig. 7) when refractive index structure parameter  $C_n^2 = 10^{-16} \text{ m}^{-2/3}$  is considered.



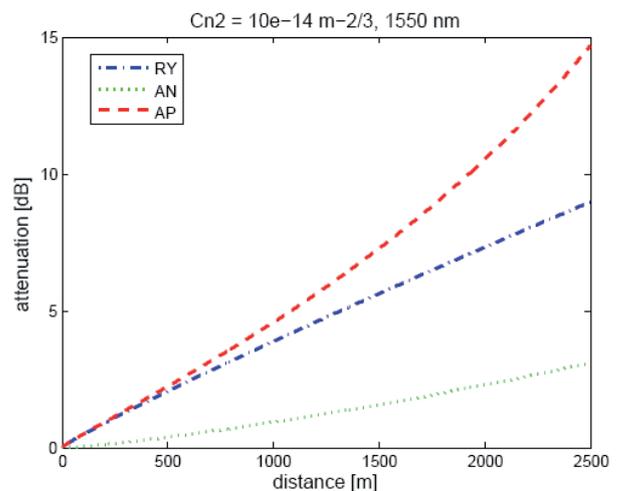
**Fig. 7.** Comparison of attenuation given by the Rytov approximation (RY), Andrews's method (AN) and the method of available power (AP) for  $C_n^2 = 10^{-16} \text{ m}^{-2/3}$  and wavelength of 1550 nm.

An analogous situation (Fig. 8) to that in the case of (Fig. 5) occurs when  $C_n^2 = 10^{-15} \text{ m}^{-2/3}$ . The difference between turbulence attenuation calculated by the Rytov approximation and that calculated by the method of available power is negligible in this case.



**Fig. 8.** Comparison of attenuation given by the Rytov approximation (RY), Andrews's method (AN) and the method of available power (AP) for  $C_n^2 = 10^{-15} \text{ m}^{-2/3}$  and wavelength of 1550 nm.

Comparing (Fig. 9) and (Fig. 6) we observe a minor difference between the turbulence attenuation determined by the Rytov approximation and that calculated by the method of available power when the 1550 nm wavelength is considered instead of the 850 nm wavelength.



**Fig. 9.** Comparison of attenuation given by the Rytov approximation (RY), Andrews's method (AN) and the method of available power (AP) for  $C_n^2 = 10^{-14} \text{ m}^{-2/3}$  and a wavelength of 1550 nm.

In all the cases presented the highest turbulence attenuation occurs when the method of available power is used while Andrews's method gives the smallest values of this attenuation in all cases. All the simulations performed were within the validity limits of the respective methods.

#### 4. Comparison of Turbulence Attenuations

It is evident that the influence of optical source wavelength is of utmost importance. This fact is well

known and it results from the equations introduced in particular presentations of the method applied for the calculation of turbulence attenuation.

For each method we compare the differences  $\Delta\alpha$  between turbulence attenuations in the case of wavelengths 850 nm and 1550 nm, when the refractive index structure parameter has the values  $10^{-16} \text{ m}^{-2/3}$ ,  $10^{-15} \text{ m}^{-2/3}$ ,  $10^{-14} \text{ m}^{-2/3}$  and the distance between the optical transmitter and receiver is from 0 m to 2500 m. This difference can be evaluated mathematically by the following equation

$$\Delta\alpha = \alpha(850 \text{ nm}) - \alpha(1550 \text{ nm}) \quad (12)$$

where  $\alpha(850 \text{ nm})$  signifies the calculated turbulence attenuation when the wavelength 850 nm is considered, and  $\alpha(1550 \text{ nm})$  represents the same parameter for calculations with the wavelength 1550 nm.

The above comparisons were of course made for calculations made by the Rytov approximation (Fig. 10), Andrews’s method (Fig. 11) and the method of available power (Fig. 12). Particular characteristics for different refractive index structure parameters are distinguished by different colors as well as by a different line style.

Calculating according to the Rytov approximation (Fig. 10) we obtain for  $C_n^2 = 10^{-16} \text{ m}^{-2/3}$  a difference between the characteristics for 850 nm and 1550 nm which is under 0.5 dB, for  $C_n^2 = 10^{-15} \text{ m}^{-2/3}$  the difference is a little higher than 1 dB. In atmospheric transmission media with a high volume of atmospheric turbulence with  $C_n^2 = 10^{-14} \text{ m}^{-2/3}$  the differences in turbulence attenuation reach as much as 4 dB for an optical path length of 2500 m.

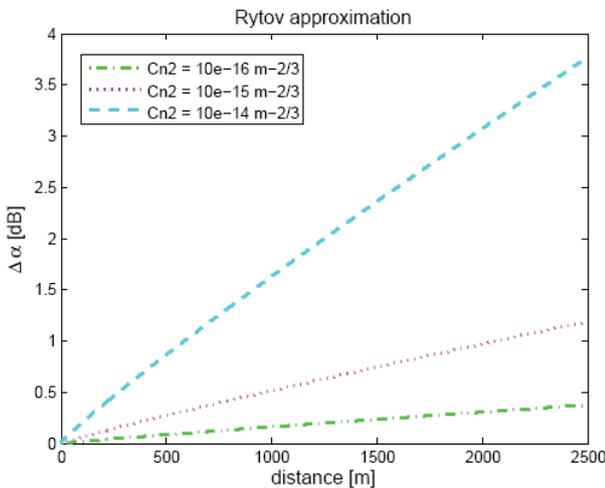


Fig. 10. Difference between attenuation of laser beam with wavelength of 1550 nm and 850 nm in turbulent atmosphere calculated by the Rytov approximation.

Considering the resultant difference between the turbulence attenuations  $\Delta\alpha$  given by Andrews’s method (Fig. 11) we can say that this difference is unimportant due to resultant values being lower than 1 dB even in the case of high turbulences and the distance between the optical transmitter and receiver increasing up to 2500 m.

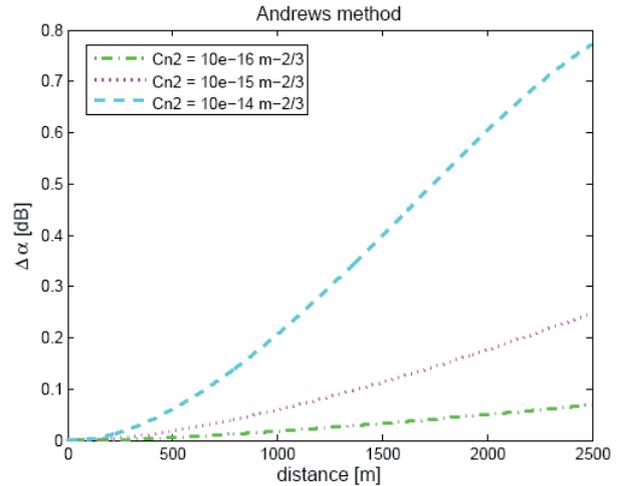


Fig. 11. Difference between attenuation of laser beam with wavelength of 1550 nm and 850 nm in turbulent atmosphere calculated by Andrews’s method.

Analyzing the results obtained by the method of available power (Fig. 12) it is evident that the difference between turbulence attenuation for low ( $C_n^2 = 10^{-16} \text{ m}^{-2/3}$ ) and medium ( $C_n^2 = 10^{-15} \text{ m}^{-2/3}$ ) turbulence is marginal in comparison with  $\Delta\alpha$  when higher atmospheric turbulences ( $C_n^2 = 10^{-14} \text{ m}^{-2/3}$ ) are taken into account. When the distance between the optical transmitter and receiver is 2500 m, the difference  $\Delta\alpha$  for a high volume of atmospheric turbulence reaches as much as 23 dB according to the calculations. Considering the method of available power the wavelength 1550 nm is of greater advantage than the wavelength 850 nm.

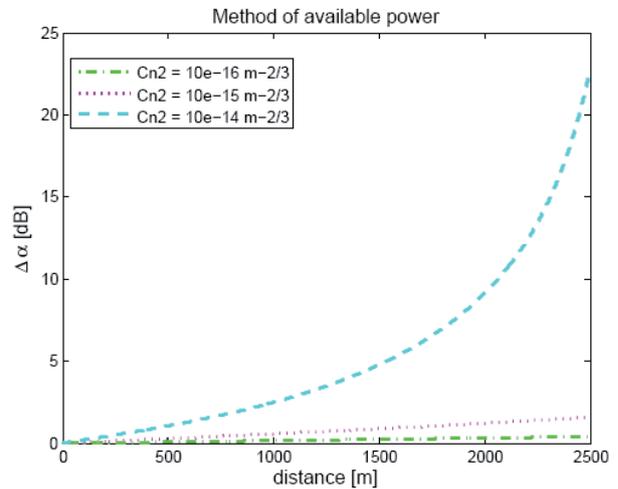


Fig. 12. Difference between attenuation of laser beam with wavelength of 1550 nm and 850 nm in turbulent atmosphere calculated by the method of available power.

## 5. Conclusion

On the basis of the calculations made, it is obvious that each of the above-mentioned methods give different results as we have expected. The reason is that each of the

methods for the calculation of turbulence attenuation is differently defined. In technical reports and specialized publications turbulence attenuation is frequently given by the structure parameter of the refractive index of atmospheric transmission medium but the method of calculation is not specified. In every case it is necessary to mention the method applied for the calculation of turbulence attenuation.

It is well known that the optical beam in the turbulent atmospheric transmission medium using the 1550 nm wavelength is less attenuated than when using the 850 nm wavelength for all the methods applied. In the case of calculating turbulence attenuation by Andrews's method we find that the difference between the attenuations of these wavelengths is minimal. When the method of available power is used, the calculated difference is very pronounced, especially for a higher volume of atmospheric turbulence. In fact, these results are understandable, because the result of Andrews's method is the lowest possible turbulence attenuation. On the other hand, the method of available power gives the worst case of turbulence attenuation as the result. Considering high atmospheric turbulences and an optical wireless link with a range of 2.5 km, we have to take into account the fact that the difference between the minimal and the maximal turbulence attenuation can be as much as 30 dB. This can be a problem in free space optical links operating in areas with a higher or unstable volume of atmospheric turbulences.

## Acknowledgements

The research leading to these results has received funding from the European Community's Seventh Framework Programme (FP7/2007-2013) under grant agreement no. 230126. The research has been supported also by the Research programmes of the Ministry of Education of the Czech Republic MSM21630513, by the Grant Agency of the Czech Republic under contracts No. 102/08/H027, No. 102/09/0550, and No. 102/08/0851, and by a grant under contract 2C06012.

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