Efficient Procedure for the Calculation of Electric and Magnetic Energies in Vacuum

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Abstract. *New expressions are derived to calculate the reactive energy stored in the electromagnetic field surrounding an electromagnetic device. The resulting expressions are 1. very simple to interpret, 2. general, 3. rigorous in terms of the currents flowing on the device, and 4. fast since they involve integrals solely over the device generating the field. The new technique is very feasible to be used in cases where the electric and magnetic reactive energies are important in practice, for example in the case of resonators, or in the case of radiating structures. Used there, they allow to study in a rigorous way the effect of the shape of the device on the amount of reactive energy, and thus for example on the Q of the device. The implementation of the new expressions in numerical CAD tools is extremely simple and straight-forward. In this paper, the expressions are derived for sources in the homogeneous medium vacuum, but this is not a fundamental restriction.*

Keywords

Electric and magnetic energy, Q factor, small antennas.

1. Introduction

The reactive energy stored in the electromagnetic field surrounding the device generating the field is an important parameter. From this parameter, important characteristics can be derived, for example the Q factor of the system. Many authors have considered the problem of determining the Q factor of an antenna. For the general case, this goes back to the paper of Collin [1], where a spherical mode decomposition is used to calculate the reactive energies. For the special case of an electrically small antenna, the paper of Chu [2], using ladder networks, is the basic paper. Many other authors have followed the same paths. Fante [3] extended the results of Collin. McLean [4] re-examined the case of small antennas. Sten studied the case of a small antenna near a ground plane [5]. The spherical mode approach actually calculates the reactive energies, both electric and magnetic, stored in the space outside a spherewith radius *a*. There are two important disadvantages linked to this approach: 1. the reactive energy within this sphere is neglected, and, related to this, 2. the exact shape of the volume of the radiating source is not taken into account. The result is that the reactive energies calculated are only approximations and that the effect of the topology of the source is hard to investigate.

Relatively recently, Geyi [6] published a technique to calculate the reactive energies taking into account the exact topology of the, in this case, small radiator considered. He used a combination of the Poynting theorem in frequency and time domain to separate the electric and magnetic reactive energy. Shlivinski performed a study of the reactive energy completely in the time domain, aiming at applications involving pulsed fields [7]. A brute force technique is used in [8], where the authors calculate the reactive energy using the FDTD method. A very complete paper is [10]. This paper gives a state-of-the-art overview of techniques and formulas to calculate impedances, bandwidths, and Q factors of antennas. However, the method suggested to calculate the reactive energies is based on the classical approach

The main goal of this research is to propose new efficient expressions for the reactive energies surrounding an arbitrary source (or device). In this paper, the arbitrary source is embedded in vacuum. However, this is not a fundamental limitation. The extension to sources embedded in complex environments, involving dielectric and magnetic media, is subject of further research.

It is assumed that all the currents are solved for the structure. This can be done using a wide range of available techniques, such as MoM (Method of Moments). This step is not explicitly considered in this paper. Then, the classical expressions for electric and magnetic reactive energy are combined with the mixed-potential integral equation expressions for the fields in terms of the currents in order to eliminate these fields. This is the “expansion step”. The “condensation step” involves the use of a series of special analytic integrals allowing to simplify the problem enormously, leading to the final expressions. The physical interpretation is that the procedure followed allows to perform the integration over observation space *analytically*. This last point is the key novelty.

2. Energies Stored

In vacuum, the energy stored in the electric field is calculated based on the classical expression

$$W^e = \int_V \frac{1}{4}\varepsilon_0(\mathbf{E}\cdot\mathbf{E}^*)dV \tag{1}$$

where V is the entire space. The procedure starts by inserting the mixed-potential integral expression formulation for the electric field generated by an arbitrary current distribution $\mathbf{J}$ inside a source volume V_m

$$\mathbf{E} = \frac{1}{j\omega\varepsilon_0}\left[k_0^2\int_{V_m}\mathbf{J}GdV_m + \int_{V_m}\left(\nabla_m\cdot\mathbf{J}\right)\nabla GdV_m\right] \tag{2}$$

where

$$G = \frac{e^{-jkR}}{4\pi R}$$

is the free space Green's function and

$$R = \sqrt{(x-x_m)^2+(y-y_m)^2+(z-z_m)^2}\ .$$

Straightforward manipulation, involving reordering the terms and changing the order of integration yields

$$\begin{aligned} W^e = \frac{1}{4\omega^2\varepsilon_0}(&\int_{V_1}\int_{V_2}(\nabla_1\cdot\mathbf{J}_1)(\nabla_2\cdot\mathbf{J}_2^*)\int_V\nabla G_1\cdot\nabla G_2^* dVdV_1dV_2 \\ &+k_0^2(\int_{V_1}\int_{V_2}\mathbf{J}_1(\nabla_2\cdot\mathbf{J}_2^*)\cdot\int_V G_1(\nabla G_2^*)dVdV_1dV_2 \\ &+\int_{V_1}\int_{V_2}(\nabla_1\cdot\mathbf{J}_1)\mathbf{J}_2^*\cdot\int_V(\nabla G_1)G_2^*dVdV_1dV_2) \\ &+k_0^4\int_{V_1}\int_{V_2}\mathbf{J}_1\cdot\mathbf{J}_2^*\int_V G_1G_2^*dVdV_1dV_2) \end{aligned} \tag{3}$$

where the subscripts 1 and 2 both replace the subscript m and indicate source coordinates, linked to the first evaluation of integral expression (2), for the electric field itself, and the second evaluation of integral expression (2), for the complex conjugate of the electric field: $V_1 = V_2 = V_m$. Using vector field algebra, (3) can be transformed into

$$\begin{aligned} W^e = \frac{1}{4\omega^2\varepsilon_0}(&\int_{V_1}\int_{V_2}(\nabla_1\cdot\mathbf{J}_1)(\nabla_2\cdot\mathbf{J}_2^*)\int_V\nabla G_1\cdot\nabla G_2^* dVdV_1dV_2 \\ &-2k_0^2\int_{V_1}\int_{V_2}(\nabla_1\cdot\mathbf{J}_1)(\nabla_2\cdot\mathbf{J}_2^*)\int_V G_1G_2^*dVdV_1dV_2 \\ &+k_0^4\int_{V_1}\int_{V_2}\mathbf{J}_1\cdot\mathbf{J}_2^*\int_V G_1G_2^*dVdV_1dV_2 \\ &+k_0^2\int_{V_1}\int_{V_2}(\mathbf{J}_1(\nabla_2\cdot\mathbf{J}_2^*)+(\nabla_1\cdot\mathbf{J}_1)\mathbf{J}_2^*)\cdot\oint_S G_1G_2^*d\mathbf{S}dV_1dV_2) \end{aligned} \tag{4}$$

where S is the surface containing V, which for entire space goes to infinity. The magnetic energy is

$$W^m = \int_V \frac{1}{4\mu_0}(\mathbf{B}\cdot\mathbf{B}^*)dV\ . \tag{5}$$

Using

$$\mathbf{B} = -\mu_0\left[\int_{V_m}\mathbf{J}\times\nabla GdV_m\right] \tag{6}$$

this gives

$$W^m = \frac{\mu_0}{4}\int_{V_1}\int_{V_2}\int_V(\mathbf{J}_1\times\nabla G_1)\cdot(\mathbf{J}_2^*\times\nabla G_2^*)dVdV_1dV_2\ . \tag{7}$$

Using vector field algebra, this can be transformed into

$$\begin{aligned} W^m = \frac{k_0^2}{4\omega^2\varepsilon_0}\Bigg(&-\int_{V_1}\int_{V_2}\left(\nabla_1\cdot\mathbf{J}_1\right)\left(\nabla_2\cdot\mathbf{J}_2^*\right)\int_V G_1G_2^*dVdV_1dV_2 \\ &+\int_{V_1}\int_{V_2}\left(\mathbf{J}_1\cdot\mathbf{J}_2^*\right)\int_V\nabla G_1\nabla G_2^*dVdV_1dV_2\Bigg). \end{aligned} \tag{8}$$

It is seen that in (4) and (8) one integral over S and two integrals over V (i.e. over entire space) remain. The key novelty is the analytical evaluation of these integrals.

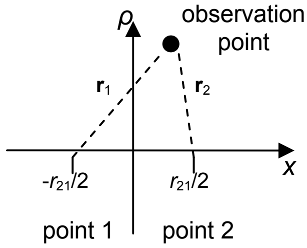

Fig. 1. Auxiliary coordinate system.

We have proven that a correct solution is obtained by choosing for each couple of points 1 and 2 an auxiliary coordinate system with the origin in the middle of the points 1 and 2 and the x-axis directed from point 1 to point 2. The proof is quite involved and beyond the scope of this paper. The surface integral yields

$$I_S = \int_S G_1G_2^*d\mathbf{S} = \frac{4\pi j}{k_0r_{21}}(\cos(k_0r_{21}) - \frac{\sin(k_0r_{21})}{k_0r_{21}})\mathbf{i}_{21}\ . \tag{9}$$

The first volume integral is

$$I_a = \int_V \nabla G_1\cdot\nabla G_2^*dV\ . \tag{10}$$

Using $(\nabla f)\cdot\mathbf{g} = \nabla\cdot(f\mathbf{g}) - f(\nabla\cdot\mathbf{g})$, this is written as

$$\begin{aligned} I_a &= \int_V(\nabla\cdot(G_1\nabla G_2^*) - G_1(\nabla\cdot\nabla G_2^*))dV \\ &= \oint_S G_1\nabla G_2^*\cdot d\mathbf{S} - \int_V G_1\nabla^2G_2^*dV. \end{aligned} \tag{11}$$

It is easily calculated that in the far field

$$\nabla G = -jk_0 G\mathbf{i}_r \quad (12)$$

so that the first term in (11) can be calculated analytically as

$$\oint_S G_1 \nabla G_2^* \cdot d\mathbf{S} = jk_0 \oint_S G_1 G_2^* dS = \frac{j\sin(k_0 r_{21})}{4\pi\, r_{21}} \; . \quad (13)$$

With r_{21} the distance between the two source points. Also, it is well-known that the free space Green's function satisfies the relation

$$\nabla^2 G = -\partial(\mathbf{r} - \mathbf{r}_m) - k_0^2 G \quad (14)$$

with ∂ the Dirac impulse function. Inserting (13) and (14) in (11) yields

$$\begin{aligned} I_a &= G_{21} + j\frac{\sin(k_0 r_{21})}{4\pi\, r_{21}} + k_0^2 \int_V G_1 G_2^* dV \\ &= \frac{\cos(k_0 r_{21})}{4\pi\, r_{21}} + k_0^2 \int_V G_1 G_2^* dV. \end{aligned} \quad (15)$$

G_{21} is G_1 evaluated at $\mathbf{r}_2$, i.e. for $R = r_{21}$. This reduces (10) to the second integral to evaluate

$$I_b = \int_V G_1 G_2^* dV \; . \quad (16)$$

For large r the integrand is decaying purely as $1/r^2$. This integral is linked to the radiation field. It is well-known that the radiation field gives an infinite contribution to the stored energies and in general does not have to be considered. A discussion on this technique can be found in [1], [6], [10]. There the problem is formally solved by subtraction of the term $\lim_{r\to\infty}(r/2c\cdot P_{rad})$ in both electric and magnetic energy.

As pointed out by Geyi [6], it is not possible to calculate numerically the stored energy in a finite sphere, subtracting the term given, with the corresponding r, and taking the limit for the finite sphere going to infinity. In this case the numerical rounding errors become increasingly significant.

Other authors treat the radiation field based on a spherical mode decomposition [1], [3], [4]. In the case that only one mode is considered, thus neglecting higher order modes, this gives rise to unique closed form expressions for the radiation field at all r-distances. It is then easy to sub-tract the corresponding energy density. The approach used in this paper is different. The technique suggested by Collin [1] is used, and the problem pointed out by Geyi is avoided by incorporating the term within the integrand, in this way subtracting the radiation energy at every point in space.

The calculation procedure starts by realizing that the complex conjugate of (16) yields the same expression, which means that the integral is real, and only the real part of the integrand has to be kept. The integral becomes

$$I_b = \int_V \mathrm{Re}(G_1 G_2^*) dV = \frac{1}{16\pi^2} \int_V \frac{1}{r_1 r_2} \cos(k_0 (r_2 - r_1)) dV \; . \quad (17)$$

The absolute maximum value of $(r_2 - r_1)$ over entire space is actually the distance r_{21} between the two source points. Since $k_0 (r_2 - r_1)$ remains finite over the entire space, the cosine can be expanded in Taylor series around $k_0 (r_2 - r_1) = t = 0$.

$$\cos t = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} t^{2n} = \sum_{n=0}^{\infty} C_n t^{2n} \; . \quad (18)$$

Normalization of the integral with respect to r_{21} through a coordinate transformation yields

$$I_b = \frac{1}{16\pi^2} r_{21} \int_x \int_y \int_z \frac{\cos(k_0 r_{21} (r_{2n} - r_{1n}))}{r_{1n} r_{2n}} dxdydz \quad (19)$$

with

$$\begin{aligned} r_{1n} &= \sqrt{(x + \frac{1}{2})^2 + (y^2 + z^2)} \\ r_{2n} &= \sqrt{(x - \frac{1}{2})^2 + (y^2 + z^2)} \end{aligned} \; . \quad (20)$$

r_{1n} and r_{2n} can be considered 'normalized' distances. Another transformation to cylindrical coordinates around the x-axis and evaluation of the resulting elementary φ integrals, and taking into account the fact that the integrand is even in x, yields

$$I_b = \frac{1}{4\pi} r_{21} \int_0^{\infty} \int_0^{\infty} \frac{\cos(k_0 r_{21} (r_{2n} - r_{1n}))}{r_{1n} r_{2n}} \rho d\rho dx \; . \quad (21)$$

The next step is using the Taylor series expansion.

$$I_b = \frac{1}{4\pi} r_{21} \sum_{n=0}^{\infty} C_n (k_0 r_{21})^{2n} \int_0^{\infty} \int_0^{\infty} \frac{[r_{2n} - r_{1n}]^{2n}}{r_{1n} r_{2n}} \rho d\rho dx \; . \quad (22)$$

A series of integrals has to be evaluated. It can be proven that taking into account the subtraction of the radiation contribution (the proof is beyond the scope of this paper), the following integrals are obtained:

$$\int_0^{\infty} \int_0^{\infty} \left[\frac{[r_{2n} - r_{1n}]^{2n}}{r_{1n} r_{2n}} - \frac{[x/r]^{2n}}{r^2} \right] \rho d\rho dx = R(2n) \quad (23)$$

where

$$\begin{aligned} r_{1n} &= \sqrt{(x + \frac{1}{2})^2 + \rho^2} \, , \\ r_{2n} &= \sqrt{(x - \frac{1}{2})^2 + \rho^2} \, , \\ r &= \sqrt{x^2 + \rho^2} \end{aligned} \quad (24)$$

are the normalized distances from the observation point considered to the source points 1 and 2, and the origin, respectively. It can be proven that $R(2n) = -1/(4n + 2)$, yielding the final result

$$I_b = -\frac{r_{21}}{8\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}(k_0 r_{21})^{2n} = -\frac{\sin(k_0 r_{21})}{8\pi k_0}. \quad (25)$$

Insertion of the expressions for the integrals I_s, I_a and I_b in the expressions for the reactive electric and magnetic energies and straightforward reordering gives

$$\tilde{W}^e = \frac{1}{16\pi\omega^2\varepsilon_0}\Big(\int_{V_1}\int_{V_2}(\nabla_1\cdot\mathbf{J}_1)(\nabla_2\cdot\mathbf{J}_2^*)\frac{\cos(k_0 r_{21})}{r_{21}}dV_1 dV_2 \quad (26)$$
$$-\frac{k_0}{2}\int_{V_1}\int_{V_2}(k_0^2(\mathbf{J}_1\cdot\mathbf{J}_2^*)-(\nabla_1\cdot\mathbf{J}_1)(\nabla_2\cdot\mathbf{J}_2^*))\sin(k_0 r_{21})dV_1 dV_2\Big)$$

$$\tilde{W}^m = \frac{1}{16\pi\omega^2\varepsilon_0}\Big(k_0^2\int_{V_1}\int_{V_2}(\mathbf{J}_1\cdot\mathbf{J}_2^*)\frac{\cos(k_0 r_{21})}{r_{21}}dV_1 dV_2 \quad (27)$$
$$-\frac{k_0}{2}\int_{V_1}\int_{V_2}(k_0^2(\mathbf{J}_1\cdot\mathbf{J}_2^*)-(\nabla_1\cdot\mathbf{J}_1)(\nabla_2\cdot\mathbf{J}_2^*))\sin(k_0 r_{21})dV_1 dV_2\Big)$$

The tilde sign above the energies indicates that the "radiation" contribution is excluded. Specific about (26) and (27) is that their first term can be identified with the "charge" and the "current" part, respectively, of the mixed potential integral expression for the electric field, but using only the real part of the Green's function. For completeness, the corresponding expression for the radiated energy is given, without proof

$$P_{rad} = \frac{1}{8\pi\omega\varepsilon_0}\Big(\int_{V_1}\int_{V_2}(k_0^2(\mathbf{J}_1\cdot\mathbf{J}_2^*)-(\nabla_1\cdot\mathbf{J}_1)(\nabla_2\cdot\mathbf{J}_2^*)) \quad (28)$$
$$\frac{\sin(k_0 r_{21})}{r_{21}}dV_1 dV_2.$$

Using the Taylor series expansion for $\cos(k_0 r_{21})$ and $\sin(k_0 r_{21})$ yields

$$\tilde{W}^e = \frac{1}{16\pi\omega^2\varepsilon_0}\Big(\int_{V_1}\int_{V_2}(\nabla_1\cdot\mathbf{J}_1)(\nabla_2\cdot\mathbf{J}_2^*)r_{21}^{-1}dV_1 dV_2$$
$$-\frac{k_0^2}{2}\sum_{n=0}^{\infty}(-1)^n\Big(\frac{1}{n+1}-1\Big)\frac{1}{(2n+1)!}k_0^{2n} \quad (29)$$
$$\cdot\int_{V_1}\int_{V_2}(\nabla_1\cdot\mathbf{J}_1)(\nabla_2\cdot\mathbf{J}_2^*)r_{21}^{2n+1}dV_1 dV_2$$
$$-\frac{k_0^4}{2}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}k_0^{2n}\int_{V_1}\int_{V_2}(\mathbf{J}_1\cdot\mathbf{J}_2^*)r_{21}^{2n+1}dV_1 dV_2\Big),$$

$$\tilde{W}^m = \frac{1}{16\pi\omega^2\varepsilon_0}\Big(k_0^2\int_{V_1}\int_{V_2}(\mathbf{J}_1\cdot\mathbf{J}_2^*)r_{21}^{-1}dV_1 dV_2$$
$$-\frac{k_0^4}{2}\sum_{n=0}^{\infty}(-1)^n\Big(\frac{1}{n+1}+1\Big)\frac{1}{(2n+1)!}k_0^{2n}$$
$$\cdot\int_{V_1}\int_{V_2}(\mathbf{J}_1\cdot\mathbf{J}_2^*)r_{21}^{2n+1}dV_1 dV_2$$
$$+\frac{k_0^2}{2}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}k_0^{2n} \quad (30)$$
$$\cdot\int_{V_1}\int_{V_2}(\nabla_1\cdot\mathbf{J}_1)(\nabla_2\cdot\mathbf{J}_2^*)r_{21}^{2n+1}dV_1 dV_2\Big),$$

$$P_{rad} = \frac{k_0^3}{8\pi\omega\varepsilon_0}\Big(\frac{2}{3}\int_{V_1}\int_{V_2}(\mathbf{J}_1\cdot\mathbf{J}_2^*)dV_1 dV_2$$
$$-k_0^2\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+3)!}k_0^{2n}\int_{V_1}\int_{V_2}(\mathbf{J}_1\cdot\mathbf{J}_2^*)r_{21}^{2n+2}dV_1 dV_2 \quad (31)$$
$$-k_0^2\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+5)!}k_0^{2n}$$
$$\cdot\int_{V_1}\int_{V_2}(\nabla_1\cdot\mathbf{J}_1)(\nabla_2\cdot\mathbf{J}_2^*)r_{21}^{2n+4}dV_1 dV_2\Big).$$

Note that keeping only the leading term of the series involved yields the expressions for the reactive energies as formulated by Geyi for small devices [6].

3. Discussion

Expressions (26) and (27) rigorously formulate the unique relation between currents and energies in a direct and explicit way. To the knowledge of the author, they are not yet available in literature. They are easy to understand and interpret, since they only involve double integrations over the source volume. The integrations concern either the current on the device, or its divergence (which corresponds to the charge on the device).

4. Applications and Examples

4.1 Analytical Calculation of Reactive Energies and Q Factors

Consider a loop with radius a, made of wire with diameter d. The loop is located in the xy plane and the origin is in the center of the loop. In cylindrical co-ordinates, the current on the loop is given by

$$\mathbf{I} = I\, e^{jm\varphi}\mathbf{i}_\varphi\,. \quad (32)$$

For $m = 0$, actually a magnetic dipole is obtained. For $m = 1$ a ring type electric dipole configuration is obtained. All integrals in (29), (30), and (31) can be evaluated analytically. Inserting the current and the known topology, using the thin wire approximation, and after some straightforward manipulation, the following expressions are obtained. For the electric dipole type (m=1)

$$\tilde{W}_e = \frac{|I|^2}{4a\omega^2\varepsilon}\Big(\ln\Big(\frac{16a}{d}\Big)-2-k^4a^4\frac{2}{3}+k^6a^6\frac{56}{135}+\ldots\Big), \quad (33)$$

$$\tilde{W}_m = \frac{|I|^2}{4a\omega^2\varepsilon}(k^2a^2(\ln(\frac{16a}{d})-2) - k^4a^4\frac{4}{3} + k^6a^6\frac{28}{45}+...), \quad (34)$$

$$P_{rad} = \frac{\pi|I|^2k^3a^3}{2a\omega\varepsilon}(\frac{1}{3} - k^2a^2\frac{2}{15} + k^4a^4\frac{11}{420} - ...). \quad (35)$$

For very small wire diameters, only the logarithmic terms have to be kept. In this case it can be seen that

$$\tilde{W}_m - \tilde{W}_e$$

goes to zero, yielding a reactance also equal to zero, for $a = \lambda/(2\pi)$. Using the expressions, the deterioration from this simple result in terms of growing wire diameters can be studied. The Q of the antenna becomes

$$Q = \frac{2\omega\tilde{W}_e}{P_{rad}} = \frac{1}{\pi(k^3a^3)}\frac{(\ln(\frac{16a}{d})-2-k^4a^4\frac{2}{3}+k^6a^6\frac{56}{135}+...)}{(\frac{1}{3}-k^2a^2\frac{2}{15}+k^4a^4\frac{11}{420}-...)}. \quad (36)$$

For the magnetic dipole (m=0)

$$\tilde{W}_e = \frac{|I|^2k^2a^2}{4a\omega^2\varepsilon}(k^2a^2\frac{2}{3} - k^4a^4\frac{8}{15} + k^6a^6\frac{32}{315} - ...) \quad (37)$$

$$\tilde{W}_m = \frac{|I|^2k^2a^2}{4a\omega^2\varepsilon}(\ln(\frac{16a}{d}) - 2 + k^2a^2\frac{4}{3} - k^4a^4\frac{4}{5} + ...) \quad (38)$$

$$P_{rad} = \frac{\pi|I|^2k^5a^5}{2a\omega\varepsilon}(\frac{1}{6} - k^2a^2\frac{1}{30} + k^4a^4\frac{1}{336} - ...). \quad (39)$$

Here, the reactance cannot be made zero for wire diameters going to zero. The Q of the magnetic dipole becomes

$$Q = \frac{2\omega\tilde{W}_m}{P_{rad}} = \frac{1}{\pi(k^3a^3)}\frac{(\ln(\frac{16a}{d})-2+k^2a^2\frac{4}{3}-k^4a^4\frac{4}{5}+...)}{(\frac{1}{6}-k^2a^2\frac{1}{30}+k^4a^4\frac{1}{336}-...)} \quad (40)$$

This example illustrates a very important conclusion. The new expressions allow to study antenna Q in terms of antenna size in a rigorous way.

Keeping only the highest order term in the expression (40) yields the approximating expression for small antennas, derived by Geyi in [6].

4.2 Numerical Calculation of Reactive Energies and Q Factors

Expressions (26), (27), and (28) are extremely easy to implement in software tools. In this section, the case of a dipole antenna is analyzed. The current on the dipole is obtained using our in-house developed tool MAGMAS [9]. MAGMAS solves the integral equations describing the structure using the method of moments. The antenna considered is a strip-dipole. The strip has length L = 1 cm and width W = 0.1 mm. For the analysis, the strip was subdivided in 100 segments along its length. A feeding current of 1 A is imposed in the middle of the antenna. Since both the "small antenna" regime and the half-wavelength dipole regime have to be covered, the dipole is analyzed in the frequency band 1 – 30 GHz. The energies,

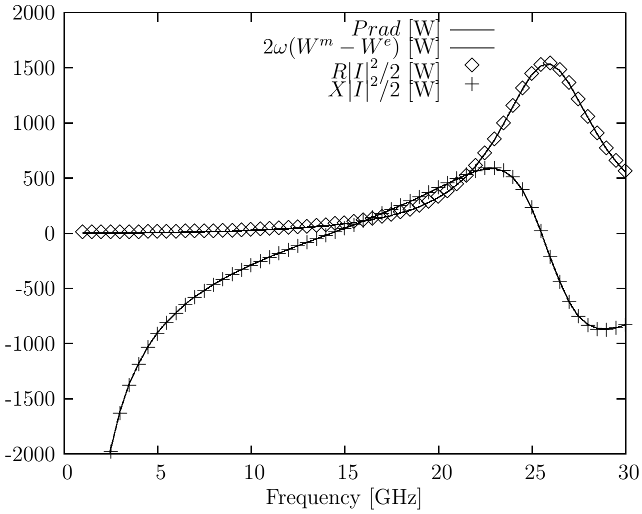

Fig. 2. Radiated power $P_{rad} = R|I|^2/2$ and difference between magnetic and electric energies $2\omega(W^m - W^e) = X|I|^2/2$ for a strip dipole antenna of 1 cm.

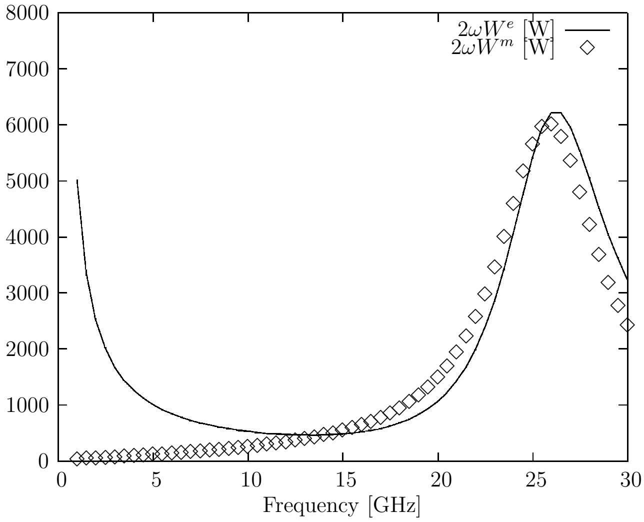

Fig. 3. Magnetic and electric energies for a strip dipole antenna of 1 cm.

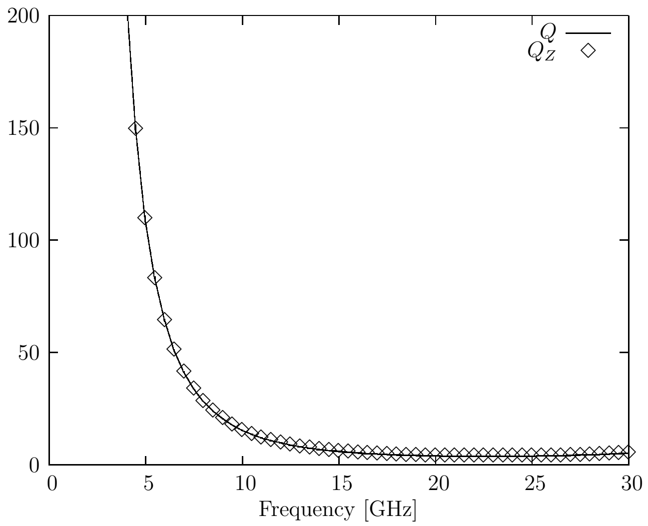

Fig. 4. Q factor for a strip dipole antenna of 1 cm, calculated with the new expressions (Q), and following the technique introduced in [10] (Qz), respectively.

calculated with the new formulas (26), (27), and (28), are studied in Fig. 2 and Fig. 3. The curves $2\omega(W_m - W_e)$ and P_r correspond perfectly with the reactance and the resistance of the dipole, quantities that are calculated in a completely different manner by the software. Below 5 GHz the antenna behaves as a small antenna. The electric and magnetic energies show the correct behavior in ω. The half-wavelength dipole behavior is observed around 15 GHz, where the electric and magnetic energies become equal, which results in a zero reactance.

References

[1] COLLIN R. E., ROTHSCHILD, S. Evaluation of antenna Q. *IEEE Transactions on Antennas and Propagation*, 1964, vol. AP-12, p. 23-27.

[2] CHU, L. J. Physical limitations on omni-directional antennas. *Journal of Applied Physics*, 1948, vol. 19, p. 1163-1175.

[3] FANTE, R. L. Quality factor of general ideal antennas, *IEEE Trans. Antennas & Propagat.*, March 1969, vol. AP-17, p. 151 to 155.

[4] MCLEAN, J. S. A re-examination of the fundamental limits on the radiation Q of electrically small antennas. *IEEE Transactions on Antennas and Propagat*ion, 1996, vol. 44, p. 672-676.

[5] STEN, J. C.-E., HUJANEN, A., KOIVISTO, P. K. Quality factor of an electrically small antenna radiating close to a conducting plane. *IEEE Transactions on Antennas and Propagat*ion, 2001, vol. 49, p. 829-837.

[6] GEYI, W. A method for the evaluation of small antenna Q. *IEEE Transactions on Antennas and Propagat*ion, 2003, vol. 51, p. 2124-2129.

[7] SHLIVINSKI, HEYMAN, E. Time-domain near-field analysis of short-pulse antennas – part II: Reactive energy and the antenna Q. *IEEE Transactions on Antennas and Propagat*ion, 1999, vol. 47, p. 280-286.

[8] COLLARDEY, S., SHARAIHA, A., MAHDJOUBI, K. Calculation of small antennas quality factor using FDTD method. *IEEE Antennas and Wireless Propagation Lett*ers, 2006, vol. 5, p. 191-194.

[9] Katholieke Universiteit Leuven, Belgium. *MAGMAS – the TELEMIC Antenna Modelling Software.* [Online] Cited 2009-11-10. Available at http://www.esat.kuleuven.be/telemic/antennas/magmas .

[10] YAGHJIAN, D., BEST, S. R. Impedance, bandwidth and Q of antennas. *IEEE Transactions on Antennas and Propagat*ion, 2005, vol. 53, p. 1298-1324.

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