

Mobile Radio Channels Modeling in MATLAB

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Abstract. *In this paper, a MATLAB based approach for mobile radio channels modeling is presented. Specifically, the paper introduces the basic concepts for modeling flat fading channels in MATLAB by means of user-defined m-files. Typical small-scale fading channel models are derived such as uncorrelated Rician fading channel and Rayleigh fading channel with Doppler shift. Further, simple and useful MATLAB constructions for approximation of cumulative distribution functions (CDFs) and probability density functions (PDFs) are also given. Finally, a MATLAB based Monte Carlo simulation example is presented, which comprises performance estimation of phase shift keying (PSK) signaling over a Rician fading channel.*

Keywords

MATLAB, fading channels, distribution, simulation.

1. Introduction

In digital communication theory the most frequently assumed model for a transmission channel is the additive white Gaussian noise (AWGN) channel. However, for many communication systems the AWGN channel is a poor model, and one must resort to more precise and complicated channel models. One basic type of non-Gaussian channel, which frequently occurs in practice, is the fading channel. A typical example of such a fading channel is the mobile radio channel, where the small antennas of portable units pick up multipath reflections. Thus, the mobile channel exhibits a time varying behavior in the received signal energy, which is called fading.

Using MATLAB for digital communication systems simulation one has the advantage of exploiting the powerful features of its Communications Toolbox along with a nice programming language. However, the Communications Toolbox of MATLAB suffers from absence of proper mobile channel models. The only available channel model in the current Communications Toolbox 2.1 is the `awgn` m-file, which is appropriate for an AWGN channel simulation. So, the users of MATLAB should build appropriate channels (i.e., m-files) in their own to reach the desired simulation model.

The paper is organized as follows. In Section 2, a brief introduction to fading channels is given. The basic

concepts for modeling flat fading channels in MATLAB are presented in Section 3. In this section, example m-files are proposed to model different types of flat fading channels. In Section 4, a MATLAB based Monte Carlo simulation example is presented, which describes the basic concepts of digital modulations performance estimation over fading channels. Finally, the concluding remarks are given in Section 5.

2. The Mobile Radio Channel

The mobile radio channel is characterized by two types of fading effects: large-scale fading and small scale fading [1], [2]. Large-scale fading is the slow variation of the mean (distant-dependent) signal power over time. This depends on the presence of obstacles in the signal path and on the position of the mobile unit. The large-scale fading is assumed to be a slow process and is commonly modeled as having lognormal statistics. Small-scale fading is also called Rayleigh or Rician fading because if a large number of reflective paths is encountered the received signal envelope is described by a Rayleigh or a Rician probability density function (PDF) [3]. The small-scale fading under consideration is assumed to be a flat fading (i.e., there is no intersymbol interference). It is also assumed that the fading level remains approximately constant for (at least) one signaling interval. With this model of fading channel the main difference with respect to an AWGN channel resides in the fact that fading amplitudes are now Rayleigh- or Rician-distributed random variables, whose values affect the signal amplitude (and, hence, the power) of the received signal.

The fading amplitudes can be modeled by a Rician or a Rayleigh distribution, depending on the presence or absence of specular signal component. Fading is Rayleigh if the multiple reflective paths are large in number and there is no dominant line-of-sight (LOS) propagation path. If there is also a dominant LOS path, then the fading is Rician-distributed. The fading amplitude r_i at the i th time instant can be represented as

$$r_i = \sqrt{(x_i + \beta)^2 + y_i^2}, \quad (1)$$

where β is the amplitude of the specular component and x_i , y_i are samples of zero-mean stationary Gaussian random processes each with variance σ_0^2 . The ratio of specular to diffuse energy defines the so-called Rician K -factor, which is given by

$$K = \beta^2 / 2\sigma_0^2 . \quad (2)$$

The best- and worst-case Rician fading channels associated with K -factors of $K = \infty$ and $K = 0$ are the Gaussian and Rayleigh channels with strong LOS and no LOS path, respectively. So, the Rayleigh fading channel can be considered as a special case of a Rician fading channel with $K = 0$. The Rician PDF is given by [3]

$$f_{Rice}(r) = \frac{r}{\sigma_0^2} \exp[-(r^2 + \beta^2) / 2\sigma_0^2] I_0 \left[\frac{r\beta}{\sigma_0^2} \right] r \geq 0 , \quad (3)$$

where $I_0[\cdot]$ is the zero-order modified Bessel function of the first kind. Now, if there is no dominant propagation path, $K = 0$ and $I_0[\cdot] = 1$ yielding the worst-case Rayleigh PDF

$$f_{Rayleigh}(r) = \frac{r}{\sigma_0^2} \exp[-r^2 / 2\sigma_0^2] r \geq 0 . \quad (4)$$

The Rician cumulative distribution function (CDF) takes the shape of [3]

$$C_{Rice}(r) = 1 - \exp(-\gamma) \sum_{m=0}^{\infty} \left[\frac{\beta}{r} \right]^m \cdot I_m \left[\frac{r\beta}{\sigma_0^2} \right] , \quad (5)$$

where $\gamma = (K + r^2 / 2\sigma_0^2)$. Clearly, this formula is more difficult to evaluate than the PDF of (3) due to the summation of an infinite number of terms. However, in practical terms it is sufficient to increase m to the value, where the last terms contribution becomes less than 0.1 percent.

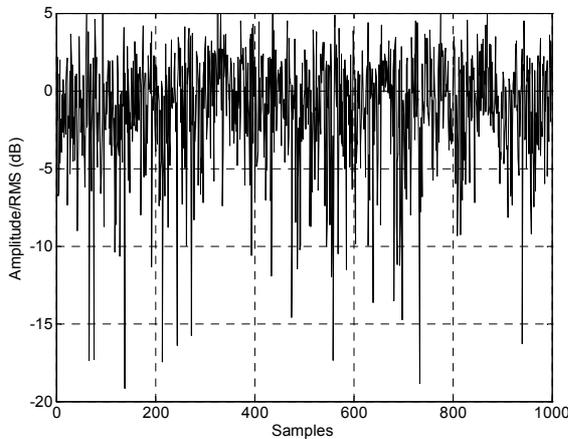


Fig. 1. A typical Rician fading envelope for $K = 5$ dB.

Having described the main fading statistics let us consider the effects of the Doppler frequency shift. Doppler shift from the carrier frequency f_c occurs when the distance between the mobile receiver and the transmitter is changing. The magnitude of the Doppler frequency shift f_m is determined by

$$f_m = \frac{v f_c}{c} , \quad (6)$$

where v is the mobile station velocity and $c = 3 \cdot 10^8$ m/s is the speed of light. The Doppler frequency f_m is often called fading bandwidth or fading fate of the channel. The rela-

tionship between f_m and the coherence time T_m of the channel (i.e., the time over which the channel's response to a sinusoid is essentially invariant) is given by [1]

$$T_m \approx \frac{0.5}{f_m} . \quad (7)$$

The Doppler power spectral density $S(f)$ of the mobile channel is often expressed as [4], [5]

$$S(f) = \frac{1}{\pi f_m \sqrt{1 - (f / f_m)^2}} . \quad (8)$$

The above equality holds for frequency shifts of f in the range $\pm f_m$ about the carrier frequency f_c .

Some important notes are in order here. First, the primary factor, which affects the performance of digital communication systems in a mobile environment, is the small-scale fading. So, we will consider Rayleigh and Rician fading channels from now on. Second, uncorrelated Rayleigh- or Rician-distributed fading sequences can be obtained directly by exploiting (1). Third, correlated fading sequences can be obtained by means of "colored" Gaussian noise processes. A well-known approach is the so-called sum-of-sinusoids method in which a "colored" Gaussian noise is approximated by a finite sum of weighted and properly designed sinusoids [5]-[8]. In this paper we opt on the modified sum-of-sinusoids method as described in [8], because of its improved statistical properties and simple software realization.

3. Fading Generation in MATLAB

Let us first consider the generation of uncorrelated Rician-distributed fading sequences in MATLAB. The mean-squared value of the Rician distribution is known to be $2\sigma_0^2(K + 1)$, where σ_0^2 is the variance of the component Gaussian noise processes in (1). Further, it is often required a Rician distribution with unit mean-squared value, i.e., $E\{r^2\} = 1$, so that the signal power and the signal-to-noise ratio (SNR) coincide. In order to meet the requirement $E\{r^2\} = 1$, the equation (1) can be written in the form

$$r_i = \sqrt{\frac{(x_i + \sqrt{2K})^2 + y_i^2}{2(K + 1)}} , \quad (9)$$

where now x_i, y_i are samples of zero-mean stationary Gaussian random processes each with variance $\sigma_0^2 = 1$. So, the desired Rician fading sequences can be generated according to (9) by using the MATLAB random number generator **randn**, which produces normally distributed random numbers. The following MATLAB m-file **rice_fading** does the task.

```
function r = rice_fading(Kdb, N, Mi)
K = 10^(Kdb/10); const = 1/(2*(K+1));
x = randn(1,N); y = randn(1,N);
r = sqrt(const*((x + sqrt(2*K)).^2 + y.^2));
rt = zeros(1,Mi*length(r)); ki = 1;
```

```

for i=1:length(r)
    rt(ki:i*Mi) = r(i); ki = ki+Mi;
end
r = rt;

```

With this MATLAB code one can obtain both the “symbol-level” and “block-level” Rician fading channels for a proper chosen interpolation factor M_i . Also, Rayleigh fading samples can be obtained for $K_{db} = -\infty$. A typical Rician fading envelope is shown in Fig.1, where the fading amplitudes are plotted in decibels (about the root-mean-squared (RMS) value).

Now, we will consider a MATLAB based approximation of the Rician CDF. In the MATLAB code below, the Rician CDF is approximated from the derived fading samples according to the m-file **rice_fading**.

```

Kdb=7; N=100000; Mi=1;
r=rice_fading(Kdb,N,Mi);
RdB = 20*log10(r);
Rt = [min(RdB):max(RdB)];
for m = 1:length(Rt)
    fade = find(RdB < Rt(m));
    Nm = length(fade); AF(m) = Nm/N;
end
semilogy(Rt,AF,'k-o'); grid;

```

In this example the Rician CDF is derived by means of an iterative construction, where at each iteration the MATLAB functions **find** and **length** are used to obtain the number of fades, which are below the specified threshold value. The obtained CDF is plotted in Fig. 2.

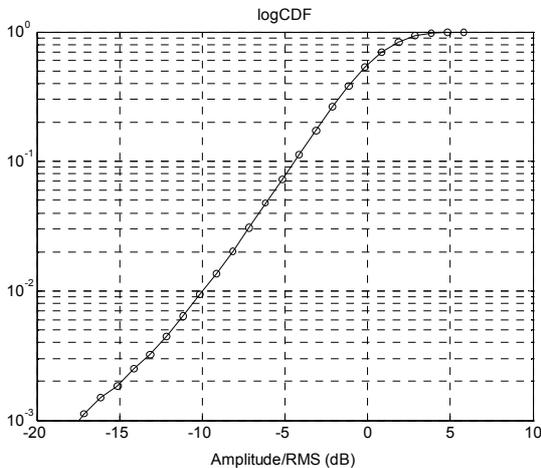


Fig. 2. An approximated Rician CDF for $K = 7$ dB.

The Rician CDF in Fig.2 is plotted on a logarithmic probability scale, which allows us to expand the tail of the CDF, where for example the 10 dB fading margin overload probability is seen to be approximately 10^{-2} .

The next MATLAB code is an example construction for comparing an approximated Rayleigh PDF with the corresponding analytical PDF given by (4). The obtained result is shown in Fig. 3.

```

N = 100000;
x = randn(1, N); y = randn(1, N);

```

```

r = sqrt(0.5*(x.^2 + y.^2));
step = 0.1; range = 0:step:3;
h = hist(r, range);
fr_approx = h/(step*sum(h));
fr = (range/0.5).*exp(-range.^2);
plot(range, fr_approx,'ko', range, fr,'k');
grid;

```

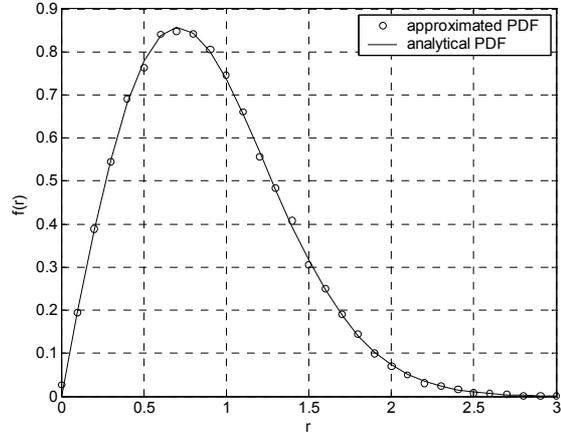


Fig. 3. The approximated and analytical Rayleigh PDF for $\sigma_0^2 = 0.5$.

In the above code the approximated Rayleigh PDF (denoted as fr_approx) is derived in MATLAB by means of the useful function **hist** (the histogram function). Then, a two-dimensional plot is used (by means of the MATLAB function **plot**) to compare the approximated PDF with the analytical PDF (denoted as fr) over a range of interest.

Finally, we will consider the generation of correlated Rayleigh-distributed fading samples by using the modified sum-of-sinusoids method as described in [8]. The quadrature components of Rayleigh fading process are given by

$$x(t) = \sqrt{\frac{2}{M}} \sum_{n=1}^M \cos(\omega_d t \cos \alpha_n + \phi_n), \quad (10)$$

$$y(t) = \sqrt{\frac{2}{M}} \sum_{n=1}^M \cos(\omega_d t \sin \alpha_n + \varphi_n), \quad (11)$$

with

$$\alpha_n = \frac{2\pi n - \pi + \theta_n}{4M}, \quad n = 1, 2, \dots, M, \quad (12)$$

where ω_d is the maximum angular Doppler frequency, ϕ_n , φ_n and θ_n are statistically independent and uniformly distributed on $[-\pi, \pi]$ for all n . The following m-file, named **ray_doppler**, exploits equations (9)-(12) for correlated Rayleigh-distributed fading samples generation in MATLAB.

```

function r = ray_doppler(fm, M, dt, N)
T = N*dt-dt; t = 0:dt:T;
c = sqrt(2/M); w = 2*pi*fm; x = 0; y = 0;
for n = 1:M
    alpha = (2*pi*n-pi+(2*pi*rand-pi))/(4*M);
    ph1 = 2*pi*rand - pi;
    ph2 = 2*pi*rand - pi;
    x = x + c*cos(w*t*cos(alpha) + ph1);

```

```

y = y + c*cos(w*t*sin(alpha) + ph2);
end
r = sqrt(x.^2 + y.^2)/sqrt(2);

```

Input arguments of the m-file **ray_doppler** are the maximum Doppler frequency (denoted as f_m) in Hz, the number of sinusoids (denoted as M), the time step (denoted as dt) and the desired fading sequence length (denoted as N). All values of ϕ_n , φ_n and θ_n are obtained by means of the MATLAB random number generator **rand**, which produces uniformly distributed random numbers. Using proper values for M (M should be not less than eight [8]), N and dt , one can obtain the desired Doppler spread. The considered approach can be extended with minor additions for correlated Rician fading samples generation.

4. Monte Carlo Simulation Example

Let us consider a transmission of digitally modulated symbols over an uncorrelated flat Rician fading channel. Then, if s_i denotes the transmitted symbol at the i th time instant, the received symbol v_i can be expressed as

$$v_i = r_i \cdot s_i + n_i, \quad (13)$$

where $\{r_i\}$ denotes a sequence of independent Rician-distributed random variables and n_i 's are samples of a complex, zero-mean white Gaussian noise process with one-sided power spectral density N_0 . A MATLAB m-file, named **psk_rice**, is given below, wherein the performance of phase shift keying (PSK) signaling over such a fading channel is simulated. It should be mentioned that the simulation example requires the Communications Toolbox of MATLAB and the m-file **rice_fading** in order to run.

```

function BER = psk_rice(logEbNo,M,Kdb)
k = log2(M); EbNolin = 10.^(logEbNo/10)
Fd = 1; Fs = 1; Nit = 100000; Ns = 1000;
Tstop = 100;
for it_snr = 1:length(EbNolin)
nstd = sqrt(1/(2*k*EbNolin(it_snr)));
bit_err = 0;
for it = 1:Nit
a = randint(Ns,1,M);
s = dmodce(a,Fd,Fs,'psk',M);
r = rice_fading(Kdb,Ns,1); v_r = s.*r';
v = v_r + nstd*(randn(Ns,1) + ...
j*randn(Ns,1));
z = ddemodce(v, Fd, Fs, 'psk', M);
errors = biterr(a,z);
bit_err = bit_err + errors;
if bit_err >= Tstop
break
end
end
BER(it_snr) = bit_err/(it*Ns*k);
end
semilogy(logEbNo, BER, 'k');
xlabel('EbNo (dB)'); ylabel('BER'); grid;

```

Input arguments of the m-file **psk_rice** are the vector \logEbNo of energy-per-bit to noise power spectral density values (in decibels), the PSK constellation size M and the

Rician K -factor K_{db} in decibels. The Communications Toolbox functions used in the simulation example are: **randint** (generate matrix of uniformly distributed random integers), **dmodce** (baseband digital modulator), **ddemodce** (digital baseband demodulator) and **biterr** (compute number of bit errors and bit error rate). Notice how the **for-end** loops are used in the simulation. The outer loop is used for signal-to-noise ratio (SNR) points enumeration, while (for each SNR point) the inner loop iterates a finite number of frames of PSK symbols until a threshold value of bit errors is reached (thanks to the **if-break-end** construction) or the total number of frames is executed. Finally, the estimated bit error rate (BER) is plotted versus the SNR points by means of the MATLAB function **semilogy**. It should be mentioned that the simulation accuracy is determined by the number of errors observed. A rule of thumb is that after 100 errors the estimated BER is within a factor 1.3 of the true BER (with approximately 99 percent confidence). So, a threshold value of 100 bit errors is used in the simulation for early stopping the inner loop iterations. One can also apply a set of SNR-dependent threshold values to avoid a time-consuming simulation. The simulation result is shown in Fig.4 for quadrature phase shift keying (QPSK) signaling and two different Rician K -factors (given in decibels).

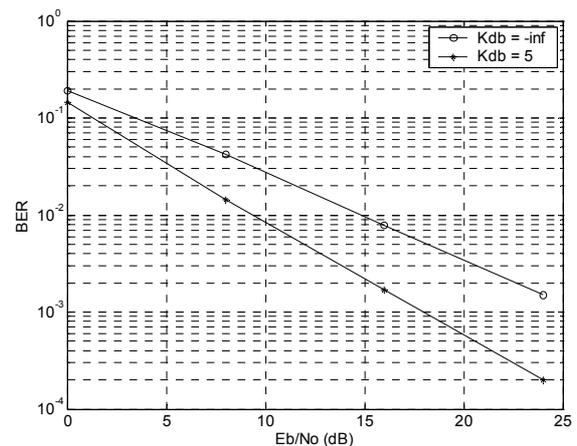


Fig. 4. Bit error rate performance of QPSK signaling over an uncorrelated Rician fading channel.

5. Conclusions

In this paper, an approach to demonstrate mobile radio channels modeling in MATLAB is presented. The basic types of small-scale fading channel models are derived and simulated using MATLAB. The developed channel models can be extended with minor additions to model other types of fading channels (for example, a correlated Rician fading channel). Further, simple and useful MATLAB constructions for approximation of CDFs and PDFs are also given. Finally, the presented Monte Carlo simulation example can be used as a framework for performance estimation of various digital modulation schemes in MATLAB.

References

- [1] SKLAR, B. Rayleigh fading channels in mobile digital communication systems, Part I: Characterization. *IEEE Communications Magazine*, 1997, vol. 35, no. 7, p. 90 - 100.
- [2] SUZUKI, H. A statistical model for urban radio propagation. *IEEE Transactions on Communications*, 1977, vol. COM-25, no. 7, p. 673 to 680.
- [3] PROAKIS, J. G. *Digital communications*. 2nd ed. New York: McGraw-Hill, 1989.
- [4] CLARKE, R. H. A statistical theory of mobile-radio reception. *Bell Systems Technical Journal*, 1968, vol. 47, no. 6, p. 957 - 1000.
- [5] JAKES, W. C. *Microwave mobile communications*. New York: Wiley, 1974.
- [6] PATZOLD, M., KILLAT, U., LAUE, F., LI, Y. On the statistical properties of deterministic simulation models for mobile fading channels. *IEEE Transactions on Vehicular Technology*, 1998, vol. 47, no. 1, p. 254 - 269.
- [7] HOEHER, P. A statistical discrete-time model for the WSSUS multipath channel. *IEEE Transactions on Vehicular Technology*, 1992, vol. 41, no. 4, p. 461 - 468.
- [8] ZHENG, Y., R., XIAO, C. Improved models for the generation of multiple uncorrelated Rayleigh fading waveforms. *IEEE Communications Letters*, 2002, vol. 6, no. 6, p. 256 - 258.

About Author

Nikolay KOSTOV was born in Varna, Bulgaria, on August 7, 1969. He received Dipl. Eng. and Dr. Eng. degrees from the National Military University of Veliko Turnovo, Bulgaria, in 1993 and 1999, respectively. He joined the Radio Engineering Department at Technical University of Varna, Bulgaria, in March 2001, where he is currently serving as an Assistant Professor. His research interests include source/channel coding, modulation techniques, and communications over fading channels.

Book Review:

Multimedia Communications. Multimedia Technology and Watermarking Dušan Levický, Technical University of Košice

The monograph *Multimedia communications* written by Dušan Levický, a professor of Electrical Engineering at the Technical University of Košice tries to cover a full spectrum of multimedia systems, methods and standards.

Many conferences and journals have been devoted to the multimedia in several last years. Numerous standards, coding methods and other techniques have been developed. It is not easy to get desired detailed information from this area or to obtain a general overview on multimedia. We suppose, this could be the main reason for writing the monograph.

The first three chapters are devoted to the multimedia systems, multimedia services and to the classification of multimedia signals.

Chapter 4 presents basics of compression methods for multimedia. The short but important last section of the Chapter 4 contains evaluation methods of the decompressed information quality. International digital facsimile coding standards are described in Chapter 5.

Chapters 6, 7 and 8 present coding of pictures and image sequences.

Existing digital video-coding standards are summarized in Chapter 9.

Chapter 10 gives an introduction to speech-coding standards, including standards for mobile communications. Audio-coding standards are given in Chapter 11.

Chapter 12 is devoted to watermarking digital image and video data.

The book contains list of references, list of standards and a useful index.

It seems, the reviewed book successfully fills the gap in the Slovak and Czech literature. Reviewers can recommend *Multimedia communications* to any engineer, technician, student and researcher interested in this subject.

LEVICKÝ, D. *Multimedia Communications. Multimedia technology and Watermarking*. Košice: Elfa s.r.o., 2003.

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