INVESTIGATING THE RÖSSLER ATTRACTOR USING LORENZ PLOT AND LYAPUNOV EXPONENTS

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Abstract

To investigate the Rössler attractor, introduced in 1976 by O.E. Rössler [3], we used Lorenz plot to show deterministic character and designated the Lyapunov exponent to show the chaotic character of the system.

Keywords

Deterministic chaos, Rössler attractor, Lorenz plot, Lyapunov exponent

1. Introduction

The Lorenz Plot was used at first by Lorenz to show the deterministic nature of investigated strange attractor in Lorenz's system [2]. Lorenz concentrated his attention upon certain partial information produced by his numerical integration scheme by constructing the following plot [1], now called Lorenz plot: He iterated his integration scheme numerically, and then plotted successive maxima by of the coordinate $z(t)$ against one another. If we denote the successive maxima by z_1, z_2,…, z_n, then one plots z_{n+1} vs. z_n. The points appear to fall on a single curve, suggesting approximately an underlying one-dimensional map $z_{n+1} = f(z_n)$. The chaotic Colpitts oscillator [5] was also analyzed using these plots in [6], [7]. Now, as we mentioned, we used this tool to investigate the Rössler attractor, using one-step Eurel's method of numerical simulation. The used simulation software is in details described in [6]. The Lorenz plot proves the deterministic character of deterministic chaotic systems, which seems in the time domain stochastic (Fig. 2 and 4). On the other hand the positive largest Lyapunov exponent [8] means the presence of deterministic chaos and also causes the strange look of the attractor. The chaotic behavior of unknown system can be proved with very high certainty, using these two tools. In this article is this procedure applied at Rössler attractor.

2. The Rössler Attractor

O. E. Rössler introduced his equation in 1976 [3]. This equation has only single non-linear term (zx). The equation has the form

$$\begin{aligned} \dot{x} &= -(y+z) \\ \dot{y} &= x + ay \\ \dot{z} &= b + z(x-c) \end{aligned} \quad . \tag{1}$$

The original classical Rössler system has such parameters $a = b = 0.2$, $c = 5.7$. The phase trajectory for these parameters occurs in Fig. 1. With parameters $a = 0.343$, $b = 1.82$, $c = 9.75$ we have so called funnel attractor (Fig. 3).

The Lorenz plots for Rössler attractor with classical parameters and with parameters causing funnel attractor are in Fig. 2 and Fig. 4. The Lorenz plot expresses deterministic character; in this case the deterministic character of successive maxima.

In Fig. 5, there is the analysis of Rössler attractor for character of the largest Lyapunov exponent. In the test are numerically calculated two identical Rössler systems with little difference only in initial condition. If the difference arises exponentially the largest Lyapunov exponent is positive. As we mentioned in [8], the positive largest Lyapunov exponent in three-dimensional systems is sufficient condition for presence of deterministic chaotic behavior. The positive character of largest Lyapunov exponent is evident from Fig. 5.

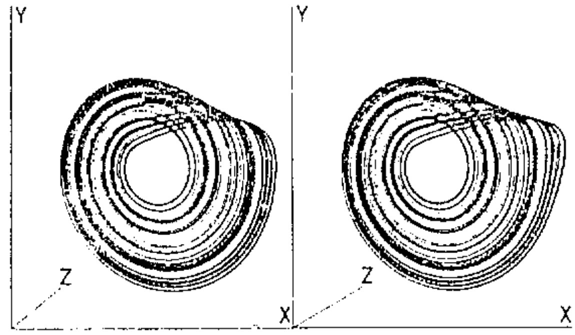

Fig. 1 Phase space trajectory of classical Rössler attractor. Stereoscopic view. Axes:-14…+14 for x and y, 0…28 for z. Assumed initial conditions: $x(0)=0$, $y(0)=-6.78$, $z(0)=0.02$. $t_{end}=339.249$. (after O.E. Rössler, 1976).

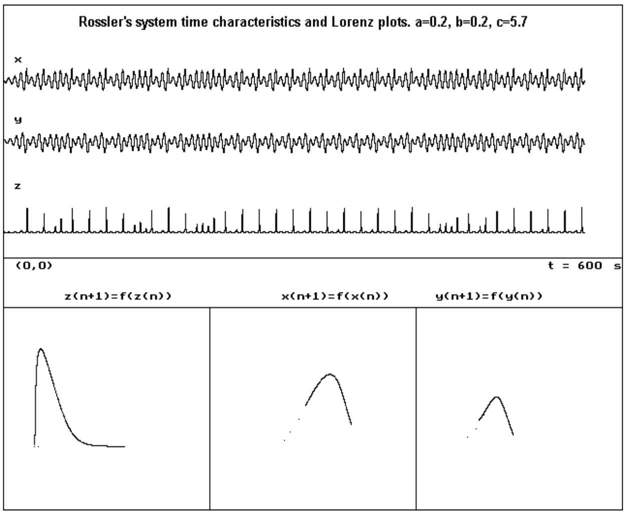

Fig. 2 Time characteristics and Lorenz plots for successive maxima for Rössler equations with original parameters.

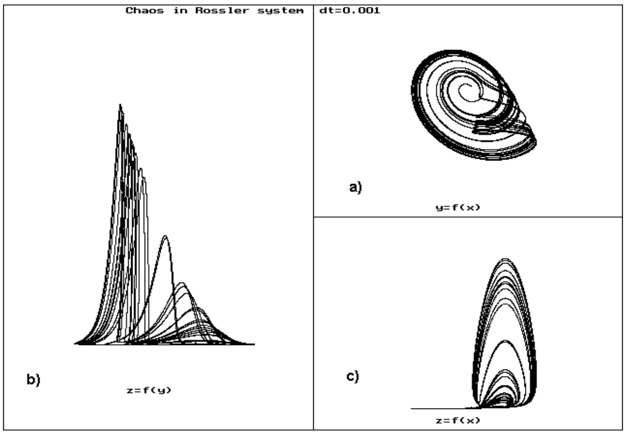

Fig. 3 Phase plain trajectories for Rössler's equations with parameters *a*=0.343, *b*=1.82 and *c*=9.75, (funnel attractor). Axes: a) -40…40 for *x* and -35…30 for *y*, b) -30…20 for *y* and -20…100 for *z*, c)-40…40 for *x* and -10…90 for *z*.

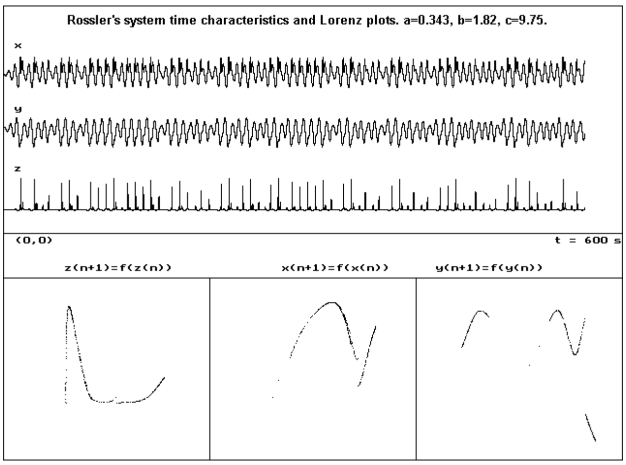

Fig. 4 Time characteristics and Lorenz plots for successive maxima for Rössler equations with parameters corresponding to funnel attractor.

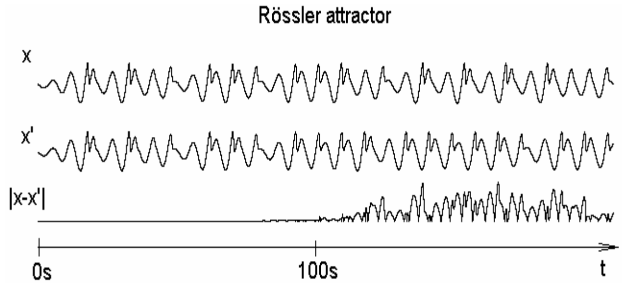

Fig. 5 Test for positive Lyapunov exponent for Rössler funnel attractor. The initial condition (IC) is **x**(0)=(1;1;1); the error in IC between *x* and *x*' is 1.10^{-4}%. The error in IC occurs at 100s, so the largest Lyapunov exponent is positive and its approximate value is 0.18 for funnel attractor and 0.09 for classical (for more details see [8]).

3. Conclusion

In Fig. 2 and 4, we showed the Lorenz plots for Rössler attractor with classical parameters and also with parameters for funnel attractor. These plots proved the deterministic character of measured system. In Fig. 1 and 3 are phase plain trajectories, where is well visible the strange attractor, specific for deterministic chaotic systems. In Fig. 5 is test for positive Lyapunov exponent, where two identical systems are simulated with small change in initial condition. We can see how the positive exponent causes that the initial error arises exponentially depending on time, in this figure. All these numerical tools proved deterministic chaotic character of examined Rössler attractor. The problems of Lyapunov exponent are in more details described in [8].

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