RAIN SCATTERING AND CO-ORDINATE DISTANCE CALCULATION

Martin HAJNÝ
Institute of the Atmospheric Physic
Bocni II/1401, 141 31 Praha 4,
Czech Republic
E-mail: hajny@ufa.cas.cz

Abstract

Calculations of scattered field on the rain objects are based on using of Multiple MultiPole (MMP) numerical method. Both bi-static scattering function and bi-static scattering cross section are calculated in the plane parallel to Earth surface. The co-ordination area was determined using the simple model of scattering volume [1]. Calculation for frequency 9.595 GHz and antenna elevation of 25° was done. Obtained results are compared with calculation in accordance to ITU-R recommendation..

Keywords

rain cell, rain, scattering, co-ordinate distance

1. Introduction

Theoretical background of using Multiple MultiPole (MMP) was described in [2]. The 3D Electrodynamics Wave Simulator [3], developed by Ch. Hafner's team at ETH Zurich and based on the Multiple MultiPole method was used for calculations.

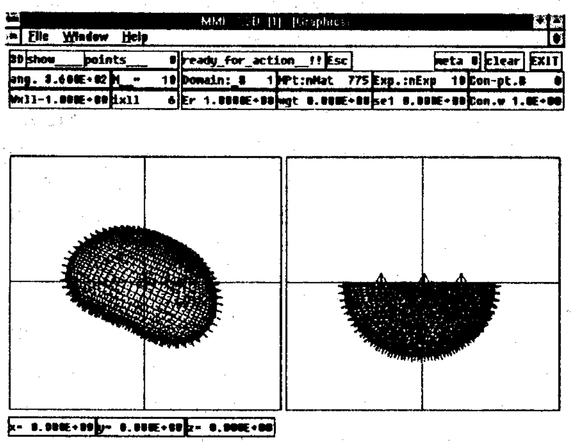

Fig. 1: The object of raindrop prepared for MMP method

Input files have been created for raindrops of the diameter from 0.5 mm to 8.0 mm with 0.5 mm step. One model of raindrop in the shape of Pruppacher-Pitter form [4] together with the location of multipoles shows Fig. 1.

2. Rain Scattering

For description of scattered field by single raindrop it is useful to define the scattering function from (2).

$$E^{s} = E^{i} f(\mathbf{K}_1, \mathbf{K}_2)\, r^{-1}\, e^{(-jkr)} \qquad (1)$$

$\mathbf{K}_1$ ... unit vector in the direction of the incident wave
$\mathbf{K}_2$ ... unit vector directed to the observation point
$f(\mathbf{K}_1, \mathbf{K}_2)$ ... scattering function (m), (cm)*
E^{s} ... electric field of a scattered wave (V/m)
E^{i} ... electric field of incident wave (V/m)
k_0 ... free-space propagation constant (m^{-1}), (cm^{-1})*
r ... distance to the observation point (m), (cm) *
* units alternatively used for computation

Bi-static scattering function describes the radiation of a single drop. Using (3) it is possible to define the bi-static cross section σ (m^2) for horizontal and vertical polarisation.

$$\sigma_{h,v}(D, \mathbf{K}_1, \mathbf{K}_2) = 4\pi \left| f_{h,v}(D, \mathbf{K}_1, \mathbf{K}_2) \right|^2 \qquad (2)$$

The bi-static cross section per unit volume of the rain area σ_{RAIN} (m^{-1}) can be calculated for given rain intensity (4):

$$\sigma_{RAIN}(R, \mathbf{K}_1, \mathbf{K}_2) = \int_0^{\infty} n(D,R)\, \sigma_{v,h}(D, \mathbf{K}_1, \mathbf{K}_2)\, \mathrm{d}D \qquad (3)$$

Where: R ... rain intensity, D... diameter of a drop
$n(D,R)$ ... drop size distribution DSD

The exponential Marshall-Palmer [5] raindrop size distribution (DSD) (5) is usually used. Parameters c, n_0 for chosen systems of units are given in Table 1.

$$n(D,R) = n_0\, e^{\frac{-c D/2}{R^{0.21}}} \qquad (4)$$

Table 1: Parameters c, n_0 for units:

R =(mm/h), D =(cm), k =(cm^{-1}), λ=(cm), f=(cm):			
n_o=(cm^{-4}):	0.08	c =(cm^{-1}):	82
R = (mm/h), D = (mm), k = (m^{-1}), λ=(m), f=(m):			
n_0= ($m^3 mm^{-1}$):	8 000	c = (mm^{-1}):	8.2

In the ITU-R documents [6] rain intensities for the required exceedance probabilities P are tabulated. These rain intensities then correspond to the reliability of link. Cumulative distribution of rain intensities for rain climatic zone **H** is given by $P(R \le 2\text{mm/h}) = 99\,\%$, $P(R \le 10\text{mm/h}) = 99.9\,\%$, $P(R \le 32\text{mm/h}) = 99.99\%$ and finally $P(R \le 83\text{mm/h}) = 99.999\,\%$.

Power scattered by rain

The scattered power is generally described by (6):

$$P_r = \frac{\lambda^2 P_t}{(4\pi)^3} \int \frac{G_t\, G_r\, \sigma_{RAIN}}{r_1^2\, r_2^2} \mathrm{d}V \qquad (5)$$

P_r ... power received in the observation point (W)
P_t ... power transmitted by transmitter (W)
G_t ... gain of transmitter antenna (-)
G_r ... gain of receiver antenna (-)
σ_{RAIN} ... bi-static cross section per unit volume (m^{-1})
r_1 ... distance from TX to the rain gravity centre (m)
r_2 ... distance from the gravity centre to RX (m)
λ ... wavelength (m)

Attenuation by diffraction over obstacle

Calculation of signal attenuation due to diffraction by Earth surface is not usually negligible for interfering emission computation. This theory was described in detail in [7], [8] and [9].

Model of scattered volume

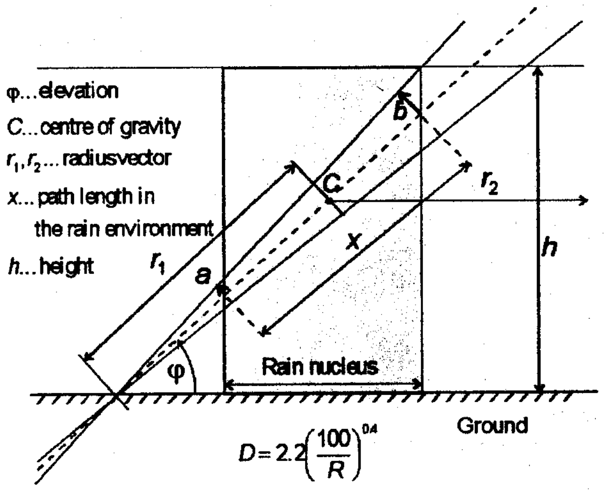

Fig. 3: Hajny-Fiser model of scattering volume

The simple Hajny-Fiser model of scattering volume [1] is based on the Assis and Einloft model [11]. The scattered volume is the intersection of antenna beam cone and rain cell. The height of the rain cell is important parameter in this model. The heights of zero isotherm (3.5±0.49) km for the average worst month for Prague were published in [12]. The worst case of height (4 km) were taken into calculation.

The distance r_1 (11) is derived from equilibrium rule (7). Figure 4 demonstrates this problem in detail.

$$r_c V = r_{c1} V_1 + r_{c2} V_2 \qquad (6)$$

Volume of 3dB antenna beam cone between transmitter and the end of rain cell:

$$V = \frac{1}{3} \pi b^2 r_b \qquad (7)$$

Volume of 3dB antenna beam cone between transmitter and the beginning of rain cell:

$$V_1 = \frac{1}{3} \pi a^2 r_a \qquad (8)$$

Volume of the intersection between an 3dB antenna beam cone and rain cell:

$$V_2 = \frac{1}{3} \pi \left(b^2 r_b - a^2 r_a \right) \qquad (9)$$

Distance from the transmitting antenna to the rain cell gravity centre:

$$r_1 = \frac{2}{3} \frac{b^2 r_b^{\,2} - a^2 r_a^{\,2}}{b^2 r_b - a^2 r_a} \qquad (10)$$

r_a... height of cone V_1
r_b... height of cone V_2
r_c... distance from TX to the volume V gravity centre
r_{c1}...distance from TX to the volume V_1 gravity centre
r_{c2}..distance from TX to the volume V_2 gravity centre
a ... radius of base of V_1 cone
b ... radius of base of V_2 cone

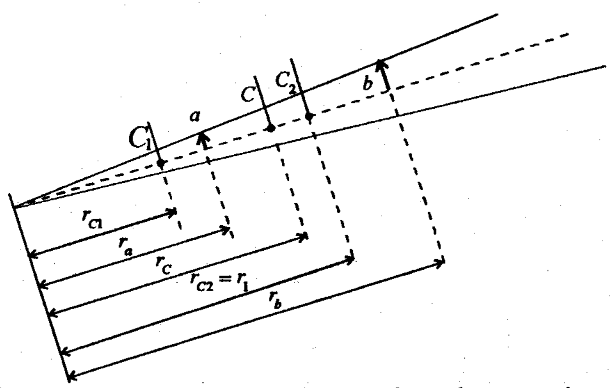

Fig. 4: Deduction of the distance from the transmitter to the rain media gravity centre.

3. Co-ordinate distance calculation and comparison of results

The problem of co-ordinate distance calculation for the propagation mode II study (scattering from hydrometeors) is described in [13]. Examples of these computations are presented for frequencies 9.595 GHz. The steps of calculation are given in Tables 3, 4, 5.

Table 3: Link parameters

frequency f	**9.595 GHz**
transmitted power P_t	15 dBW
diameter of TX antenna	6.35 m
diameter of RX antenna	2.4 m
gain of TX antenna G_t	53.5 dB
gain of RX antenna G_r (G_T)	45 dB
elevation of TX antenna φ	25°
RX noise temperature T_{rx}	160 K
reference bandwidth B	30 MHz
latitude of earth station ζ	50° 06' 05''
longitude of earth station δ	14° 26' 02''
radio-climatic zones	A2
rain climatic zone	H
percentages of time p	0.01 %
corresponding rain intensity R	32 mm/h

Table 4: Co-ordinate distance calculation ITU-R model:

link noise contribution N_L	1 dB
link performance margin M_S	4 dB
noise temperature of RX system T_e	160 K (T_{rx})
equivalence factor W	0 dB
threshold interference level $P_r(p)$	-129.0 dBW
min. permissible transmission loss L	144.0 dB
oxygen specific attenuation β_o	0.00693 dB/km
water vapour specific attenuation β_{vz}	0.00606 dB/km
max. hydromet. scatter distance d_{m2}	318.71 km
co-ordination distance d_r	**296.8 km**
offset of centred point Δd	**8.32 km**

Table 5: Co-ordinate distance calculation Hajny-Fiser model:

height of zero isotherm h * alternative height of zero isotherm	4000 m 3500 m*
antenna beam width ϑ	0.3445°
scattering cross section σ_{RAIN} 1) horizontal 2) vertical polarisation	1) $1.9692\cdot10^{-5}$ m^{-1} 2) $1.0774\cdot10^{-5}$ m^{-1}
diameter of rain cell D	3 470.3 m
distance to the gravity centre r_1	6 993.0 m 5 990.9 m*
height of the rain gravity centre h_1	2 955.4 m 2 531.9 m*
height of the RX antenna h_2	10 m
scattered volume V	$2.5326\cdot10^7$ m^3 $1.8164\cdot10^7$ m^3 *
horizon ray from the rain gravity centre d_a	194.2 km 179.7 km*
horizon ray from the RX antenna d_b	11.3 km
diffraction attenuation A	1) 16.89 dB 2) 14.47 dB
total oxygen attenuation β_o	1) 1.33 dB 2) 1.30 dB
total water vapour attenuation β_{vz}	1) 1.16 dB 2) 1.14 dB
co-ordination distance in the direction of the main antenna beam interfering station:	1) **228.19 km** 2) **222.90 km** 1) **215.06 km*** 2) **209.73 km***
shift of the gravity centre in the direction of the main antenna beam:	6.34 km 5.43 km*

Note: *) alternative height of zero isotherm h = 3500 m
1) horizontal polarisation
2) vertical polarisation

Comparison of obtained results for typical exceedance probabilities f = 9.595 GHz and φ=25° calculated by ITU-R Rec. with results calculated by CP51A09 model is given in Table 9. The relative difference of these models is also shown in percentages.

Table 9: Co-ordination distance d_r (km) calculated in the direction of the main beam of transmitting antenna in accordance with the ITU-R model [13] and CP51A09 [1] model (f = 9.595 GHz, elevation φ = 25°).

COORDINATION DISTANCE (km) / DIFFERENCE (%)					
% of time	ITU-R Rec.	Height of zero isotherm h = 4 000 m		Height of zero isotherm h = 3 500 m	
		HP	VP	HP	VP
0.1%	**272.06 km**	214.19 km	209.77 km	201.45 km	197.00 km
	0 %	**21.3 %**	**22.9 %**	**27.5 %**	**25.9 %**
0.01%	**296.8 km**	228.19 km	222.90 km	215.06 km	209.73 km
	0 %	**23.1 %**	**24.9 %**	**27.5 %**	**29.3 %**
0.001%	**309.48 km**	232.88 km	226.63 km	219.82 km	213.52 km
	0 %	**24.7 %**	**26.8 %**	**29.0 %**	**31.0 %**

Angular dependence of interferences in horizontal plane

The rain-scatter co-ordinate contour is determined by the ITU-R Rec. as a circle having radius d_r, and a centre pointed offset from the earth station along the main beam azimuth to the satellite as the distance Δd. In case of these calculations it was d_r=295.8 km and Δd=8.32 km. Used values: p=0.01 % (zone H: R=32 mm/h), P_t=15.0 dBW, D_{TXant}=6.35 m, D_{RXant}=2.4 m h_{RXant}=10 m, (G_t=53.5 dB, G_r=45.0 dB, for f=9.595). Results based on the model of scattered volume for the horizontal and vertical polarisation of transmitted wave are given in Fig. 6 and 7.

4. Conclusions

The values of bi-static scattering cross section per unit volume were calculated for Pruppacher-Pitter raindrops form [5], using the MMP numerical method.

The new model of rain scattering volume was proposed to estimate the level of interference signal. This model is based on Assis - Einloft prediction method where the concentric external rain cylinder having small residual rain rate is omitted. The results of co-ordinate distance calculations were compared with ITU-R method and differences were observed. The angular (azimuth) dependence of co-ordinate distance reflecting the radiation of a rain cell depends also on the chosen polarisation. Details are described in [14].

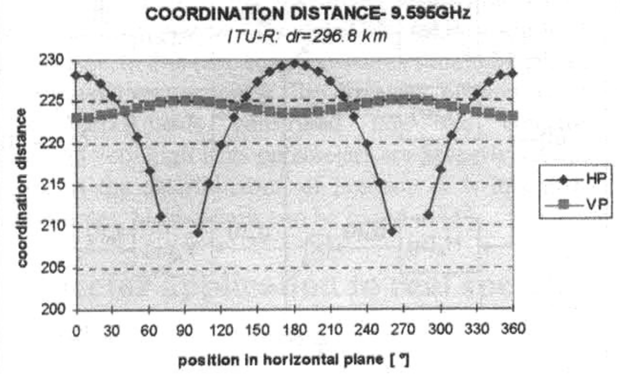

Fig. 6: Angular dependence of co-ordination distance (in the horizontal plane) for horizontal and vertical polarisation of transmitted wave, f=9.595 GHz.

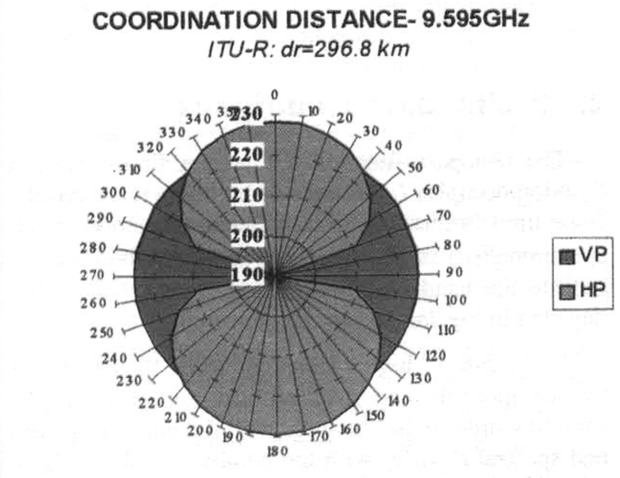

Fig. 7: Angular dependence of co-ordination distance (in the horizontal plane) for horizontal and vertical polarisation of transmitted wave, f=9.595 GHz.

Acknowledgements

The author would like to thank supervisor Dr. Miloš Mazánek from CTU Prague for his support during PhD. study as well as Dipl. Ing. J. Valenta from TESTCOM Prague for his help with co-ordinate distance calculations.

References

[1] Hajny, M.- Mazanek, M.- Fiser, O.: Bi-static Scatering Function - Radiation Patern Calculation, MC5 Meeting, COST 255, Vigo, May 1998, CP51A09

[2] Bazant, L.:The Computation of Forward Scattered Functions for Pruppacher-Pitter Raindrop Forms at 37 GHz, Radioengineering, April 1996, Vol. 5, No. 1, pp. 29-33

[3] Hafner, CH.- Bolmholt, L.: The 3D Electrodynamic Wave Simulator," New York, John Wiley and Sons, 1993

[4] Pruppacher, H. R.- Pitter, R., L.: A Semi-Empirical Determination of the Shape of Cloud and Rain Drops, Journal of the Atmospheric Sciences, January 1971

[5] Marshall, J., S.- Palmer, W., K.: The Distribution of Raindrops with Size, Journal Meteorology, No. 5, pp. 165-166, 1948

[6] CCIR Report 563-3, Doc.: Radiometrical data, pp. 82 Geneva April 1988

[7] Propagation in non-ionized media, Doc. 5/204-E, Conclusions of the interim meeting of study group 5, CCIR, Geneva, April 1988

[8] Schejbal, V.: Propagation over Irregular Terrain, Radioengineering, Vol. 6, No. 1, pp. 19-22, April 1997

[9] Hajný, M.- Mazánek, M.- Fišer, O.: Ku-band scattering parameters calculated by MMP method, First International Workshop on Radiowave propagation Modelling, 28-29 ctober 1998, ESTEC, Noordwijk, Netherlands, C255p18

[10] Hajný, M.- Mazánek, M.- Valenta, J.- Kvičera, V.: Prediction of Electromagnetic Wave Scattering for Co-ordinate Distance Calculation, 1st International Workshop on Radiowave Propagation Modelling, COST 255 project, ESTEC, Noordwijk, Netherlands October 1998, C255p42

[11] Assis, M., S.- Einloft, C., M.: A simple method for estimating rain attenuation distribution, Conference URSI, p. 301, La Baule 1977

[12] O. Fiser, "Interaction of Electromagnetic Field with a Rain Medium with Respect to the Satellite Link Attenuation Prediction" (in Czech), PhD. theses, Czech Technical University, Prague, 1986

[13] Determination of Co-ordination area of an earth station operating with a geostacionary space station and using the same frequency band as a system in a terrestrial service, Doc. 12/1007-E, draft revision of ITU-R IS.847, Geneva, November 1993

[14] Hajny, M.: Prediction of electromagnetic waves scattering in the rain volume, (in Czech), PhD. theses, Czech Technical University, Prague 1998

About author

Martin HAJNY was born in Náchod, in 1970. He received the MSc. degree in electrical engineering from the Faculty of Electrical Engineering at Czech Technical University in February 1996. Since March 1996 he has been Ph.D. student at this faculty. He took part the COST 235 and COST 255 project.