APPROXIMATION OF THE CHARACTERISTIC FUNCTIONS - USING THE FUZZY APPROACH

POKORNÝ, M.
VŠB - Technical University
Faculty of Electrical Engineering and Informatics,
Dept. of Measurement and Control,
17. listopadu, 708 33 Ostrava - Poruba
Czech Republic

Abstract

To approximate the real characteristic functions of the non-linear electronic elements various mathematical methods are used. Such methods offer the analytic approximating mathematical expressions as their results. One of the interesting non-conventional method for the approximation task is the approach of the fuzzy sets theory and fuzzy multivalue linquistic logic. To apply of this method we obtain the approximation functions of the non-linear fuzzy rule based form. To design of such fuzzy model we use the approaches of the fuzzy non-linear regression analysis. The article describes the structural and .parameters identification of such model and introduces the results of the numerical experiments.

Keywords:

fuzzy regression analysis, fuzzy conditional rule, fuzzy non-linear model

1. Introduction

To analyze of the non-linear electronic elements it is necessary to perform the approximation of its characteristic functions using the appropriate analytic expressions. The approximating function have to be a good substitution mainly in the field of the characteristic where the operating point of the non-linear element occurs. The rough approximations are often non-sufficient namely in this cases in which the low-level signals are consider.

To approximate the real non-linear characteristic functions using the mathematical analytic function a few classical methods exists. One of the interesting non-conventional methods for approximation is the method based on the fuzzy non-linear regression analysis. To apply this method we can to simply describe very strong non-linearity using the fuzzy *IF-THEN* rule based model. This method is effective namely in the multi-dimensional cases. To identify the structure and parameters of such fuzzy model we use the sampled data obtained by measuring of the element under study.

For example - the characteristic functions of the semiconductor triode are of the form of the multi-dimension functions. The output characteristic of such semiconductor triode is of the form

$$i_C = f(i_B, u_{CE}).$$

This non-linear dependence we can approximately substitute using two linear functions:

$$i_C = k_1.i_B + k_2.u_{CE} \qquad (1)$$

for the working field of the characteristic and

$$i_C = k_3.u_{CE} \qquad (2)$$

for the saturation field of the characteristic.

The fields of validity of the partial approximation functions *(1)* and *(2)* are described such

$$i_C = \begin{cases} k_1.i_B + k_2.u_{CE},\ 0 \le i_B \le 1/k_1(k_3 - k_2).u_{CE},\ u_{CE} \ge 0 \\ k_3.u_{CE},\ 1/k_1(k_3 - k_2).u_{CE} < i_B,\ u_{CE} \ge 0. \end{cases}$$

The approximation of the output characteristic function $i_C = f(i_B, u_{CE})$ is realized using two linear functions. The global approximation function is non-smooth and namely in the surroundings of the breakpoint is not adequate to the original characteristic function.

The above mentioned fuzzy approach makes possible to aggregate of both of the expressions (1),(2) into one smooth dependence using the fuzzy non-linear regression method method described in the next chapter.

2. Fuzzy non-linear approximation method

Let us consider the *n*- dimensional non-linear function of the form

$$y = f(x_j),\ j = 1,2,\ldots,n.$$

This function is described using the data set of the *m*- measured data

$$[x^0_{i1}, x^0_{i2}, \ldots, x^0_{in}; y^0_i],\ i = 1,2,\ldots,m.$$

Let us divide the n- dimensional input space of the input variables x_j into partial subspaces. In every partial subspace r, $r = 1,2,...,R$, a partial approximate linear function is valid. Such function is defined as the linear regression dependence of the form

$$y_r = k_{r0} + k_{r1}x_1 + k_{r2}x_2 + ... + k_{rn}x_n,\ r = 1,2,...,R. \quad (3)$$

Therefore, if the multi-dimensional non-linear relation exists, then in the various input variables subspaces exist the various linear regression relations (3).

To perform the smooth connections of all R- partial approximation relations (3) the fuzzy approach for the diversification of the input variables space is needed and the fuzzy logic method to aggregate the partial relations is used.

Every partial input subspace r is determined using the intervals of the input variables x_j in which the appropriate partial linear regression relation $y_r = f_r(x_j)$ is defined. This process is realized using the *IF-THEN* rules. The definition of the r-th partial subspace is realized using the structure of the antecendent part of the r-th rule

$$(x_1 is A_{r1})\ and\ (x_2 is A_{r2})\ and ...\ and\ (x_n is A_{rn})$$

where the A_{rn} are the fuzzy sets representing the linquistic fuzzy intervals of the n-th input variable in the r-th subspace. The consequent part of the r-th rule includes the appropriate linear function (3). Then the R_r rule is of the form

$$R_r:\ IF(x_1 is A_{r1})\ and\ (x_2 is A_{r2})\ and ...\ and\ (x_n is A_{rn})$$
$$THEN\ (y_r = k_{r0} + k_{r1}x_1 + k_{r2}x_2 + ... + k_{rn}x_n) \quad (4)$$

Thus, it is necessary to divide the input variables space and to define appropriate partial linear regression functions. The final fuzzy model of the global non-linear approximated function is created using the set rules (4) of the form

$$R_1:\ IF(x_1 is A_{11})\ and\ (x_2 is A_{12})\ and ...\ and\ (x_n is A_{1n})$$
$$THEN\ (y_1 = k_{10} + k_{11}x_1 + k_{12}x_2 + ... + k_{1n}x_n)$$
$$R_2:\ IF(x_1 is A_{21})\ and\ (x_2 is A_{22})\ and ...\ and\ (x_n is A_{2n})$$
$$THEN\ (y_2 = k_{20} + k_{21}x_1 + k_{22}x_2 + ... + k_{2n}x_n) \quad (5)$$

.

.

.

$$R_R:\ IF(x_1 is A_{R1})\ and\ (x_2 is A_{R2})\ and ...\ and\ (x_n is A_{Rn})$$
$$THEN\ (y_R = k_{R0} + k_{R1}x_1 + k_{R2}x_2 + ... + k_{Rn}x_n)$$

The model *(5)* is performed using the sampled data in the procedures of the structural and parameter identification described in the next chapter.

3. Identification of the fuzzy non-linear model

To identify the fuzzy non-linear regression model two processes of the identification are used, namely model structure and model parameters identification. Each of them concern either the premise or the consequent part of rules.

So the identification of the premise structure consists of the algorithm choosing the optimal input variables and the algorithm of the optimal partition of their input fuzzy space. The identification process is starting with the linear fuzzy model (input space undivided). Then the input variables are consequently chosen, their space is hierarchically divided and the partial sub-models are constructed. The sub-models are identified and the value of their lost function (the fitting criterion *FCR*) is calculated in the same time. To measure the fitting of the non-linear regression model the simple Kondo *FCR* criterion have been used of the form [1]

$$FCR = (100/N)\ \sum_{i=1}^{m} |y^0_i - y_i\ |/y^0_i \quad (6)$$

where $i = 1,2,...,m$ are number of samples. The optimal premise structure is achieved in this one with the smallest FCR value.

The vagueness of this fuzzy model is expressed using the sum J

$$J = c_1 + c_2 + ... + c_n.$$

The value of J have to be as small as possible. The identification of the consequent parameters k_{r0}, k_{rj} are computed by the linear programming method using the transformed variables z_j for the objective function *min J* and the constraints [2]

$$\alpha^T z_i + (1 - A) \sum_{j=1}^{n} c_j\ |z_{ij}\ | \geq y_i + (1 - A)e_i$$

$$-\alpha^T z_i + (1 - A) \sum_{j=1}^{n} c_j\ |z_{ij}\ | - y_i + (1 - A)e_i$$

$$z_{ij} = w_r x_{ij}$$

where $i = 1,2,...,m$ is number of the sampled data and w_r is the weight coefficients calculated by formula

$$w_r = \min_j\ [\mu_{rj}(x^0_j)].$$

The $\mu_{rj}(x^0_j)$ are the grades of membership of the observed data x^0_j to the fuzzy sets A_{rj} as the truth values of the rules.

There is necessary to notice, that it is possible to calculate the regression coefficients k_j of the fuzzy form $k_j(\alpha_j, c_j)$, where the α_j is the center value and c_j is the width of its fuzzy interval. The regression coefficient k_j is then expressed of the triangular fuzzy number form. In the mentioned approximating method the width c_j is not used and the regression coefficient k_j is of the crisp form $k_j \equiv \alpha_j$.

The structure identification of the consequent is performed using the simple comparation method. If the value of regression coefficients k_{rj} are lower than determined by decision-maker limit *KMIN*, then the variable x_j is implicitely eliminated from the consequent.

To identify the premises parameters the Sugeno-Kang method is used [1]. If the membership function of the fuzzy input variable values are formed by the straight lines, the parameters of a premise variable are the coordinates of the breakpoints described as a_k.

The premise parameters a_k are adjusted so that the error of the model output decreases using the observed data $[x_1^0, x_2^0, , ... x_n^0; y^0]$. Setting the error at zero, we can obtain the equation represents a linear line *e* as a set of optimal points which coordinates are the optimal values a^*_k. If the origin

values of the values a_k represent point W^0, then the corresponding optimal point W^*, representing the optimalised values a^*_k, is obtained as the nearest point to the point W^0 on the optimal line *e*.

4. Computer program module

To realize the discused approximation method the special computer program module called *NEFRIT (The Non-Linear Effective Fuzzy Regression Identification Tool)* have been used [2].

5. The practical numerical experiments

Two numerical experiments have been performed and the fuzzy approximation process have been tested.

A simple one-dimensional task

Let us consider a simple one-dimensional non-linear dependence $y = f(x)$ - see the *line 1* in *Fig.1*. Let the space of the independent variable x is divided into two parts, namely

x is *SMALL* (*S*),

x is *BIG* (*B*).

Let in subspaces S and B are defined the linear regression functions $y_1 = k_{10} + k_1x$ and $y_2 = k_{20} + k_2x$ respectively. Thus, we can define the non-linear regression model by describing the real function $y = f(x)$ using two Sugeno-Takagi conditional rules (4), $r = 1,2,$ of the form

R_1: IF (x is S) THEN $(y_1 = k_{10} + k_1x)$

R_2: IF (x is B) THEN $(y_2 = k_{20} + k_2x)$.

The global value of dependent variable y is computed using the partial values y_1 and y_2. To compute the partial values we have to put in the partial regression function the transformed variable z

$$y_r = k_{r0} + k_r z,\ r = 1,2,$$

$$z = w_r x,$$

where w_r is a weight coefficient computed for each of rules by formulas

$$w_1 = \mu_S(x^0)$$

$$w_2 = \mu_B(x^0)$$

as the grades of membership of the input data x^0 to linquistic value (fuzzy set) S and B respectively. The values w_1, w_2 present the truth values of the rules R_1 and R_2. The global output value y we obtain as the weighted average value by the formula

$$y = (w_1y_1 + w_2y_2) / (w_1 + w_2).$$

This fuzzy method gives the possibility of the smooth connection both of the approximating functions $y_1 = k_{10} + k_1x$ and $y_2 = k_{20} + \gamma + k_2x$ without using the third function $y_3 = k_{30} + k_3x$ which is necessary when a classical approximation method is used.

In the real experiment a simple V/A-characteristic $i = f(u)$ have been approximated - see *Fig.1*.

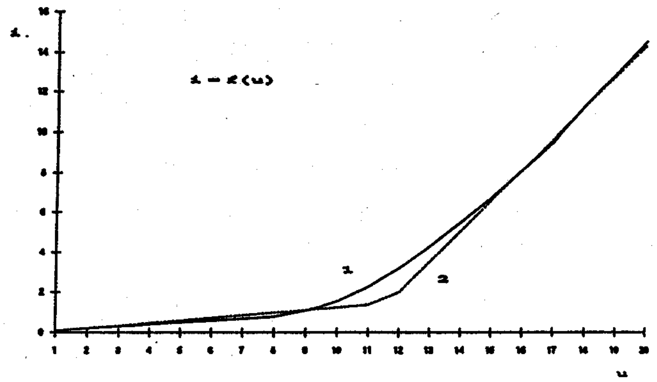

Fig.1

The input variable space have been divided into two subspaces - *SMALL (S), BIG (B)*. In every subspaces have been identified appropriate regression function $i_1 = f(u)$, $i_2 = f(u)$. The identification process have been performed using *20* sampled data. The final fuzzy regression model of the dependence $i = f(u)$, including two *IF-THEN* rule, have been identified of the form

R_1: *IF (u is S) THEN* $i_1 = -0,028 + 0,128u$

R_2: *IF (u is B) THEN* $i_2 = -16,462 + 1,538u$. (7)

The parameters a_k of the broken-line fuzzy sets S and B are of the form

u: S: $a_1 = 8,00$, $a_2 = 14,90$,

B: $a_3 = 8,10$, $a_4 = 15,00$.

The accuracy of the approximation is expressed using the fitting criterion *FCR* (6)

FCR = 1,83%.

The approximating function (7) is expressed in *Fig.1* using the *line 2*. To improve the value of the fitting criterion *FCR* we have to use the higher number of the linquistic values A_j of the input linquistic variables x_j - for example SMALL, MEDIUM, BIG. [Using the program system NEFRIT this number is limited ($j = 2$)].

A two-dimensional task

Now, provided the multi-dimensional case, it is needed to divide the fuzzy input space into suitable number of subspaces to be adequate sufficient. Then the fuzzy non-linear approximating model is created using *R*- rules of the form (5)

R_1: *IF* $(x_1$ *is* $A_{11})$ *and* $(x_2$ *is* $A_{12})$ *and ... and* $(x_n$ *is* $A_{1n})$

THEN $(y_1 = k_{10} + k_{11}x_1 + k_{12}x_2 + \ldots + k_{1n}x_n)$

R_R: *IF* $(x_1$ *is* $A_{R1})$ *and* $(x_2$ *is* $A_{R2})$ *and ... and* $(x_n$ *is* $A_{Rn})$

THEN $(y_R = k_{R0} + k_{R1}x_1 + k_{R2}x_2 + \ldots + k_{Rn}x_n)$

where A_m is the fuzzy set representing the linquistic value of the n-th input variable in the r-th rule. The premise part of each rules determines the subspace and the consequent involves linear approximating function which is valid in appropriate r-th subspace.

Now the global output value y is calculated using the partial output values y_r by formula

$$y = \sum_{r=1}^{R} w_r y_r / \sum_{r=1}^{R} w_r, \qquad r = 1,2,\ldots,R.$$

A two-dimensional task is presented using the parametric characteristic function $i_C = f(i_B, u_{CE})$. The identification process have been performed using *100* sampled data. Two-dimensional input space (variables i_B, u_{CE} with fuzzy linquistic values *SMALL (S), BIG (B)*) have been in the final optimal model divided into three subspaces and three *IF-THEN* rules have been identified of the form

R_1: *IF* $(u_{CE}$ *is* $S_2)$ *THEN* $i_C = 0,000 + 0,030 i_B + 0,162 u_{CE}$

R_2: *IF*$(i_B$ *is* $S_1)$*and*$(u_{CE}$ *is* $B_2)$ *THEN* $i_C = 0,171 + 0,878 i_B + 0,000 u_{CE}$

R_3: *IF*$(i_B$ *is* $B_1)$*and*$(u_{CE}$ *is* $B_2)$ *THEN* $i_C = 1,484 + 0,443 i_B + 0.062 u_{CE}$

The parameters of the broken-line approximated fuzzy sets S_1, S_2 and B_1, B_2 are of the form

i_B: S_1: $a_{11} = 3,70$, $a_{12} = 6,85$; u_{CE}: S_2: $a_{21} = 4,05$, $a_{22} = 13,60$,

B_1: $a_{13} = 4,15$, $a_{14} = 7,30$ B_2: $a_{23} = 4,05$, $a_{24} = 20,78$.

The fitting of the approximation is expressed using the fitting criterion *FCR*

FCR = 9,84%.

To improve the value of the fitting criterion *FCR* we have to use the higher number of sampled data or to increase the number of the linquistic values A_j as in the above mentioned case.

6. Conclusions

One of the fields, in which the fuzzy approaches are used, is the modelling of the non-linear systems behaviour. Such the fuzzy non-linear regression analysis appears to be one of the suitable methods for the approximating of the characteristic functions of the non-linear electronic elements. The numerical experiments give the results similar to the classical methods. The fittness of the approximation model to be sufficient if the appropriate number of the sampled data and the appropriate number of the linquistic values of the input linquistic variables are used. The mentioned fuzzy method is useful namely in the multi - dimensional cases.

References

[1] Takagi,T.,Sugeno,M.: Structure Identification of Fuzzy Model, Fuzzy Sets and Systems, 28, 1988

[2] Pokorný,M.: Vyuzití fuzzy metod v regresní analýze, Doctoral Thesis, Technical University of Brno, FEI, ÚAMT, 1994

About author, ...

Miroslav Pokorný was born in 1941. He received the Dipl.Ing. degree in 1963, PhD degree in 1994, both from the Technical University in Brno. Now he is Associate Professor of measurement and control systems with the Technical University in Ostrava. His main interests are in the fuzzy sets theory and its applications to the intelligent control systems.