LATTICE ALL-POLE STRUCTURE OF DIGITAL FILTERS AND ITS ZERO INPUT RESPONSE

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Abstract

The lattice structures for linear prediction were formulated in speech processing by Makhoul [1]. The LPC coders are often used in the transmission of speech signals. Such systems include a filter in the receiver that produces an approximation of a speech waveform. The parameters of this filter are extracted from the speech signal in the vocoder for frames of N samples. The lattice all-pole structure with two multipliers frequently occurs in such vocoders. The signal flow graphs (SFG) are very useful for a computation of the transfer function from the source nodes to the sink nodes.

Keywords:

Digital Filter, Lattice All-Pole Structure, Speech Synthesis, Signal Flow Graph .

1. Introduction

Linear prediction methods model the signal spectrum by an all-pole structure with a digital filter transfer function given by

$$H(z) = \frac{a_M z^M}{z^M + b_{M-1}z^{M-1} + b_{M-2}z^{M-2} + \ldots + b_1 z + b_0} = \frac{a_M z^M}{B_M(z)} \ . \tag{1}$$

It is assumed, without any loss of generality, that the leading coefficient b_M of $B_M(z)$ is one.

Makhoul showed the existence of a class of such lattice structures all of which have the following properties in common: (1) the resulting all-pole linear prediction filter is guaranteed to be stable (if for example the absolute values of every reflection coefficient are less than one); (2) stability and frequency responses are less sensitive to finite wordlength computations than canonic forms.

2. State-space description of the lattice all-pole structure

The lattice structure involves a cascade of particular sections. The ith section, referred to as **a digital two-pair**, is completely characterized by a parameter k_i, for $i = 1, 2, \ldots, M$. These parameters are known as **reflection coefficient**. The SFG of the ith section is shown in Figure 1.

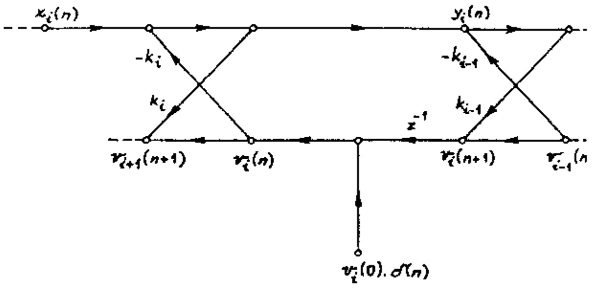

Fig. 1
The ith lattice section with two multipliers.

The difference state equations of the ith section have the form

$$y_i(n) = x_i(n) - k_i v_i(n) \ ,$$
$$v_i(n+1) = (1 - k_{i-1}^2) v_{i-1}(n) + k_{i-1} y_i(n) \ , \tag{2}$$
$$v_i(n) = (1 - k_{i-1}^2) v_{i-1}(n-1) + k_{i-1} y_i(n-1) + v_i(0)\delta(n)$$

....

$$v_{i+1}(n+1) = (1 - k_i^2) v_i(n) + k_i x_i(n) \ .$$

Consider the number of sections required to realize the transform function (1) is M. The SFG of the Mth order lattice all-pole structure we can see in Figure 2.

The state difference equations can be written as

$$v_1(n+1) = -k_1 v_1(n) - k_2 v_2(n) - \ldots - k_M v_M(n) + x(n) ,$$
$$v_2(n+1) = (1 - k_1^2) v_1(n) - k_1 k_2 v_2(n) - k_1 k_3 v_3(n) - \ldots$$
$$\ldots - k_1 k_{M-1} v_{M-1}(n) - k_1 k_M v_M(n) + k_1 x(n) \ ,$$

$$v_3(n) = (1-k_2^2)v_2(n) - k_2k_3v_3(n) - k_2k_4v_4(n) - \ldots$$

$$\ldots - k_2k_{M-1}v_{M-1}(n) - k_2k_Mv_M(n) + k_2x(n)\ ,$$

....

$$v_{M-1} = (1-k_{M-2}^2)v_{M-2}(n) - k_{M-2}k_{M-1}v_{M-1}(n)\ldots$$

$$-k_{M-2}k_Mv_M(n) + k_{M-2}x(n)\ , \tag{3a}$$

$$v_M(n+1) = (1-k_{M-1}^2)v_{M-1}(n) - k_{M-1}k_Mv_M(n)\ldots$$

$$+k_{M-1}x_{M-1}(n)\ .$$

....

$$y(n) = -k_1v_1(n) - k_2v_2(n) - \ldots - k_Mv_M(n) + x(n)\ . \tag{3b}$$

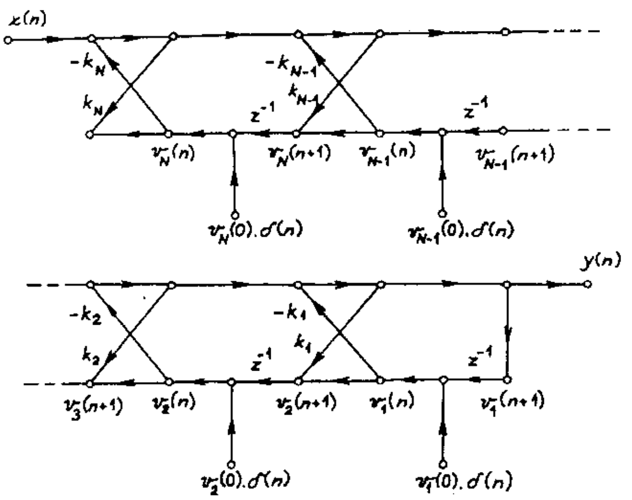

Fig. 2
Mth order lattice all-pole structure of digital filters.

These state difference equations (3) can be arranged into the matrix form - **the discrete state dynamic equations**

$$\mathbf{v}(n+1) = \mathbf{A}\,\mathbf{v}(n) + \mathbf{B}\,x(n)\ ,$$

$$y(n) = \mathbf{C}\,\mathbf{v}(n) + \mathbf{D}\,x(n)\ . \tag{4}$$

For example, the matrix of reflection coefficients **A** has the form

$$\mathbf{A} = \begin{bmatrix} -k_1 & -k_2 & -k_3 & \ldots & -k_{M-2} & -k_{M-1} & -k_M \\ (1-k_1^2) & -k_1k_2 & -k_1k_3 & \ldots & -k_1k_{M-2} & -k_1k_{M-1} & -k_1k_M \\ 0 & (1-k_2^2) & -k_2k_3 & \ldots & -k_2k_{M-2} & -k_2k_{M-1} & -k_2k_M \\ \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \ldots & (1-k_{M-2}^2) & -k_{M-2}k_{M-1} & -k_{M-2}k_M \\ 0 & 0 & 0 & \ldots & 0 & (1-k_{M-1}^2) & -k_{M-1}k_M \end{bmatrix}$$

It is clear that matrix equations (4) are convenient in the case of a computer solution. If somebody wants to process the input data in line procedure, then he must use a special hardware, which for example is a digital signal processor. Therefore, the state difference equations (3) can be solved with the help of the unilateral z-transform and can be written into a more suitable form including the initial conditions $v_1(0), v_2(2), \ldots, v_M(0)$

$$v_1(n) = -k_1v_1(n-1) - k_2v_2(n-1) - \ldots$$

$$\ldots - k_Mv_M(n-1) + x(n-1) + v_1(0)\delta(n)\ ,$$

$$v_2(n) = (1-k_1^2)v_1(n-1) - k_1k_2v_2(n-1) -$$

$$k_1k_3v_3(n-1) - \ldots - k_1k_Mv_M(n-1) +$$

$$k_1x(n-1) + v_2(0)\delta(n)\ ,$$

$$v_3(n) = (1-k_2^2)v_2(n-1) - k_2k_3v_3(n-1) -$$

$$k_2k_4v_4(n-1) - \ldots - k_2k_Mv_M(n-1) +$$

$$k_2x(n-1) + v_3(0)\delta(n)\ ,$$

$$\ldots \tag{5}$$

$$v_M(n) = (1-k_{M-1}^2)v_{M-1}(n-1) - k_{M-1}k_Mv_M(n-1) +$$

$$k_{M-1}x(n-1) + v_M(0)\delta(n)\ .$$

Figure 2 shows the SFG according to equations (3) or (5). The SFG is useful for computing the transfer function from some source nodes to a sink node. To do this, it is convenient to use **Mason' gain formula**. If $Y(z)$, resp. $X(z)$ are the z-transform of $y(n)$, resp $x(n)$, then the solution of the difference equations (3) can be written as

$$Y(z) = H(z)X(z) + \frac{1}{B_M(z)}\sum_{i=1}^{M} H_i(z)v_i(0)\ . \tag{6}$$

The first term of the equation (6) is usually called the zero state response (ZSR), ([2]). The total response $Y(z)$ can be considered as the sum of the ZSR and the zero input response (ZIR) (the remaining term of (6) with an addition). There is an example of the transfer functions computation in the last part of this paper, which combines the above results.

3. Example

It is required to compute the coefficients of all transfer functions of (6), when the reflection coefficients of the fourth-order lattice structure are given

$$k_1 = 0{,}4382651448\ ,\quad k_2 = -0{,}8889359911$$

$$k_3 = 0{,}9702683906\ ,\quad k_4 = -0{,}9788207224$$

Figure 3 gives the lattice all-pole structure of the fourth order.

The z-transform of output signal has the form

$$Y(z) = H(z)X(z) + \frac{1}{B_4(z)}\sum_{i=1}^{4} H_i(z)v_i(0)\ .$$

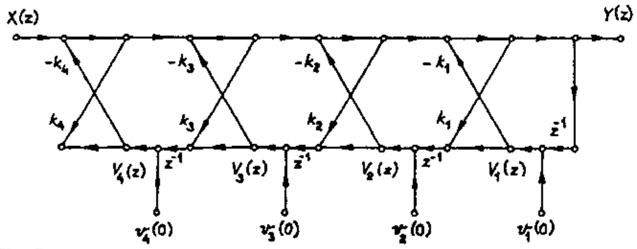

Fig. 3
Fourth-order lattice all pole structure.

The coefficients of all transfer functions can be obtained by using the Mason's gain formula. The transfer function $H(z)$ is equal to

$$H(z) = \frac{a_4 z^4}{z^4 + b_3 z^3 + b_2 z^2 + b_1 z + b_0} = \frac{a_4 z^4}{B_4(z)} \quad .$$

$a_4 = 1$,

$b_4 = k_1 + k_1 k_2 + k_2 k_3 + k_3 k_4 = -3,180635683$,

$b_3 = k_2 + k_1 k_3 + k_1 k_2 k_3 + k_2 k_4 + k_1 k_3 k_4 +$

$k_1 k_2 k_3 k_4 = 3,861190467$,

$b_2 = k_3 + k_1 k_4 + k_1 k_2 k_4 + k_2 k_3 k_4 = -2,112154181$,

$b_1 = k_4 = 0,4382651448$.

The other transfer fuctions of the initial conditions have these coefficients

$H_1(z) = -(c_4 z^4 + c_3 z^3 + c_2 z^2 + c_1 z)$,

$c_4 = k_1 = -0,9788207224$,

$c_3 = k_2 + k_1 k_2 k_3 + k_1 k_2 k_4 + k_1 k_3 k_4 = 1,779617403$,

$c_2 = k_3 + k_2 k_3 k_4 = -1,266942524$,

$c_1 = k_4 = 0,4382651448$.

$H_2(z) = -(d_4 z^4 + d_3 z^3 + d_2 z^2)$,

$d_4 = k_2 = -0,9702683906$,

$d_3 = k_2 + k_2 k_3 k_4 = -1,266942524$,

$d_2 = k_4 = 0,4382651448$.

$H_3(z) = -(e_4 z^4 + e_3 z^3)$,

$e_4 = k_3 = -0,8889359911$,

$e_3 = k_4 = 0,4382651448$.

$H_4(z) = -(f_4 z^4)$,

$f_4 = k_4 = 0,4382651448$.

4. Conclusion

To better understand the influence of the initial conditions in the LPC synthesis of speech signal, we can use the SFG description and the solution of difference equations by means of z-transform. The Mason's gain formula is very useful for the determination of all transfer functions.

5. References

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[2] SIEBERT, W. McC.: Circuits, Signals, and Systems. The MIT Press, McGraw-Hill Book Company, Inc., New York, 1986.