SOME PROBLEMS IN ELABORATION OF EQUIVALENT ELECTRICAL SCHEMATICS OF LINEAR MECHANICAL SYSTEMS

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Abstract

The solution of different acoustical signal processing problems in electroacustic transducers are usually based on the method of electromechanical analogies.

Finding of equivalent electrical schematics of mechanical systems as a planar graphs has some limitations in first analogy, (force - voltage, velocity - current). Systematic analysis and general conclusions about solvability of this problem are made on the basis of Kuratowski's Theorem. Some examples of solution are included.

Keywords:
electromechanical analogy, equivalent electrical network, (circuit) electromechanical transducer

1. Introduction

The method of electromechanical analogies is of very great importance in electroacoustics. Using this method we can draw the analogue equivalent electrical networks for such complicated devices as microphones, loudspeakers and loudspeaker systems, headphones, accelerometers and a lot of other devices based on the transformation of electrical signals into mechanical and vice versa by using of electromechanical transducers. In this equivalent electrical circuit we can solve all problems of signal processing including signal transmission and transformations which are important in electroacoustics. In addition we can use this method for solution of the variety of pure mechanical vibratory problems in engineering, building acoustics and room acoustics. The equivalent circuits approach is suitable for theoretical analysis of the sound transmission and sound reduction processes in different building structures, as for example partitions, windows etc. as well as for the computation and measurements evaluation of the sound transmission loss values.

The solution of mechanical vibratory problems similarly as the solution of problems in electrical circuits, results in the system of linear differential equations. Mathematical formalism for modeling these physically different processes are the same.We receive in this way the same equations for linear mechanical systems as well as for linear electrical circuits. This is the actual base for introducing of electromechanical analogies and equivalent electrical circuits of mechanical systems. This is of very great importance, as we can in this way use the more elaborated theory and methods of solwing electrical networks for solution linear mechanical systems. We can also apply this method in the field of electromechanical transducers, on mechanical side of which we use in most cases membrane as linear mechanical system.

2. Electromechanical analogies

If we go out from basic electrical elements capacity C, inductivity L and resistance R in electrical networks and basic physical quantities voltage U and current I we can introduce two possible analogies:

- force (F) — voltage (U)
- velocity (V) — current (I)
- compliance (C_m) — capacity (C)
- mass (m) — inductivity (L)
- mech. resistance (r) — elec. resistance (R)

This analogy is more frequently used in comparison with the second one, which is based on relations:

- force (F) — current (I)
- velocity (V) — voltage (U)
- compliance (C_m) — inductivity (L)
- mass (m) — capacity (C)

- mech. resistance (r) elec. resistance (R)

If we suppose harmonic electrical and mechanical signals the definitions of electrical and mechanical impedances Z_e and Z_m are possible in form

$$Z_e = \frac{U}{I} \tag{1}$$

and

$$Z_m = \frac{F}{V} \ , \tag{2}$$

where U, I, F, V, Z_e, Z_m are in general complex quantities.

In the first analogy we can find the correspondence

$$Z_e \rightarrow Z_m \tag{3}$$

and in the second analogy

$$Z_e \rightarrow \frac{1}{Z_m} \ . \tag{4}$$

It is important to say that the application of the first analogy results in the dual topology of the equivalent electrical circuit in comparison with the topology of the original mechanical network. On the other side application of the second analogy does not change the topology of the equivalent circuit but we can find here the inversion of impedances according to relation (4).

3. Restrictions of first analogy

In the case when we use the first analogy for to drawing analogue electrical circuits of simple mechanical systems, we usually does not meet any serious problems. We have only to elaborate easy and connvenient procedure how to find the correct dual topology of the schematics of mechanical systems. Such methods and procedures are described in literature [4], [5], [6]. But in more complicated cases we can meet some problems.

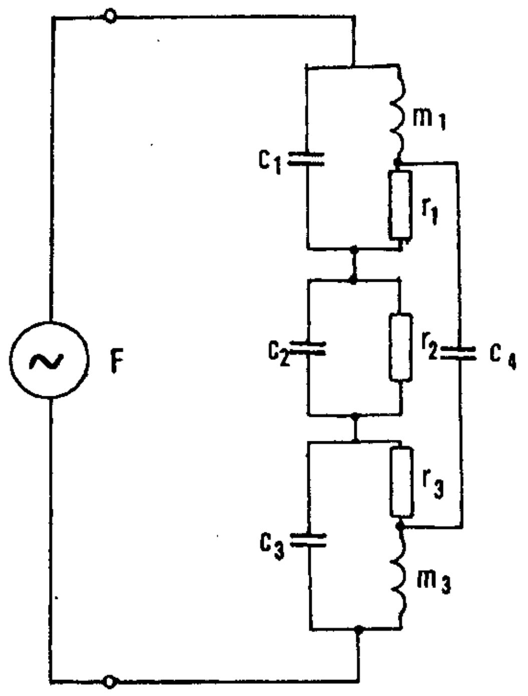

Fig.2
Equivalent electrical circuit of mechanical system on Fig.1

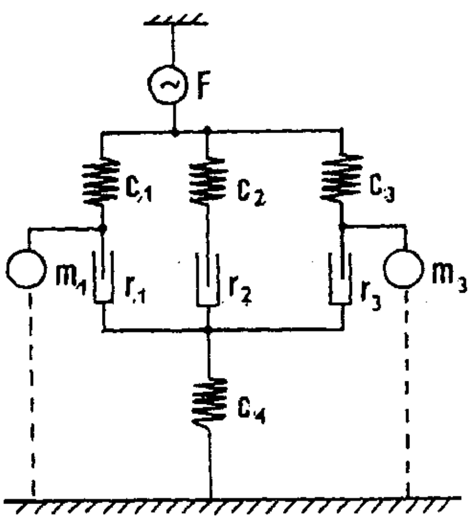

Fig.1
Mechanical schematic on the level of planar graph

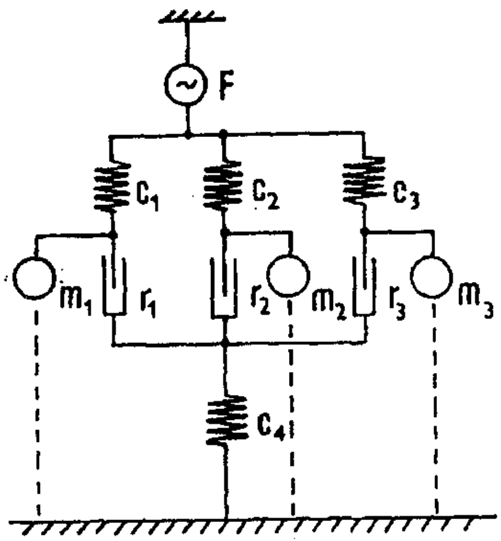

Fig.3
Mechanical schematic on the level of non planar graph

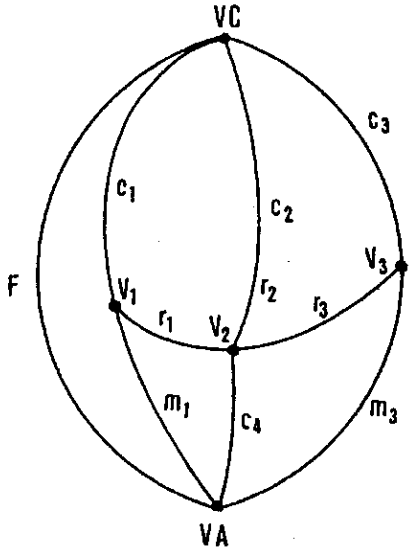

Fig.4
Graph diagram for mechanical system on Fig.1

Let us first consider the mechanical circuit in Fig.1 where the usual graphical symbols for the mechanical compliance, mechanical resistance and mass are applied.

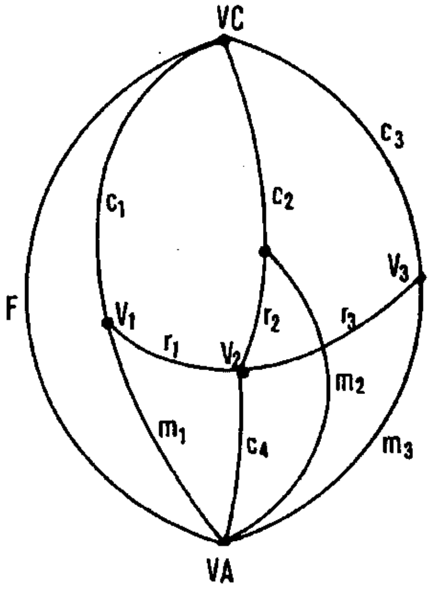

Fig.5
Graph diagram for mechanical system on Fig.3

The equivalent analogue electrical circuit is in Fig.2.

Now when we add the third mass m_3 according to Fig.3. we shall not be able to find and draw the planar analogue electrical circuit for this mechanical system.

The mention about the possibility of such problems we can find in [5], but it lacks for a systematic teoretical analysis of the problem as well as conclusions. In fact this problem can be solved by using theory of graphs and Kuratowski's Theorem [1], [2], [3]. If we depict topology of mechanical schematics in Fig.1 and Fig.3 by the graph diagrams we receive Fig.4 and Fig.5.

We can see that the graph diagram of mechanical system in Fig.1 is the planar graph in contrary with the graph diagram in Fig.5 which is actually non planar graph. According to Kuratowski's Theorem in this case equivalent dual electrical schematics does not exist [1].

4. Conclusion

The statement, that the mechanical schematics with non planar topology have not the equivalent electrical networks is very valuable, as we have at our disposal the objective method to ascertain for which mechanical schematics it is not possible to find and draw equivalent electrical circuits on the basis of first analogy. This method defines the class of topologies which lead to problems in first analogy. In similar cases we have only the possibility to apply the second analogy. This is equal from the point of view of solvability. In this analogy mechanical schematics and equivalent electrical schematics have the same topology so that equivalent electrical schematics must exist.

5. References

[1] Ducko, A. - Pichňa, R.: Some limitations of electromechanical analogies. Elektrotechnický časopis, Vol.42, 1991, N.11-12, pp. 606-608.

[2] Recski, A.: Matroid Theory and its Applications in Electric Network Theory and in Statistics. Budapest, Akademiai Kiado, 1989.

[3] Shu-Park, Ch.: Introductory Topological Analysis of Electrical Networks. Holt, Rinchart & Winston, Inc., New York, 1969.

[4] Merhaut, J.: Teoretické základy elektroakustiky. Praha, Academia , 1971.

[5] Ducko, A.: Elektroakustika I. ES SVŠT, Bratislava, 1990.

[6] Skudrzyk, E.: Simple and Complex Vibratory Systems. The Pennsylvania State University Press, 1968.

[7] Simonyi, K.: Theoretische Elektrotechnik, 8. Auflage. Berlin, VEB Deutscher Verlag der Wissenschaften, 1980.

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Aristid Ducko was born in 1932 and received his MSc degree in electrical engineering from Slovak Technical University in Bratislava in 1954 and Ph.D. degree in electroacoustics from Czech Technical University in Prague in 1969. He is now the head of Department of Radioelectronics, Faculty of Electrical Engineering, Slovak Technical University in Bratislava.