Fractal Dimension Analysis And Fractal Properties of Financial Markets
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ABSTRACT
Financial markets are a subject of many studies and research purposes. The ability to correctly observe and describe the dynamics of financial markets would mean the possibility to predict its future behaviour. Many approaches and methods have been used to achieve this objective. In recent history it has come to light that many natural phenomena could be described using fractal geometry. Alongside with the fractals can be often found the application of fuzzy logic and also the theory of chaos. These three concepts are recently very powerful in describing complex system and getting closer to understanding them. This paper deals with the basic concepts of fractal analysis and also fuzzy logic and the possibility to use those to observe financial markets and analyze their behaviour. The purpose of this is to present the reader with introduction to the problematics and propose starting concepts to begin the research into the matter.

INTRODUCTION
Understanding and predicting of development on financial market has been a subject of great interest for several decades. Many techniques and approaches has been used for this purpose. And with ever growing technological progress more and more sophisticated methods are being examined. Numerous statistical and mathematical methods are being implemented within various algorithms and in powerful computing devices. The focus of many scientists, developers and also professional traders recently turned to artificial intelligence and also neural networks. These are believed to posses the ability to predict the behaviour on any financial markets through complex analysis of data and the ability to learn and develop itself. While many people believe it is possible to gain an edge through this methods, others remain sceptical. This paper deals with the one of said method to forecast behaviour on markets through complex analysis using fractals and fuzzy logic depending on the fact that financial markets posses many properties of fractal and e analised as such.

1 HISTORY OF FRACTALS AND FRACTALS IN NATURE
For most part of the modern history everything that was necessary to be described by mathematics and geometry was sufficiently described by Euclidean geometry. Euclidean geometry was published by greek mathematician Euclides and is based on axioms and definitions. In 19th century new approach has come to light. Karl Weierstrass, Helge von Koch and Benoit Mandelbrot are names that are considered to be the very beginning of the fractal geometry. Mandelbrot's name is one of the most associated with the discovery of this new approach which is was heavily caused by his access to computers and also the knowledge of his fellow colleagues research. As Mandelrot was forced to leave his native Germany during war he moved to New York. There he started working in IBM in Thomas J. Watson's research centre, where he was given free will as where he will aim his research. In 1967 his paper *How Long Is the Coast of Britain? Statistical Self-Similarity and Fractional Dimension* was published. In this paper he claimed that the coast is showing self-similarity. After the paper he returned to work with computers and along with the contribution of work of Julia and Fatou he was for the first time able to see how these sets of self similar looked like in their limits. He was able to produce the Mandelbrot set, formally described as $M = \{c \in \mathbf{C} \mid f_c^{(n)}(z) \text{ is finite as } n \rightarrow \infty\}$.

When one zooms in the Mandelbrot set, one can observe that this object is indeed self similar. Also if one zooms in further in certain sections around the edges one can get Julia sets.

Benoit Mandelbrot is considered the inventor of fractal geometry and he also popularized it heavily thorough is numerous applications of it to many other areas of science. In the paper *How Long Is the Coast of Britain?* fractals are used to represent natural phenomena such as coastlines, trees, snowflakes which would be described in Euclidean geometry with great difficulties. Fractal geometry also plays an important role, alongside the chaos theory, in physics, medicine and population dynamics.

Fig. 1. The Mandelbrot set (Trochet, 2009)

2 FRACTAL DIMENSIONS

As mentioned above the fractal geometry serves the purpose of helping better comprehension of complex system found in nature which are characterized by its fractal properties. Among these properties are self-similarity and self-affinity and they can be found in nature textures. This concept was a great help to create models of nature objects because it can characterize chaotic systems.

Fractal dimensions describe self similarity or also structural complexity of many different natural phenomena such as temporal signals. Determination of these is based on the information sets. Fractal analysis as a whole is being used to describe dynamic systems and has found many application thorough various fields. One of applications can be seen in the analysis of a time series where a fractal dimension is able to describe regularities in longer time frames.

There are more ways to determine the fractal dimension. One of the most intuitive methods to do so is shown in the Fig.2. below and is called the box method.
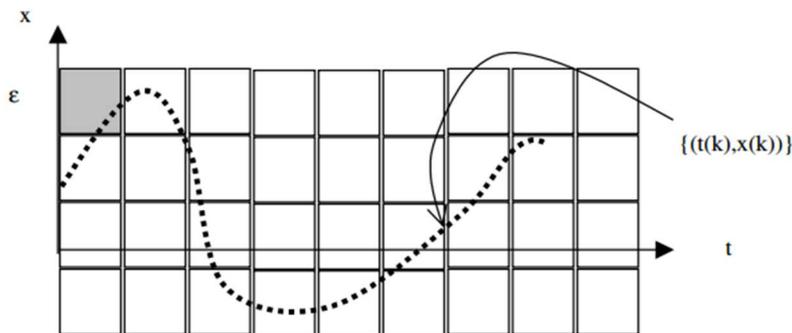

Fig. 2. The box method for determination of fractal dimension (Pedrycz, Bargiela, 2001).

In the case of higher dimension rate there unfortunately arise problems with computing power. This is usually met with an approach of constructing hyperspheres around the individual points of data sets.
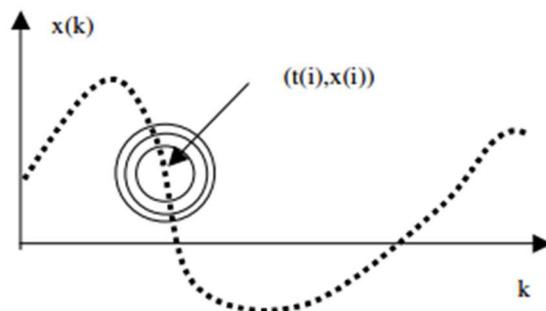
Fig. 3. An example of box counting leading to the determination of the fractal dimension (Pedrycz, Bargiela, 2001).

To show the relationship between the fractal dimension and chaos, there is a way to describe a chaos level while using the fractal dimension. However the theoretical background covering the fractal analysis and its possible applications is insufficient to help with the evaluation of the fractal dimension, more so if there is a multidimensional set being evaluated. (Dohnal, 1991)

Another method for estimating fractal dimension was proposed by Zeng, Koehl and Vasseur in the paper *Design and implementation of an estimator of fractal dimension using fuzzy techniques* (Zeng, Koehl, Vasseur, 2000). This paper focuses on estimating the fractal dimension of one-dimensional fractals. The approach considers the real fractal dimension as a continuous function and is approximating this function to obtain estimations of a fractal dimension by using fuzzy logic controllers.

3 FUZZY KNOWLEDGE

Knowledge found in the field of engineering and complex research are rather symbolic and quantitative. To perform analysis of a knowledge set that is symbolic one needs to take into account the structure of the knowledge base. Structure can be divided into two different types - the relative structure, which has relevance to a specific query and the absolute structure, which is a default property of the knowledge set. Any problem or situation that comes with the roots in real life have the same knowledge as these and can be described as fitting into more categories at the same time, which a different degree of fitting into each category. Any operation with this quantitative knowledge requires a certain amount of manipulation with them to put them in a form where it is better possible to analyze them. Every manipulation with these knowledge sets comes with the price of loss information and therefore the efforts are often being made to make as little manipulation as possible. To achieve this there is a need to use a very wide selection of quantitative calculi, such as fuzzy, rough sets or probability (Dohnal, 1991). From these calculi it was proven in practice by usage in industrial business that fuzzy models are very convenient for that purpose, especially for being easy to understand and apply with no particularly deep knowledge of mathematics. Also they can be easily and quickly altered.

The theory of fuzzy set is based on the fact that the elements that humans thinking is formed by are words, not a numbers. Therefore it cannot be quantified. The fuzzy sets allow us the possibility of non exact data values to be analyzed. It is said that it allow the existence of a certain uncertainty.

4 FRACTAL PROPERTIES OF FINANCIAL MARKETS

Fractal geometry has been shown and proved to be efficient tool for description of many effects found in nature that are hard to define with standard geometry. The possibility to use fractals to understand and describe financial markets has been examined by many.

Similarly to the coastline previously compared to fractal by Benoit Mandelbrot the development of price on financial markets represented by a curve can be observed as a fractal on a long term period. Price development is essential element of the development on financial markets, however beside it there is an element of trading volume. Trading volume is an exact count of stocks, or commodity, that changed their owner in a predefined timeframe and it is an important part of financial markets to observe in order to understand its dynamics. To give an example of the role of traded volume in the history of stock and commodity market, there is an old Wall Street saying " It takes volume to make price move". Multifractal structure of 1- minute traded volume time series was examined by Moyano, Souza and Queiros on the data from July 1st 2004 to December 31th 2004 (Moyano, Souza, Queiros, 2006).

5 CASE STUDY AND DISCUSSION

The study carried by Salim Lahmiri in April 2016 (Lahmiri, 2016) is examining a behaviour on the financial markets of crude oil and its chaotic properties. The study focused on two time periods, before and after the international crisis in 2008. Method used in this paper was a Lyapunov exponent which was estimated for prices, returns and volatilities on the said market. While the measured values indicate no chaos exists in the prices and returns in either of said time periods, this paper presents strong evidence of the dynamics of chaos in volatilities after the international crisis.

The data used for this paper were obtained from Federal Bank of Saint Luis and the market price of said commodity, which is crude oil, is represented in Fig. 4, the returns are in Fig 5. and finally volatilities are represented in Fig 6.
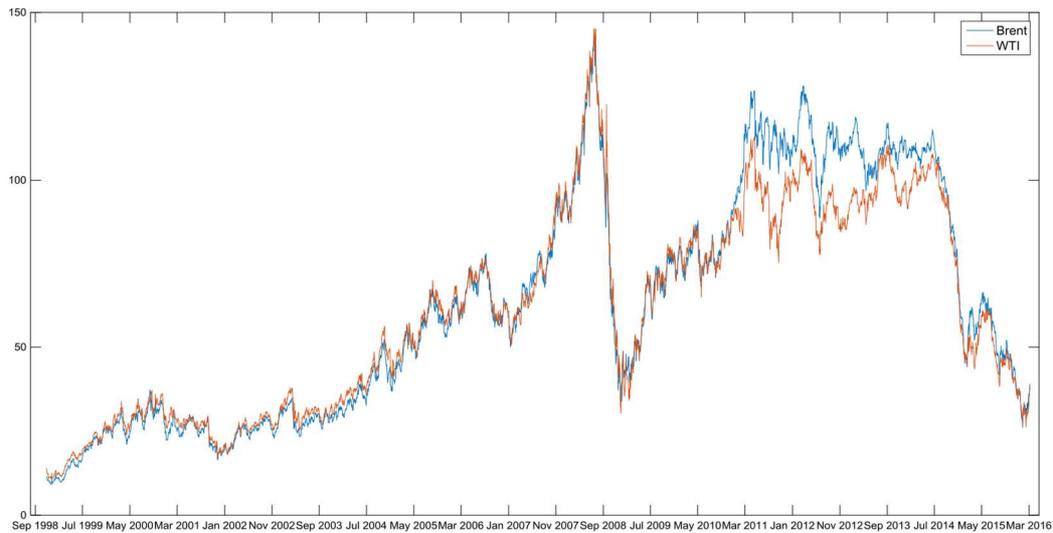
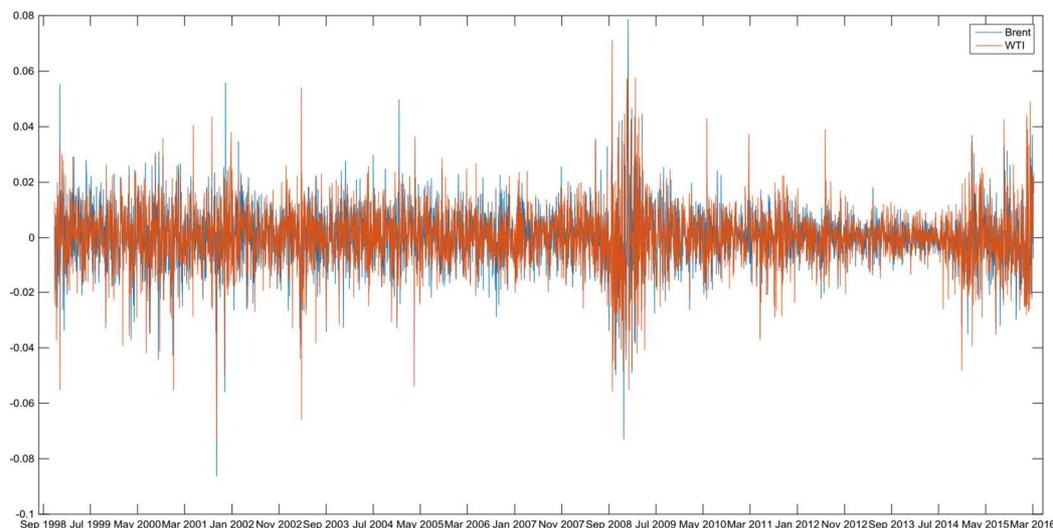Fig. 4. Evolution of WTI and Brent prices (Lahmiri, 2016)

Fig. 5. Evolution of WTI and Brent returns (Lahmiri, 2016)

The figures representing prices and returns propose that these are stable systems, consider the chaos theory. And it is possible to state that these are predictable since they are not chaotic.

The figure showing the volatility series clearly presents that the volatili is not a chaotic system until the year 2008 where the financial crisis appeared. However after the year 2008 it appears to be largely chaotic, therefore an unstable system which could be proclaimed as unpredictable due to its chaotic behaviour.

To conclude it, financial crisis does not cause unpredictability of pricer nor return because they were not chaotic before the crisis and not after it. Howerew in the case of volatility it is so, that the financial crisis can inflict the behaviour of chaotic system in the market and therefore make it unpredictable.
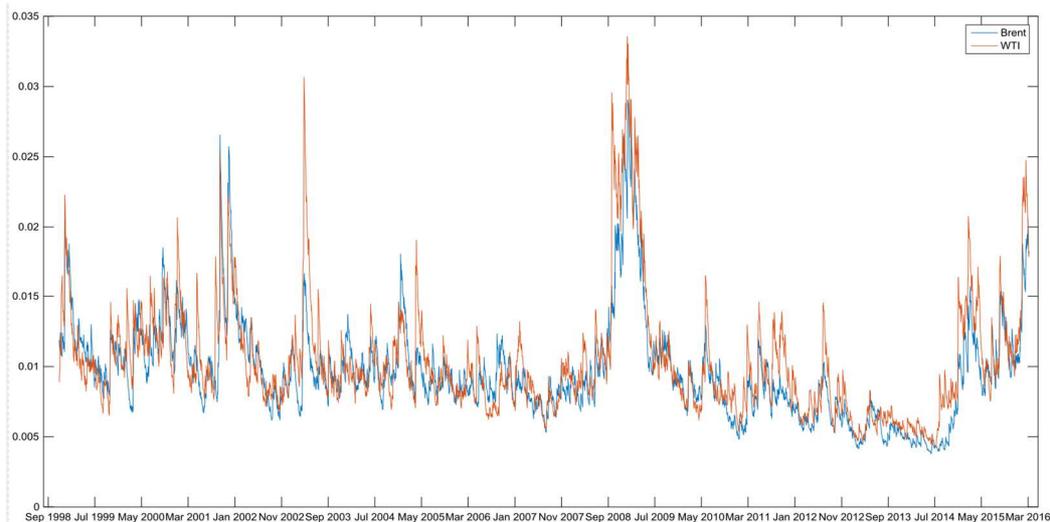
Fig. 6. Evolution of WTI and Brent volatilities (Lahmiri, 2016)

This study provided strong evidence for that fact that even though the empirical data suggests otherwise, there is an unquestionable presence of chaos in the crude oil markets after the international crisis, precisely in the volatility. The financial crisis has a strong effect on the behaviour of financial markets and its dynamics in the sense of making them them more unpredictable.

6 CONCLUSION

This paper deal with methods that are being currently studied to be able to improve understanding of the financial market dynamics. Concretely it dealt with the possibility to apply fractal analysis and fuzzy logic on said markets pro deepen the knowledge of its behaviour and the ability to predict it.

Multiple studies and researchers have proposed papers and studies showing the fractal properties and also chaotic behaviour on financial markets. There is also a study proving that financial crisis can inflict chaotic behaviour in previously stable volatility of crude oil. Many other studies have proved to find fractal properties on a various timeframes of financial markets curves.

In conclusion, findings in this paper show that there is indeed potential to using fractals, fractal dimension, fuzzy logic and also chaos theory to study and analyze markets in order to deepen the understanding of its behaviour.

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Statement of originality

I, Marie Jedličková, do hereby declare that this paper and all material contained in it is my original work. This paper has not been submitted for any other purposes. I certify that the intellectual content of this paper is the product of my own work performed by me under the guidance and advice of my faculty advisor.

Marie Jedlickova
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