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Comparison of analytical and numerical methods applied to lateral torsional buckling of beams

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Abstract

The paper is focused on the numerical evaluation of transversally loaded beams of one symmetric and four asymmetric cross-section shapes susceptible to lateral torsional buckling. Several analytic methods used for evaluation of the beam bending resistance in design software were evaluated. Analytic methods and results are compared to each other and finite element models using shell and solid elements.

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Keywords: Beam; steel; lateral torsional buckling

1. Introduction

The purpose of this paper is analysis of behavior of beam with asymmetric cross-section susceptible to lateral torsional buckling subjected to bending. The buckling resistance of asymmetric cross-section is not included in Cl. 6.3 of EC 3 [1]. Such beam can be assessed using geometrically nonlinear analysis. The paper contains a comparison of various model types. Results of Scia Engineer [2] and RFEM [3] using analysis of bar model, RF-FE-LTB [4], RFEM using shell elements and ANSYS [5] using solid elements are included.

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Nomenclature

B	bimoment
E	modulus of elasticity
$e_{0,d}$	design value of maximum amplitude of an imperfection
I_w	warping constant
k	factor for $e_{0,d}$
L	beam length
m_k	uniformly distributed torsional moment
M_{Ed}	design bending moment
M_{Rd}	design bending resistance
q	uniformly distributed load
w	sectorial area of checked fiber
y'	eccentricity of load to center of shear
z_j	constant of asymmetry of cross-section
θ	rotation about a longitudinal axis
σ_w	secondary torsional stress
σ_x	normal stress

The numerical investigation serves as a complement to the experiments conducted at Brno University of Technology in the year 2016. The test equipment and verification methodology of flexural torsional beam buckling has been developed by J. Melcher and M. Karmazínová [6, 7, 8].

2. Methods

Beams with welded cross-sections are used for the investigation of behavior during lateral torsional buckling. All beams are 6 m long from steel grade S235 (modulus of elasticity $E = 210$ GPa, Poisson ratio $\upsilon = 0.3$, density $\rho = 7850$ kg/m^3). The beams are simply supported; both ends are supported laterally and warping is allowed. One doubly symmetric and four asymmetric cross-sections are used in the study. The upper flanges of asymmetric cross-sections are shifted by 20, 40, 60, and 80 mm (see Fig. 1). Beams are loaded by gravity and a uniformly distributed load. The line of application of the uniformly distributed load is at the top area of the top flange at the connection with the web (see Fig. 2). No partial load coefficients are considered in this study. The load acts downward and does not change its direction during deformation.

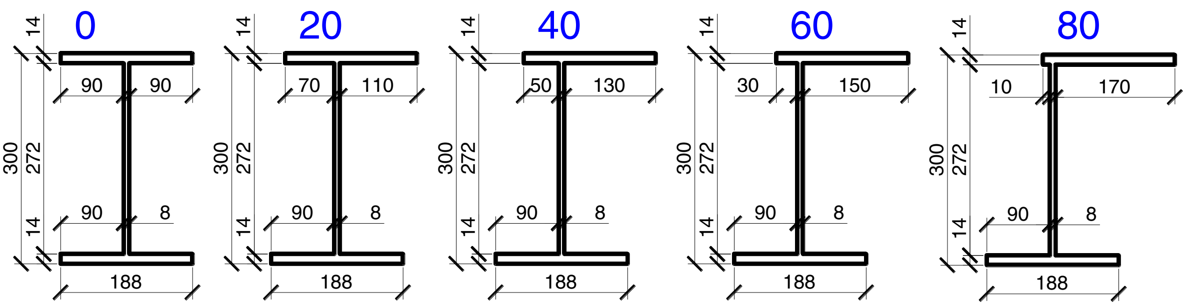

Fig. 1: Investigated cross-sections and their labeling

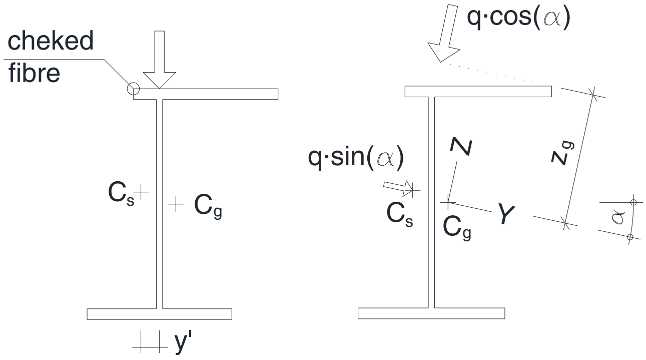

Fig. 2: Line of application, direction of uniformly distributed load, the most stressed fiber; division of load into the direction of principal axes

2.1. Bar model

Engineering design software [2, 3] assessment is the main topic of this chapter. Beam is characterized by gravitational axis. Both equations (6.56) and (6.57) in Cl. 6.3.2 in EC 3 [1] are used. The buckling curve c is selected for beam "0", and curve d for beams with asymmetric cross-sections. The output of the software is in the form of utilization. Linear analysis is performed. In the software, the load aims to the shear center, so in the case of asymmetric cross-sections and eccentric loading regarding the center of shear, complete assessment is obtained by adding the lateral effect (load passes through the center of shear – determined by software) and torsional effect caused by bimoment (uniformly distributed torsional moment – determined manually) together. Linear superposition is used.

The stress of torsion is determined as follows. The bar is subjected to the uniformly distributed torsional moment:

$$m_k = q \cdot y' \tag{1}$$

Rotation about longitudinal axis *x* is solved by software. Its shape is substituted by sinus function. The value of its second derivation at half span is put into the formula for bimoment (secondary torsional moment) and secondary torsional stress obtained for checked fiber. Checked fiber is assumed as the most stressed due to the lateral effect and that is why normal stress σ_x caused by torsion is evaluated for this fiber and not for its maximum. Utilization is determined as a ratio of this combined stress to the yield strength. The torsional stress is manually determined by these equations.

$$\theta_{(x)} = \theta_{max} \cdot sin\frac{\pi \cdot x}{L} \tag{2}$$

$$\theta''_{(L/2)} = -\frac{\pi^2}{L^2} \cdot \theta_{max} \cdot sin\frac{\pi \cdot L/2}{L} = -\frac{\pi^2}{L^2} \cdot \theta_{max} \tag{3}$$

$$B_{(L/2)} = -E \cdot I_w \cdot \theta'' = -E \cdot I_w \cdot (-\frac{\pi^2}{L^2} \cdot \theta_{max}) = \frac{\pi^2}{L^2} \cdot E \cdot I_w \cdot \theta_{max} \tag{4}$$

$$\sigma_w = \frac{B}{I_w} \cdot w = \frac{\pi^2 \cdot E \cdot I_w \cdot \theta_{max}}{L^2 \cdot I_w} \cdot w = \frac{\pi^2}{L^2} \cdot E \cdot \theta_{max} \cdot w \tag{5}$$

The software is set as follows. Elastic critical moment is solved by Galea method in Scia [2]. RFEM [3] uses Eigenvalue analysis. Neglecting the bending about a weak axis is set to 0.0 and modified Cl 6.3.4 in EC 3 is used instead. This method is not in accordance with EC 3, it is based on the dissertation of J. Naumes [9]. Destabilization impact of loading and equation (6.56) in EC 3 are set in both cases.

2.2. RF-FE-LTB model

The main interest of this part is strength assessment of bar model with initial imperfection according to Cl. 5.3.4 (3) in EC 3 [1] – the equivalent initial bow imperfection of the weak axis of the profile is considered and torsional imperfection is neglected. Add-on module of RFEM software [4] divides a bar to 1D elements with 7 degrees of freedom in each node. – 3 displacements, 3 rotations and warping. The module takes loading position into account. A cross-section is described by governing points. Normal, shear and equivalent stress according to von Mises theory are determined in these points.

The solution is geometrically nonlinear, iterative. Assumptions of computing are small deformations, physical linearity and Bernoulli beam theory. Element length of 25 cm is used.

Beams are initially curved perpendicularly to the weak axis. There are two sets of imperfections. In the first case, the amplitude is taken directly from Cl. 5.3.4 (3). Imperfection is given as $k \cdot e_{0,\mathrm{d}}$. A value of $e_{0,\mathrm{d}}$ is taken from Tab. 5.1 in EC 3. Coefficient $k = 0.5$ is recommended. Final amplitudes are 15 mm and 20 mm for curves c and d, respectively. In the other case, the optimum amplitude is derived from bar model of the beam "0" described in Chapter 2.1. Model with the optimal amplitude in this module has the same maximum elastic load resistance as established on the bar model. Coefficient k depends on the applied equation used in the bar model analysis – see Table 1. Final amplitudes are 30 mm and 40 mm for curves c and d, respectively.

2.3. Shell element model

The main subject of this chapter is strength assessment with spatial imperfection, both deflection and torsion, with amplitude according to Cl. 5.3.4 (3) [1]. Scaled first buckling mode is used to achieve this in RFEM software [3]. Beam consists of shell elements of 2 cm in size. Beam is supported at the point of the intersection of the web and the bottom flange. Lateral stability is provided by the support at the intersection of the web and the top flange blocking lateral displacement only (see Fig. 3). Overlapping of the areas of the web and flanges causes higher stiffness of the cross-section. However, this effect is negligible and can be attributed to the neglected area of welds. Geometrically nonlinear analysis is performed with Newton-Raphson method and one increment. Normal stress is observed. Optimal amplitude is studied in the case of beam "0" as well as in the previous chapter. For all beams, initial imperfections with amplitude $k \cdot e_{\mathrm{o,d}}$, where $k = 0.5$ as recommended by EC 3, is used.

This model serves as a controlling model of an accuracy of linear superposition of lateral and torsional effects as used for bar models. Linear superposition used for bar models is compared to the model of shell elements in the case of beam "60". Two lines of application of the uniformly distributed load are investigated – first passes through the center of shear, the second through the intersection of the web and the flange. The second load causes additional torsional stress. The normal stress caused by the first load passing the center of shear is added to the calculated stress σ_{w} caused by torsion as described in Chapter 2.1. This accumulated stress is compared to the normal stress caused by the second load causing bimoment.

2.4. Solid element model

ANSYS® Academic Research, Release 16.2 software [5] is used for the model of beam comprised of solid elements. 8-node structural element type SOLID 185 is chosen. The mapped mesh and boundary conditions can be seen on Fig. 3. Using the symmetry, only half of the beam is modelled. Bilinear material model with von Mises yield criterion with yield strength 235 MPa and slope of the yielding plateau E / 10 000 is assumed. Eigenvalue analysis is performed and the geometry is updated with the shape of the 1^{st} positive buckling mode. The maximum deflection is chosen 15 mm as suggested by EC 3 [1]. Then the geometrically nonlinear analysis is performed. The beam is loaded by gravity in the first step of the nonlinear analysis and nodal load at the top of the web in the downward direction in the second step. Elastic resistance is determined by reaching the yield strength at the middle of the beam. The plastic resistance is reached when the solver stops converging, meaning the force can no longer be increased.

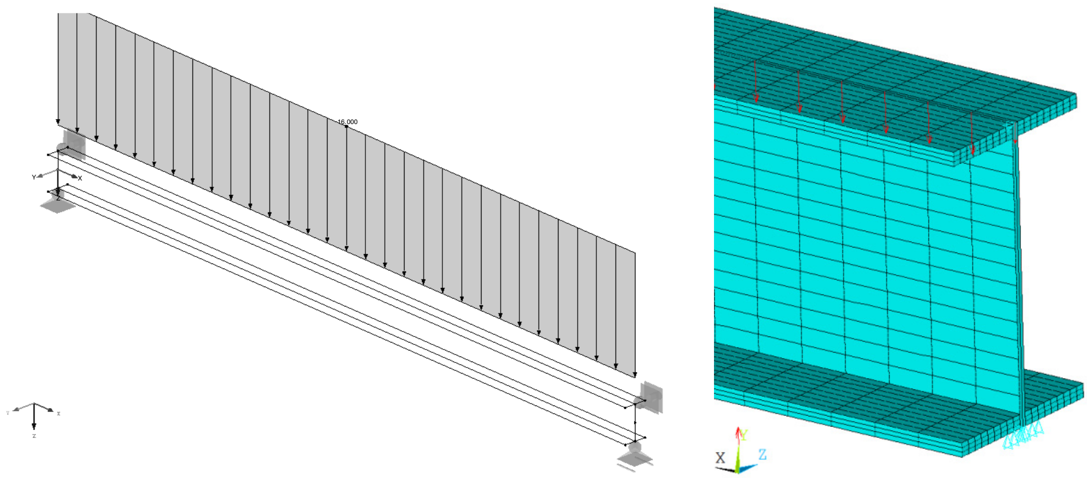

Fig. 3: Boundary conditions – on the left: shell element model in RFEM; on the right: solid element model in ANSYS

3. Results and discussion

3.1. Bar model

There is an exact correspondence among hand calculations as suggested by software help [2, 3] and software outputs of beam “0” for both equations (6.56) and (6.57) in EC 3 [1]. Software outputs of beam “60” were manually checked and they corresponded exactly as well.

Scia approaches asymmetric cross-sections as following. The uniformly distributed load is divided into the directions of principal axes (see Fig. 2). Bending about weak axis z is assumed as simple bending. Bending about strong axis y is assumed with a possibility of lateral torsional buckling. Lateral torsional resistance is determined according to Cl. 6.3.2.2 in EC 3. Cross-sectional characteristics of principal axes are used. The distance of the most distant point of the cross-section to the center of shear is used for the destabilization effect of load position in lateral torsional buckling evaluation. However, load leads to the shear center and stress caused by bimoment has to be added manually. This length is measured in principal axes direction. Constant of asymmetry of cross-section z_j is taken into account. Bending about strong and weak axes are put together using equations (6.61) and (6.62) in EC 3 without axial force. Note that the software uses procedures in EC 3 which are not verified for asymmetric cross-sections.

RFEM divides load to principal axes direction and assumes load position the same way as Scia. However, the assessment is provided using formulas in [9]. This method is similar to the general method in Cl. 6.3.4 in EC 3, and according to the author of this dissertation, it is usable for asymmetric cross-sections. Unless it is allowed in RFEM, asymmetric cross-sections are described as “Impossible to assess”.

3.2. RF-FE-LTB model

Recommended imperfection by EC 3 forms the least safe results. Multiplier k = 0.5 is equivalent of equation (6.57), k = 1.0 is equivalent to (6.56) (see Table 1).

Table 1. Coefficients k

Equation used on the bar model [1]	Coefficient k [-]	Normal stress [MPa]
(6.56)	1.0	240
(6.57)	0.5	235

3.3. Shell element model

Optimal amplitudes of imperfections are suggested according to Table 2.

Stress from load aiming to the center of shear is 199 MPa. Manually determined torsional stress σ_w is 34 MPa. Totally 233 MPa. Stress from the interaction is 235 MPa. Linear superposition is regarded as possible.

Table 2. Coefficients k

Equation used on the bar model	Coefficient k [-]	Normal stress [MPa]
6.56	0.6	231
6.57	1/3	235

3.4. Solid element model

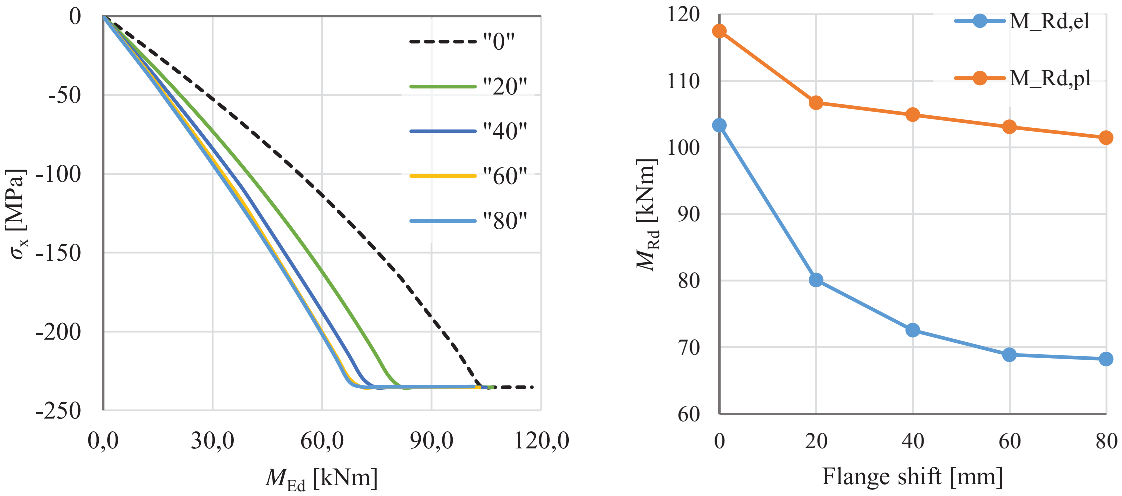

Fig. 4. Longitudinal stress at the checked fiber; elastic (M_Rd,el) and plastic (M_Rd,pl) bending resistance for beams “0” – “80”.

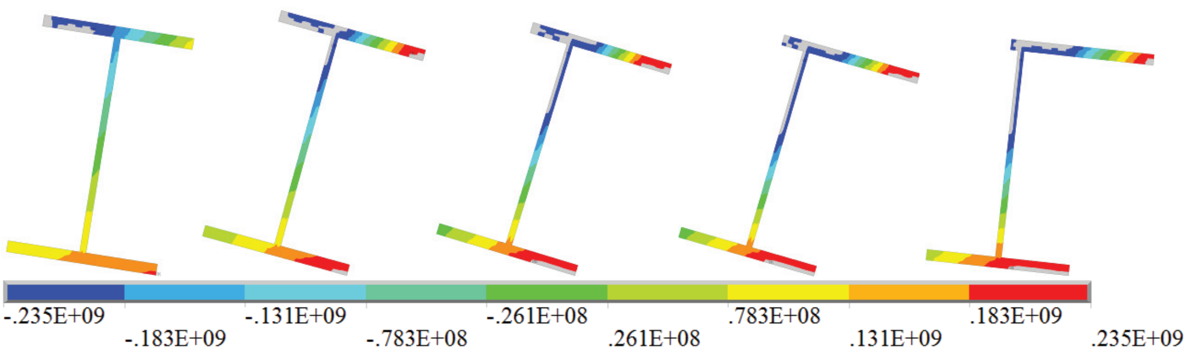

Fig. 5. Longitudinal stress σ_x [Pa] at the plastic resistance M_{Rd} (greyed areas are heavily yielded) on deformed beams (true scale)

Using solid element model and geometrically and materially nonlinear analysis, it is possible to determine both elastic and plastic moment resistance (see Fig. 4). There is a large drop in elastic resistance at the transition from doubly symmetric to asymmetric cross-section. The drop is not so significant for the plastic resistance. It seems that asymmetric sections have higher plastic reserve after first yielding is reached. Due to geometrical nonlinearity, the increment of stress with increasing bending moment slightly rises in the checked fiber until yield strength is reached. The longitudinal stress at the moment of collapse is plotted on Fig. 5.

3.5. Comparison of models

Curves of bar models create the lower boundary of the comparison of methods, meaning they provide the safest results. Both models with spatial initial imperfections – the models with shell elements and solid elements – give almost the same results. Nonlinear models are considered as more accurate because of the real position of load. Nonlinear models have hypothetic asymptote and differentiates among the resistances are decreasing (see Fig. 6). The reason seems to be the dominant effect of torsion from eccentricity to shear center. Initial amplitude affects mostly doubly symmetric beam “0”.

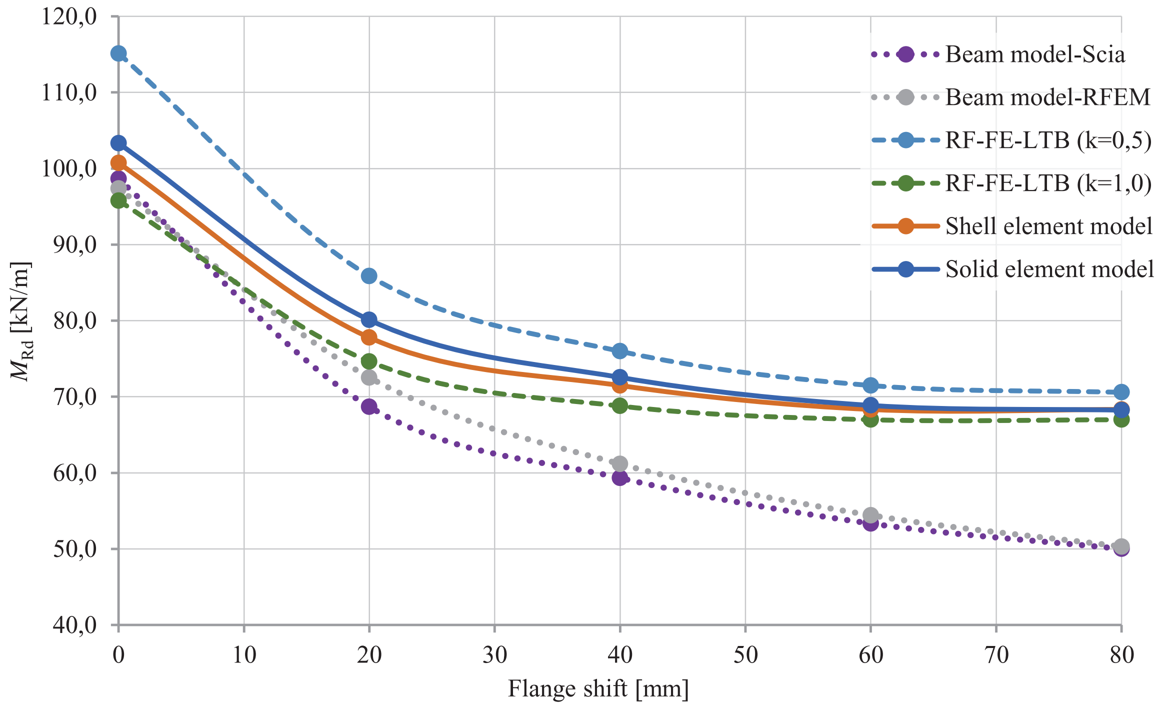

Fig. 6: Overview of elastic buckling resistance of beams

4. Conclusion

Despite different formulas used by engineering design software, the results are very similar, and the bending resistance has a decreasing trend with increasing flange shift. Curves define the safest assessment. Principles of both methods are the same – bending about both axes with a possibility of stability loss. Scia software does not respect the limitation of clauses in EC 3. Chapter 6.3.2 and 6.3.3 is focused on symmetric sections only. RFEM does not use mentioned chapters but requires more professionalism in setting options and uses a method which has not been widely accepted yet.

Add-on RF-FE-LTB provide results which are in better agreement with shell and solid element models. However, using recommended factor for amplitude k = 0.5 yields presumably unsafe results. Amplitude of the imperfection $e_{0,d}$ was taken from the recommendation for doubly symmetric cross-section and therefore models are regarded as the approximate solution. The amplitude of asymmetric sections may be different, especially due to residual stress from welding. Experiments and advanced numerical modelling with a residual stress of welding will be performed in the future.

Acknowledgements

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