Coarse Time Synchronization Utilizing Symmetric Properties of Zadoff-Chu Sequences

Jiri Blumenstein, Marek Bobula

***Abstract*—We demonstrate a novel metric for coarse time synchronization suitable for wireless communication. The novel metric benefits from a symmetry of Zadoff-Chu sequences in the time domain. The time domain symmetry is obtained after a simple manipulation of the sequence polarity. As opposed to the well-known methods, with the utilization of the Zadoff-Chu sequence symmetry property, more correlation terms are possible to be exploited. Thus, the novel metric provides higher correlation gain and notable improvement in the probability of correct time synchronization. The method is evaluated via Monte-Carlo simulations utilizing unique word single carrier/frequency domain equalization frames.**

***Index Terms*—Synchronization, UW-SC/FDE, Zadoff-Chu sequences, time-domain symmetry.**

I. Introduction

EVEN though being an integral part of wireless communication systems for decades, time synchronization still represents an open research question. For coarse time synchronization, among other sequences, the Zadoff-Chu sequences (ZCs) [1] are especially used due to their favorable autocorrelation properties. Moreover, ZC belongs to the family of constant amplitude zero autocorrelation (CAZAC) waveforms implying a good peak-to-average power ratio (PAPR) and beneficial channel estimating capabilities. On the other hand, the symbols of the ZC sequence are not selected from a common quadrature amplitude modulation (QAM) symbol alphabet, which may be a drawback in certain situations.

In this paper we consider a single carrier system (SC) with frequency domain equalization (FDE) similarly as in the case of Long-Term Evolution (LTE) uplink [2]. As opposed to LTE, where a cyclic prefix (CP) is used, we utilize the concept of unique word (UW), proposed in [3], [4] and originally intended for orthogonal frequency-division multiplexing (OFDM). The synchronization performance of the UW-SC-FDE approach is compared with other methods in [5]. In CP-OFDM, the cyclic prefix is filled with a random sequence, whereas UW is designed as deterministic. Therefore, UW can be selected optimally for appropriate tasks like synchronization and/or channel estimation [3]. The concept of UW is similar to the approach presented in [6] and [7], where cyclic prefixes together with suffixes and postfixes, respectively, are elaborated.

Jiri Blumenstein is with the Department of Radioeletronics, Brno University of Technology (BUT), Brno, Czech Republic, email: blumenstein@vutbr.cz

Marek Bobula is with RACOM s.r.o., Nové Město na Moravě, Czech Republic, email: marek.bobula@racom.eu.

This research was financed by GACR project 17-18675S *Future transceiver techniques for the society in motion*, with partial support of LO1401 (INWITE). The support from RACOM s.r.o. is also gratefully acknowledged.

In this paper, the well-known coarse time synchronization methods by *Schmidl and Cox* [8] and derived methods [9]–[11] are further researched while a novel correlative metric is shown. This novel metric utilizes the time-domain symmetry of the ZC sequence and therefore achieves higher correlation gain compared to the state-of-the art methods. Another approach to the coarse synchronization problem, which produces a timing metric without notable sidelobes, is presented in [12]. In essence, it is a differential cross-correlation with ZC weighted by pseudo noise (PN) sequences.

The time-domain symmetry of the ZC sequence is exploited in [13], where symmetric samples are added prior to multiplication with its replica, thus the multiplication complexity is reduced. In [14] a signature format based on symmetric ZC sequences is proposed to deal with the frequency offset being a multiple of subcarrier bandwidth. In this paper, we exploit this symmetry for synchronization purposes. The contribution of the letter is following:

- Demonstration of the novel correlative metric for coarse time synchronization.
- Comparison with well-known methods by *Bhargava et al.* and *Serpedin et al.* showing superior performance in the AWGN channel and partly also in the intersymbol interference (ISI) channel.
- The novel method is not sensitive to a carrier frequency offset (CFO) and provides better in-phase and quadrature sampling resolution, thus improving FDE capabilities.
- We evaluate the influence of the ZC root index influence on the timing accuracy.

II. System model

A. Synchronization sequences

Each time instant n of the complex valued ZC sequences $z(n) \in C^{1\times N_{ZC}}$ is described by:

$$z(n) = \begin{cases} \exp\left(-\mathrm{j}\frac{\pi u n(n+1)}{N_{ZC}}\right) \text{ for odd } u, \\ \\ \exp\left(-\mathrm{j}\frac{\pi u n^2}{N_{ZC}}\right) \text{ for even } u, \end{cases} \tag{1}$$

where u is the root index and it holds that $0 < u < N_{ZC}$.

B. Synchronization frame designs

As a benchmark, we utilize the methods proposed in [9], [10] which we refer to as *Bhargava et al.* and *Serpedin et al.* The synchronization frame design of the proposed method is different compared to the benchmark methods. The conventional approach is depicted in Fig. 1a. In particular, the difference to the proposed method is that in [9], [10] authors use frame schemes [+B+B-B-B] or [+B+B-B+B], respectively, where B stands for the ZC sequence of the length $L = N_S/2$. This means that within one synchronization frame we need to subsequently transmit four ZC replicas with different polarity.

In Fig. 1b, we propose the novel synchronization frame such that the scheme is [+A-A]. Now, A represents a ZC sequence with length $L' = N'_S/2$, i.e., we transmit only two ZCs, however, each sequence is double the length of the sequence used in the conventional schemes $L' = 2L$. Therefore, the in-phase and quadrature sampling is improved and, as ZC sequences of synchronization frames are often used for channel estimation, this capability is potentially enhanced.

C. Unique word guard interval

In order to maintain the possibility to perform cyclical convolution and to transform ZCs into the frequency domain via fast Fourier transform (FFT), we utilize the concept of UW. The common approach with CP could be also used, however, in our case, given that CP would be composed from the partial replica of the synchronization sequence, the correlation metrics may select a timing instant corresponding to CP instead of the intended synchronization sequence. Therefore, in the simulations, we utilize a vector of zeros with a length of twelve samples as the UWs.

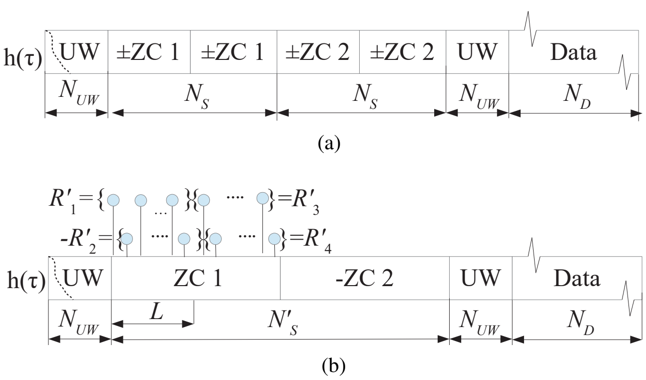


Fig. 1: Schemes of the synchronization frame designs, (a) the methods by *Bhargava et al.* or *Serpedin et al.* (the frames differ in the polarity of the third ZC sequence), (b) The proposed method. The total length of the synchronization frames is equal. The depicted polarity of the $\mathbf{R}'_i$ segments corresponds to the case of odd u.

D. Timing metrics

1) State-of-the art: The received signal, in the case of the benchmark methods, is written as:

$$\mathbf{R}_i = \{r(k+(i-1)L), ..., r(k+iL-1)\},\ i = [1,2,3,4], \tag{2}$$

where $L = \frac{N_S}{2}$ is the length of the received signal segment $\mathbf{R}_i$, and i indexes signal sections used for the correlation metric according to:

$$\mathbf{M}_\mathrm{B} = \frac{|\mathbf{R}_1^\mathrm{H}\mathbf{R}_2 + \mathbf{R}_3^\mathrm{H}\mathbf{R}_4|}{|\mathbf{R}_2|^2 + |\mathbf{R}_4|^2} \tag{3}$$

for the *Bhargava et al.* method. Here, $()^\mathrm{H}$ is the Hermitian transpose. Please note that four $\mathbf{R}_i$ segments are utilized to determine $\mathbf{M}_\mathrm{B}$. Now, the *Serpedin et al.* method utilizes a more complex timing metric written as:

$$\begin{aligned}\mathbf{M}_\mathrm{S} =& \frac{|\mathbf{R}_1^\mathrm{H}\mathbf{R}_2 + \mathbf{R}_3^\mathrm{H}\mathbf{R}_4 - \mathbf{R}_2^\mathrm{H}\mathbf{R}_3|}{3(|\mathbf{R}_3|^2 + |\mathbf{R}_4|^2)} \\ &+ \frac{|\mathbf{R}_1^\mathrm{H}\mathbf{R}_3 + \mathbf{R}_2^\mathrm{H}\mathbf{R}_4| + |\mathbf{R}_1^\mathrm{H}\mathbf{R}_4|}{3(|\mathbf{R}_3|^2 + |\mathbf{R}_4|^2)},\end{aligned} \tag{4}$$

which means that twelve $\mathbf{R}_i$ segments are used. The timing metrics are depicted in Fig. 3b. Please note that we evaluated the methods by *Serpedin et al.* and *Bhargava et al.* not only for the ZC sequences; however, simulations were done also for pseudo-noise sequences and for Legendre sequences resulting in insignificant performance deviations.

2) The novel timing metric: The novel timing metric utilizes different indexing of the received signal. It is written as:

$$\begin{aligned}\mathbf{R}_{1,2} =& \mathbf{R}'_1 || \mathbf{R}'_2 \text{ where} \\ & \mathbf{R}'_1 = a_1\{r(k), ..., r(k+L/2-1)\} \\ & \mathbf{R}'_2 = a_2\{r(k+1), ..., r(k+L/2)\} \\ \mathbf{R}_{3,4} =& \mathbf{R}'_3 || \mathbf{R}'_4 \text{ where} \\ & \mathbf{R}'_3 = a_3\{r(k+L/2), ..., r(k+L-1)\} \\ & \mathbf{R}'_4 = a_4\{r(k+L/2+1), ..., r(k+L)\} \\ \mathbf{R}_{5,6} =& \mathbf{R}'_5 || \mathbf{R}'_6 \text{ where} \\ & \mathbf{R}'_5 = a_5\{r(k+L), ..., r(k+2L-1)\} \\ & \mathbf{R}'_6 = a_6\{r(k+L+1), ..., r(k+L+L/2)\} \\ \mathbf{R}_{7,8} =& \mathbf{R}'_7 || \mathbf{R}'_8 \text{ where} \\ & \mathbf{R}'_7 = a_7\{r(k+L+L/2), ..., r(k+2L-1)\} \\ & \mathbf{R}'_8 = a_8\{r(k+L+L/2+1), ..., r(k+2L)\},\end{aligned} \tag{5}$$

where $a_l,\ l \in [1, ..., 8]$ is an element of vector:

$$\mathbf{a} = \begin{cases}[+1,-1,+1,+1,+1,-1,+1,+1] \text{ for odd } u \\ [+1,-1,+1,-1,-1,+1,-1,+1] \text{ for even } u\end{cases} \tag{6}$$

and where $||$ stands for vector concatenation. Please note that the vector $\mathbf{a}$ controls the polarity of the signal segments $\mathbf{R}'_i$. Then, for one certain time instant, i.e., the intended synchronization moment, it holds that:

$$\mathbf{R}'_i = \pm\mathbf{R}'_{i+2},\ i \in [1,2,3,4,5,6], \tag{7}$$

which we refer to as the time domain symmetry. It holds for both the real and imaginary parts of $\mathbf{R}'_i$. In fact, Eq. (7) means that every odd and even $\mathbf{R}'_i$ segment are equal. Thus, each concatenated segment $\mathbf{R}_{i,i+1}$, constructed in Eq. (5), contains one $\mathbf{R}'_i$ segment, which is equal to a different $\mathbf{R}'_j$ $(i \neq j)$ segment included in a different concatenated $\mathbf{R}_{j,j+1}$ segment. Therefore, we can construct the correlation terms $\mathbf{C}_p$ as:

$$\begin{aligned}&\mathbf{C}_1 = \mathbf{R}_{1,2}^\mathrm{H}\mathbf{R}_{3,4},\ \mathbf{C}_2 = \mathbf{R}_{5,6}^\mathrm{H}\mathbf{R}_{7,8},\ \mathbf{C}_3 = \mathbf{R}_{1,2}^\mathrm{H}\mathbf{R}_{5,6}, \\ &\mathbf{C}_4 = \mathbf{R}_{3,4}^\mathrm{H}\mathbf{R}_{7,8},\ \mathbf{C}_5 = \mathbf{R}_{1,2}^\mathrm{H}\mathbf{R}_{7,8},\ \mathbf{C}_6 = \mathbf{R}_{3,4}^\mathrm{H}\mathbf{R}_{5,6}, \\ &\mathbf{C}_7 = (\mathbf{R}_{1,2}||\mathbf{R}_{3,4})^\mathrm{H}(\mathbf{R}_{5,6}||\mathbf{R}_{7,8}),\end{aligned} \tag{8}$$

where the number of $\mathbf{R}_{i,i+1}, i \in [1,2,...,7]$ signal segments used for the correlations is sixteen. It is worth noting that the two ZC sequences utilized in the proposed synchronization frame are divided into four $\mathbf{R}_{i,i+1}$ segments where the length of an individual $\mathbf{R}_{i,i+1}$, segment equals to $L'/2$, i.e., the length L of the ZC sequence utilized in the benchmark schemes. Thus, the total synchronization frame size is equal among the proposed scheme and the schemes proposed by *Bhargava et al.* and *Serpedin et. al*. The individual correlations $\mathbf{C}_p,\ p \in [1, ..., 7]$ are depicted in Fig. 2. The correlation terms $\mathbf{C}_1$, $\mathbf{C}_2$ and $\mathbf{C}_5$ create either three equal peaks or a slightly broadened peak in $\mathbf{C}_6$. The correlation terms $\mathbf{C}_3$ and $\mathbf{C}_4$ both exhibit a plateau, similarly as the metric proposed by *Schmidl and Cox*. If the two terms $\mathbf{C}_3$ and $\mathbf{C}_4$ are summed, one distinct peak occurs (as utilized in the method of *Faulkner et al.* [15]).

In its non-normalized form, the metric is written as:

$$\mathbf{M} = |\mathbf{C}_P| + \sum_{p=1}^{P-1} |\mathbf{C}_p + \mathbf{C}_{p+1}|,\ \text{where}\ P = 7. \tag{9}$$

3) Note on the correlation gain: The benefit of the proposed timing metric is higher correlation gain compared to the benchmark methods. For example, suppose $N_S = 32$ symbols, i.e. $L = \frac{N_S}{2} = 16$. This means that for calculating $\mathbf{M}_\mathrm{B}$ we use $4 \times L = 64$ symbols and for $\mathbf{M}_\mathrm{S}$ we utilize $12 \times L = 192$ symbols. For determining the novel metric $\mathbf{M}$ we use $16 \times L'/2 = 256$ symbols. This property clearly enhances the noise tolerance of the novel timing metric.

E. ISI channel model and the carrier frequency offset

To emulate multipath propagation, we utilize exponentially decaying normally distributed complex channel taps to convolve with the transmitted signal. The length of such channel impulse response is 20 samples. With 6 samples per symbol, this channel causes ISI over 3.33 symbols. For comparison, we also use the additive white Gaussian noise (AWGN) channel.

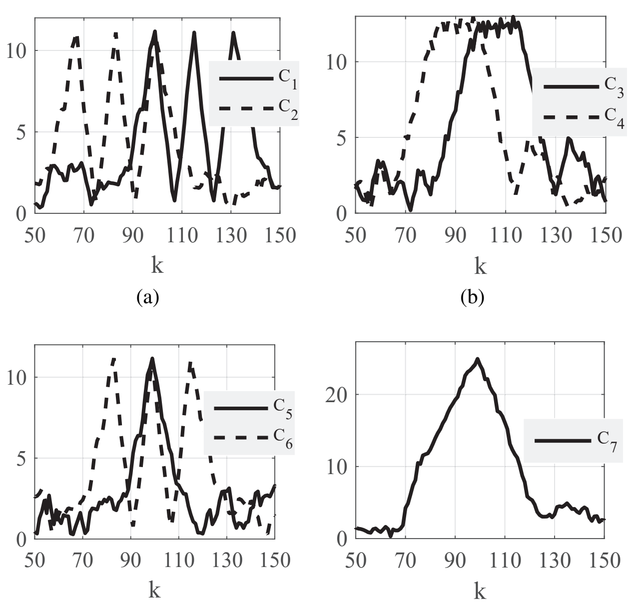


Fig. 2: Individual correlation terms of the proposed metric.

Considering a time invariant CFO, the presented method, similarly as the benchmark methods by *Bhargava et al.* and *Serpedin et al.*, is not influenced by the frequency offset and channel phase. This is due to the fact that the CFO influences both ZC 1 and ZC 2 sequences equally. The received signal with the CFO is written as: $r'(k) = r(k)\exp(\mathrm{j}2\pi\epsilon k/N)$, where ϵ is the CFO and the correlation terms form Eq. (8) are written as: $\mathbf{C}_p = \exp(\mathrm{j}2\pi\epsilon k/N)(\mathbf{R}_{i,i+1}^{\mathrm{H}}\mathbf{R}_{j,j+1}), \forall\{p, i, j\}$. Subsequently, as $|\exp(\mathrm{j}2\pi\epsilon k/N)| = 1$, the absolute value operations in Eq. (9) ensures the insensitivity of the metric $\mathbf{M}$ to the CFO.

III. Simulation results

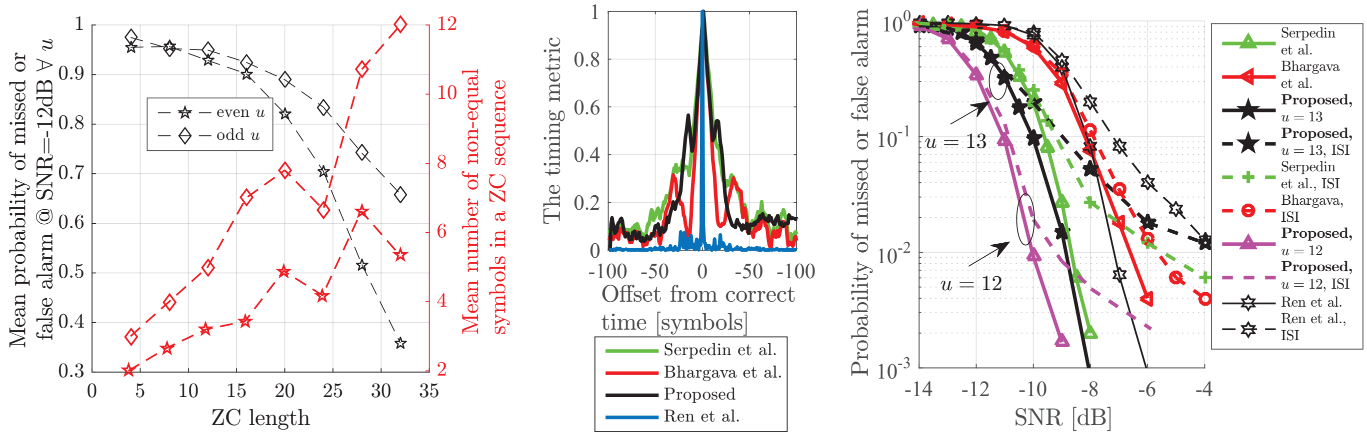
The main simulation results are plotted in Fig. 3c. It shows that the proposed method is superior in the AWGN channel to the methods of *Serpedin et al.*, *Bhargava et. al* and *Ren et al.*. This is namely due to higher correlation gain. The parameter set of the investigated synchronization setup is in Table I.

The method by *Bhargava et al.*, performing worst in solely AWGN, provides better results in ISI from SNR of -6 dB. The method by *Serpedin et al.* performs better in the SNR region from -9.5 dB to -6 dB while the proposed method is superior in the SNR region from -14 dB to -9.5 dB. This holds for the odd u. For the even u, the proposed method outperforms the benchmarks both in AWGN and ISI environment.

1) Root index parity: The influence of the root index parity is evaluated via Monte-Carlo simulations and is depicted in Fig. 3a. It is seen that if averaged over all the even or odd root indexes (for the specified ZC length), the even root indexes result to overall better performance. Here it is interesting to point out, that the mean number of non-equal symbols contained in the ZC generated with odd u is significantly higher then the mean number of non-equal symbols generated with even u (see Fig. 3a). This property enables the higher noise resistance of ZCs generated with the even root indexes.

If we compare e.g. the odd root indexes $u = [1, 3, 5, ..., 15]$, as seen in Fig. 4, we see slightly different results for the individual odd root index realizations. The difference between the best- and the worst-case root index realization is; however, in the order of 0.01 probability of the missed or false alarm at the 9 dB signal-to-noise ratio (SNR) level. While all the depicted 95% confidence intervals include all the simulated odd root indexes, we conclude very weak sensitivity on the root index selection within both root index parity groups.

2) ZC sequence length influence: Another influence on the false or missed alarm probability has the ZC length. Apart from the evident fact, that the longer the ZC, the higher the correlation gain, it is worth mentioning that only the ZC lengths equal to the multiples of four (i.e., $L = [4, 8, 12, ...]$) provide sharp peak in the proposed metric $\mathbf{M}$. The other even ZC lengths (i.e., $L = [2, 6, 10, ...]$) create a two-symbol wide plateau, thus the resulting synchronization instant is not recognized. Notable cases are the prime ZC lengths which provide equal number of ZC symbols for both the odd and even root indexes. In addition, the prime ZC lengths results to the highest number of non-equal ZC symbols, thus they are not well suitable for the synchronization (see Fig. 3a). Contrary,



(a) (b) (c)

Fig. 3: (a) Influence of the ZC root indexes parity and ZC length. (b) Comparison of normalizer timing metrics (c) The curves of the missed and false alarm probabilities for both AWGN and ISI channels. Utilized root indexes $u = 12$ and $u = 13$.

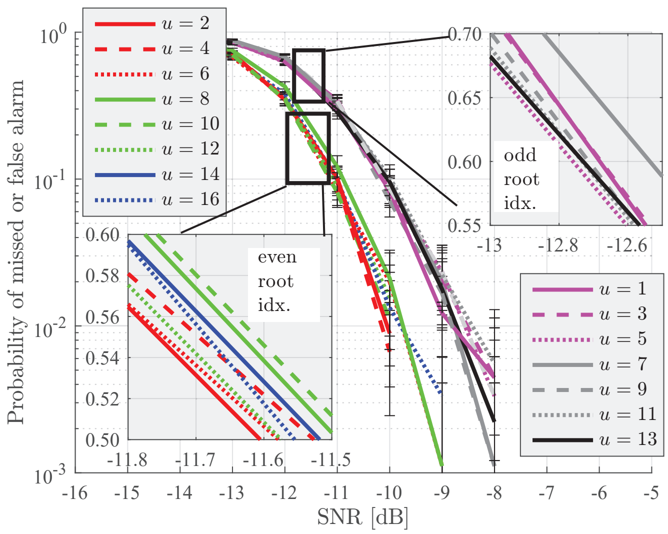


Fig. 4: Influence of the root index selection and parity on the probability of a missed or false alarm of the proposed method. The curves are equipped with 95% confidence intervals. The ZC length is sixteen symbols.

for the FDE purposes, the prime ZC lengths are attractive due to higher resolution sampling.

TABLE I: Parameter set of the investigated synchronization frame setup

Sequences	Zadoff-Chu
N_S/ length	32 symbols
N_S' length	64 symbols
Channels	(1) AWGN, (2) ISI (20 samples)
Pulse shaping	RRC (roll-off $= 0.25$)
Samples per symbol	6

3) Computational complexity: The relative computational complexity is evaluated with respect to the length of the ZC sequence L and without considering the normalization. Note that we mark the number of multiplications as $(\cdot)$ and the number of additions as $(+)$. For *Bhargava et al.* we have $12L(\cdot)$ and $11L(+)$, for *Serpedin et al.*: $36L(\cdot)$, $39L(+)$ while the proposed method requires $60L(\cdot)$ and $53L(+)$ operations.

IV. Conclusion

The novel timing metric has been proposed and it shows a superior behavior in the AWGN and ISI channel due to higher correlation gain. The novel metric utilizes ZC sequences with twice the length of the benchmark methods; however, the total synchronization frame length is preserved as the number of the required sequence repetitions is halved. The proposed method is invulnerable to the CFO and the phase shift of the channel.

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