A Robust Threshold for Iterative Channel Estimation in OFDM Systems

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Abstract. *A novel threshold computation method for pilot symbol assisted iterative channel estimation in OFDM systems is considered. As the bits are transmitted in packets, the proposed technique is based on calculating a particular threshold for each data packet in order to select the reliable decoder output symbols to improve the channel estimation performance. Iteratively, additional pilot symbols are established according to the threshold and the channel is re-estimated with the new pilots inserted to the known channel estimation pilot set. The proposed threshold calculation method for selecting additional pilots performs better than non-iterative channel estimation, no threshold and fixed threshold techniques in poor HF channel simulations.*

Keywords

Adaptive threshold, additional pilot symbols, error correction decoder, soft outputs, HF.

1. Introduction

Channel estimation in orthogonal frequency division multiplexing (OFDM) systems is generally based on the known pilot symbols, which are inserted to the data packets to sample the time varying and frequency selective channel. It is crucial to estimate the channel as accurate as possible in order to acquire the expected advantages of OFDM systems. Pilot symbol assisted (PSA) channel estimation approach is commonly used in OFDM systems due to its simple implementation and the ability to ensure adequate estimation accuracy. In PSA channel estimation approach, after estimating the channel at the known pilot symbol positions, the estimates are interpolated to obtain the channel information at the relevant data positions [1]. Estimation performance can be improved by adding more pilot symbols in order to sample the channel more densely; however this approach results in reduced data rate or expanded bandwidth. On the other hand, it is possible to exploit the forward error correction (FEC) decoder soft outputs for iterative channel estimation and decoding rather than adding more channel estimation pilots.

Improving the channel estimation iteratively by using the soft outputs of an error correction decoder is known as turbo equalization [2]. The original turbo equalization approach performs well when the channel is known at the receiver; however, especially for fading channels, accurate channel estimation is hard to obtain and the expected success of the approach cannot be achieved with imperfect channel information. In the literature, iterative channel estimation and decoding for slow fading channels [3, 4], for fast fading channels [5], for fast flat fading channels [6], and for frequency selective fast fading channels [7] have all been studied and iterative estimation and decoding techniques have shown to improve performance.

A simple and straightforward solution to improve the channel estimation performance in PSA systems is to employ the entire set of the error correction decoder outputs [8]. However, utilizing the error correction decoder output symbols without any selection criteria does not always improve the channel estimation performance due to poor quality of some output symbols [9]. A solution to mitigate the degrading effects of the poor quality output symbols is given in [10] by introducing a threshold test. The symbols, whose reliabilities are higher than this threshold value, are selected as the additional pilots for the iterative channel estimation. Another threshold based approach is studied in [11] in which the initial estimates obtained from the known pilots replace the poor quality decoder output symbols, instead of inserting zeros, to keep the filter coefficients fixed.

Therefore, it is evident that picking only the reliable decoder outputs as the additional pilot symbols for the iterative channel estimation should improve the system performance. In literature, utilizing only the reliable decoder output symbols for iterative channel estimation and decoding are studied [10-13]. A heuristic threshold adjustment approach is proposed in [13] to obtain a predetermined number of reliable pilots by decreasing the fixed threshold in small steps. Although performance of the threshold adjustment approach is better than that of a fixed threshold, selection of the predetermined number of reliable pilots is troublesome and small step sizes increase the complexity [13].

Since the symbol reliability values may change for different data packets according to the code structure, number of iterations and channel conditions, establishing an adaptive threshold value for each packet is the main motivation of this study in order to improve the system performance. To the best knowledge of the authors, no such adaptive and robust threshold calculation method exists for the selection of reliable decoder output symbols. In this paper, therefore, we propose a novel adaptive threshold computation method to establish reliable decoder output symbols that will be inserted to the known channel estimation pilot symbols set in order to improve the bit error rate (BER) performance of pilot symbol assisted coded OFDM systems. Threshold computation method depends only on the statistics of each packet and does not require any adjustment or predetermined value to select the reliable decoder output symbols, which makes the proposed threshold robust and adaptive for any channel conditions.

In the proposed threshold computation technique, the minimum and the standard deviation of the symbol reliability values of each packet are calculated. Then, a particular threshold is determined by utilizing these adaptive values for the corresponding packet. Finally, the symbols whose reliabilities are higher than this threshold are chosen as the reliable symbols. The selected additional pilot symbols are included to the set of known channel estimation pilots and more accurate channel estimation is obtained via interpolation using the extended channel estimation pilots set. The proposed technique improves the bit error rate performance over non-iterative, no threshold, and fixed threshold cases in poor high frequency (HF) channel simulations.

This paper is organized as follows. Model for the pilot symbol assisted iterative channel estimation in OFDM systems is given in section 2. Proposed threshold calculation method for the described system model is explained in section 3. Simulation results for the proposed approach are presented in section 4. Finally, section 5 gives the conclusion.

2. System Model

Pilot symbol assisted OFDM system model and the iterative channel estimation process for the proposed threshold selection approach is given within this section.

Block diagram of the transmitter is shown in Fig. 1. K information bits are encoded using a rate R_c code such as low-density parity-check (LDPC) code [14] or any linear block code. The $N=K/R_c$ encoded bits are interleaved over multiple blocks to mitigate the effects of fading. Interleaved coded bits are grouped in m bits and then mapped to obtain the complex 2^m quadrature amplitude modulation (QAM) data symbols according to the constellation diagram. Finally, at the transmitter side, after inserting the pilot symbols to the data subcarriers, the inverse fast Fourier transform (FFT) is performed to create the time waveform. Besides, although each packet contains pilot tones for packet detection and frame synchronization, perfect synchronization is assumed at the receiver side.

As illustrated in Fig. 2, at the receiver side after packet detection, FFT of each symbol is calculated. Then, the FFT demodulator extracts distorted data and pilot symbols for channel equalization and estimation.

Following the notation in [1], channel estimation pilots inserted to the data symbols are denoted by $x[n_p,k_q]$ where $n_p=pd_t$ and $k_q=qd_f$. p and q are the closest integers according to the d_t and d_f, pilot intervals in the time and frequency domain, respectively. Similarly, as information bits are transmitted in packets, the k-th data in the n-th packet is shown by $x[n,k]$.

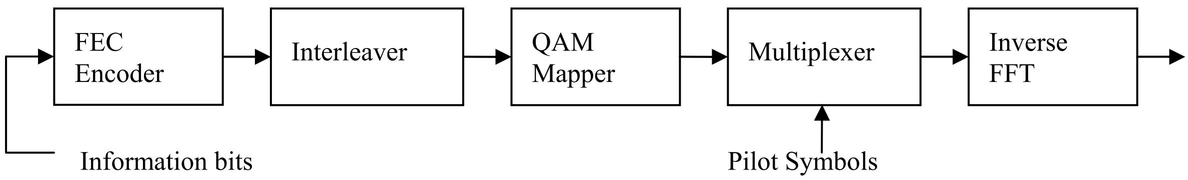

Fig. 1. Transmitter block diagram of the PSA OFDM system.

Following the notation in [1] again, the received signal, $y[n,k]$, is denoted by

$$y[n,k] = h[n,k]x[n,k] + z[n,k] \tag{1}$$

where $h[n,k]$ and $z[n,k]$ are the channel frequency response and the complex valued additive Gaussian noise samples at the related positions, respectively. Channel is estimated at the known pilot symbol positions by

$$\tilde{h}_p[p,q] = \frac{y[n_p,k_q]}{x[n_p,k_q]} = h[n_p,k_q] + \frac{z[n_p,k_q]}{x[n_p,k_q]} \tag{2}$$

where $\tilde{h}_p[p,q]$ and $h[n_p,k_q]$ are the channel frequency response estimates and channel frequency response of the k-th tone in the n-th packet at the known pilot positions, respectively. Now the estimates are ready to be interpolated to obtain the channel frequency response at the data positions.

After interpolating the estimates to obtain channel information at the relevant data positions by

$$\hat{h}[n,k] = \sum_p \sum_q w[n,p;k,q]\tilde{h}_p[p,q] \tag{3}$$

where $w[n,p;k,q]$ is a weighting function used in the interpolation process. Demodulated data symbols are then, as shown in Fig. 2, equalized by simple channel inversion operations by

$$d[n,k] = y[n,k] / \hat{h}[n,k]. \quad (4).$$

After channel equalization data symbols are de-interleaved, before a powerful forward error correction decoding algorithm is performed. The decoder is initialized with the soft inputs calculated in the form of log-likelihood ratios (LLR) and the decoder delivers soft outputs. The decoding process is stopped either the maximum number of decoder iterations is met or the syndrome check is satisfied. The syndrome check is only satisfied when the decoding algorithm is accomplished with no errors.

Finally, hard decisions are obtained at the decoder output either the stopping criteria are met or the maximum number of turbo iterations is reached. No action is taken for iterative channel estimation process if the syndrome check is satisfied.

The iterative channel estimation process, shown in Fig. 2 with dashed blocks, is performed if the syndrome check is not satisfied when the maximum number of decoder iterations is reached. Decoded data symbols are interleaved before calculating the symbol reliabilities of an entire data packet. Then, an adaptive symbol threshold value is established for the corresponding data packet to select the additional pilots for the iterative channel estimation. Data symbols, whose reliabilities are higher than this threshold, are selected as the additional pilots, which will be inserted to the interpolation set in order to improve the channel estimation performance. The decoding process is repeated with the new channel estimates.

Iterative channel estimation process is stopped either the syndrome check is satisfied at the decoder output or the maximum number of turbo iterations is met.

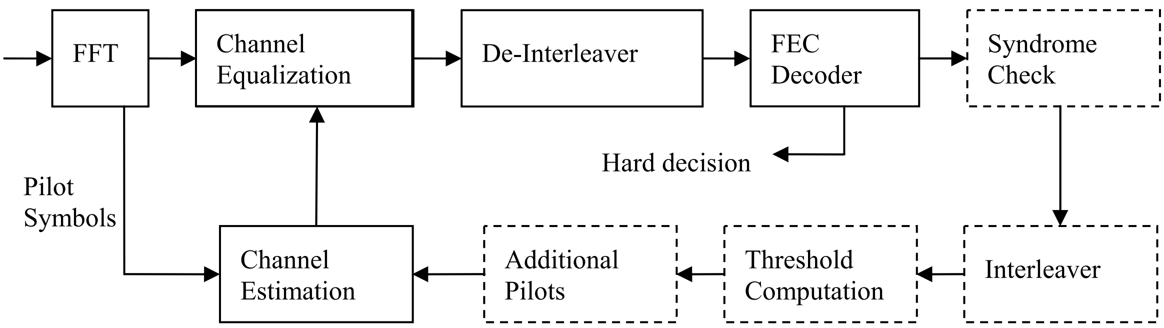

Fig. 2. Receiver block diagram of the system for the proposed threshold selection approach.

The reliability of a symbol is defined as the reliability of the least reliable bit composing m bits of a 2^m-QAM symbol where m bits of the 2^m-QAM symbol can be defined as $q=[c_1,\ldots,c_m]$. A posterior probability of a bit c_i is given with $P(c_i|y)$ where y is the received code word. The log-likelihood ratio of $P(c_i|y)$ is defined by

$$\lambda(c_i) = \log\left(\frac{P(c_i = 0 \mid y)}{P(c_i = 1 \mid y)}\right). \quad (5)$$

As the LLR may take values in the $(-\infty, +\infty)$ range, establishing a threshold in such a wide margin may be troublesome and using a narrow range is more convenient. Therefore, the reliabilities of each symbol, $R(q)$, are calculated by mapping the corresponding bit log-likelihood ratio, $\lambda(c_i)$, with the hyperbolic tangent function, which is shown in equation (6).

$$R(q) = \tanh(|\lambda(c_i)|). \quad (6)$$

Now, the symbol reliabilities, $R(q)$, are mapped to the [0,1] interval which is a more appropriate way to establish a threshold for different conditions.

3. Proposed Threshold Calculation Approach

In OFDM systems information bits are transmitted to the receiver in packets. Depending on the code structure, number of iterations and the channel conditions, the symbol reliabilities may differ among the packets. Therefore, to improve the channel estimation for each data packet, a robust threshold, which can be used for different packets, should be determined.

The decoder output symbols whose reliabilities are higher than the threshold value are selected as the additional pilots for each data packet for re-estimation. On the other hand, since the channel estimation performance can be improved with more reliable additional pilots, the threshold will be chosen in such a way that as many reliable decoder output symbols as possible can be picked. No action is taken for iterative channel estimation process if the syndrome check is satisfied which means the decoding process is completed with no errors.

Obtaining the probability distribution of the decoder output symbol reliabilities should help to establish a model for a robust threshold selection method. However, it is not an easy task to acquire the probability distribution of the decoder output symbol reliabilities from the distribution of the decoder output LLRs since an assumption for this distribution is not known. Therefore, utilizing a quantile-quantile (Q-Q) plot, an empiric distribution test for random variables, of the symbol reliabilities may lead to establish a model for threshold computation. In Fig. 3, some Q-Q plots are given for different data packets to illustrate the distribution of symbol reliability values. Since the symbol reliability values vary among the packets, it is clear that an adaptive threshold computation method is necessary to establish a robust threshold value in order to select sufficient number of additional reliable decoder output symbols for channel estimation.

Consequently, we propose to establish an approach for threshold computation that includes some statistical properties of the decoder output symbol reliabilities. Thus, the computed threshold is adaptive and robust for each packet that has different minimum, maximum, mean and standard deviation values as shown in Fig. 3. Although it is not clear which statistical values should be utilized to calculate the threshold, we propose to use the minimum and the standard deviation of the symbol reliability values for each packet, which both lie between zero and one (Fig. 3).

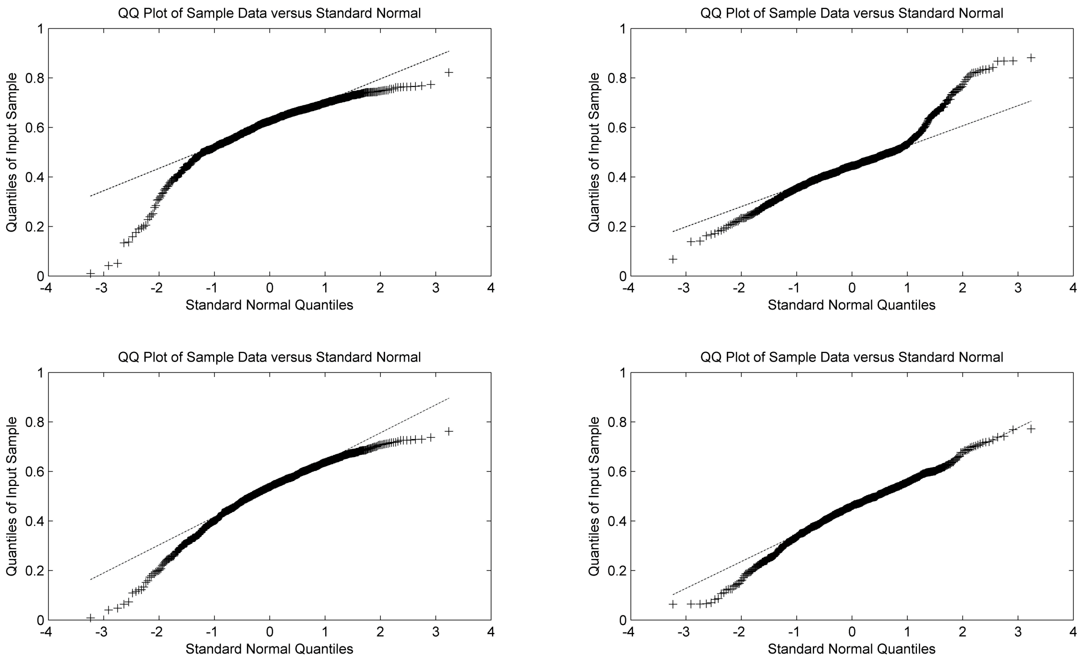

Fig. 3. Q-Q plots of standard normal vs. reliability of the decoder output symbols.

The adaptive threshold, *th*, for each data packet is calculated by

$$th = \omega + \beta.\sigma \tag{7}$$

where ω and σ are the minimum and the standard deviation of decoder output symbol reliabilities for each data packet, respectively. Besides, β is a positive real coefficient that is used for more accurate representation of the adaptive threshold computation approach. The optimum value for β is the question of interest for the proposed approach. However, as the decoder output symbol reliabilities depend on both the number of iterations and the channel conditions, it is hard to set an optimum value for β. Nevertheless, it is evident that selecting as many reliable decoder output symbols as possible should improve the system performance.

4. Simulation Results

In this section, we demonstrate the performance of the proposed adaptive threshold calculation method for iterative channel estimation over HF channels. In recent years HF communication has gained considerable attention because of renewed interest in military and commercial use of the spectrum between 3 MHz and 30 MHz. HF channel is characterized as a multipath time varying environment that produces both time and frequency dispersion. International Telecommunication Union Radio Communication sector (ITU-R) defines the standard poor HF channel [15] as a disturbed channel in mid latitudes, which has several spectral notches in the signal's spectrum. Poor HF channel, used in our simulations, is characterized by two independently fading paths with equal mean attenuation, equal frequency spreads of 1 Hz and 2 ms differential time delay between the two paths [15].

As described in section 2, at the transmitter, K source bits are encoded using a rate 3/4 column and row regular LDPC code whose parity-check matrix is generated randomly. Coded bits are interleaved over five blocks, resulting in a block size of $5N$ bits. 64 QAM symbols are generated using a gray coded square QAM constellation. Waveform requirements are chosen as specified in [13]. Finally, one thousand data packets, which are sufficient to compare the results, are generated and passed through a poor HF channel simulator. At the receiver side, a normalized LDPC code decoding algorithm [16] is used. The decoder is stopped either the maximum number of decoder iterations is met or the syndrome check is satisfied.

BER performance of the system with different β coefficients at various signal-to-noise ratios (SNR) is given in Fig. 4. The number of maximum decoder iterations is set to 20. As illustrated in the figure, the receiver system has the best BER performance when the coefficient β is equal to 1. Performance degrades with increased β because the number of selected additional pilots is getting smaller even though the pilot symbols have higher reliabilities. On the other hand, the performance also degrades when β is equal to zero since all symbols, except the one whose symbol reliability is the minimum among the all decoded output symbols, are selected. Besides, when the threshold exceeds a value, as an example, $\beta \geq 3$, there will be no significant change in the degraded bit error rate performance of the receiver.

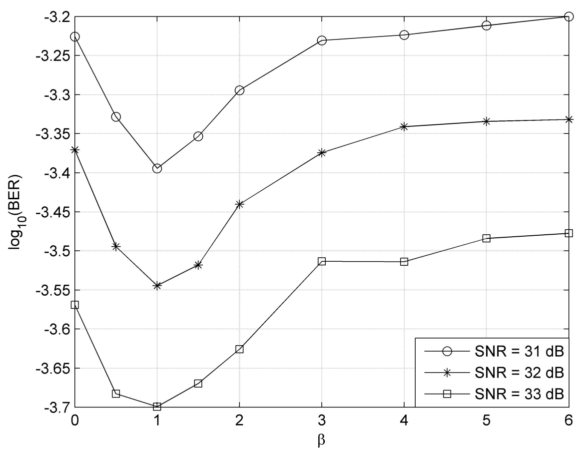

Fig. 4. Bit error rate vs. β for maximum 20 decoder iterations.

The proposed threshold calculation method is also tested with different number of maximum decoder iterations and the results are given in Fig. 5. The system performance is similar for different number of maximum decoder iterations while the threshold is calculated with β=1. However, when the threshold is calculated with different β coefficients, the performance degradation gets higher with the small number of maximum decoder iterations. Therefore, it can be claimed that the importance of the threshold increases in the case of less decoder iterations.

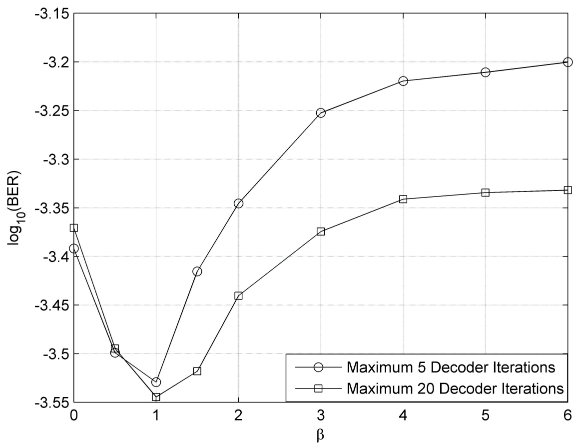

Fig. 5. Bit error rate vs. β for different decoder iterations at SNR=32 dB.

Bit error rate performance of the receiver system is given in Fig. 6 and Fig. 7 in which the maximum number of decoder iterations are set to 5 and 20, respectively. In both figures, "no turbo equalization" refers to the case where the channel estimation and decoding is performed only once. Utilizing the entire set of decoder outputs symbols as the additional pilots is referred to as "no threshold" whereas "fixed threshold" refers to the case of a predetermined threshold value. Our technique, using robust and adaptive threshold for iterative channel estimation, is referred to as "proposed threshold".

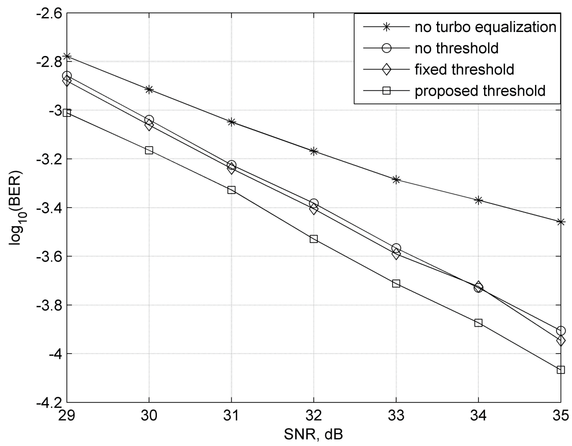

Fig. 6. Bit error rate vs. SNR for maximum 5 decoder iterations for the proposed threshold model, β=1.

Iterative channel estimation improves the system bit error rate performance, as shown in Figs. 6 and 7. Performance of the proposed threshold results in better performance than both the "no threshold" and "fixed threshold" cases though fixed threshold has a marginal performance improvement over the no threshold case as shown in Figs. 6 and 7. Performance improvement of the iterative channel estimation for five decoder iterations when the logarithm of the bit error rate is chosen as -3.4 is discussed. As shown in Fig. 6, no threshold case improves the system performance approximately 2 dB and our proposed threshold has one more dB gain according to the no turbo equalization case. In addition, as illustrated in Fig. 7, the performance improvements for 20 decoder iterations are less than 2 dB and 1 dB for no threshold case and proposed threshold case according to the no turbo equalization, respectively. As a result, the performance improvement of the proposed method for the small number of decoder iterations is higher than that of more decoder iterations.

Performance of the "fixed threshold" is similar to that of "no threshold" in our simulations. No threshold case utilizes all the decoder output symbols as additional pilots whereas fixed threshold eliminates some of the unreliable decoder outputs according to that fixed threshold. However, as shown in Fig. 3, it is hard to determine a proper unique fixed threshold that improves the performance for all different data packets since the statistics of the decoder output symbol reliabilities may change among these packets. On the other hand, performance of our proposed threshold computation method is better than that of the fixed threshold because the proposed threshold is now adaptive and robust for different data packets.

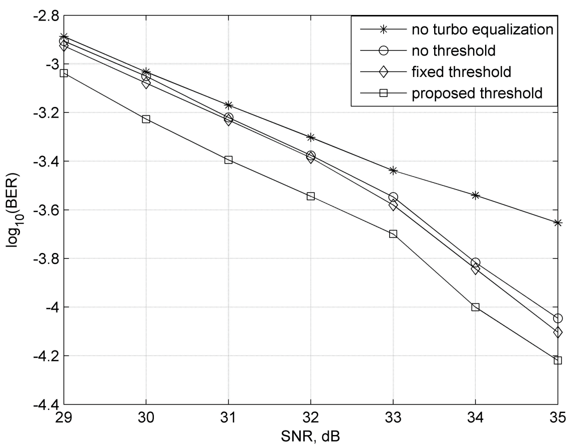

Fig. 7. Bit error rate vs. SNR for maximum 20 decoder iterations for the proposed threshold model, β=1.

The maximum number of turbo iterations is five for all cases given in the simulation results section.

5. Conclusion

A robust and adaptive threshold computation method has been proposed for pilot symbol assisted iterative channel estimation and decoding of coded OFDM based systems. Decoder output symbols, whose reliabilities are higher than the computed threshold, are selected as the additional pilots symbols that are inserted to the channel estimation pilot set in order to obtain more accurate channel estimates. Simulation results show that the proposed threshold computation method improves the bit error rate performance of the system over no threshold and fixed threshold cases.

At first glance, employing as many pilots as possible from the decoder outputs should be considered as a better choice for iterative channel estimation. On the contrary, getting as many reliable additional pilots as possible improves the system performance according to the simulation results. In addition, since the decoder output symbol reliabilities may change for different data packets and the unreliable decoder output symbol reliabilities may degrade the system performance, it is crucial to establish an adaptive threshold for each data packet in order to select the additional pilot symbols. Consequently, since the adaptive threshold depends only on the statistics of each data packet, the proposed threshold computation method is robust for different channel conditions. Moreover, the proposed threshold computation method may also be applicable for different system parameters with the help of a simple, yet effective coefficient, β.

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