LINE FITTING USING HOUGH-LIKE PROCEDURE

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Abstract

The Hough Transform is an image processing algorithm which is used to extract geometric primitives from digital images. It has a number of desirable properties, typically robustness to noise and data occlusion. In this paper a new approach to line fitting problem based on Hough Transform is presented. A critical comparison is made with the more traditional least squares method and potential benefits arising from the application of the proposed HT method is illustrated by examples.

Keywords

Line fitting, least squares method, impulsive noise, Hough transform, accumulator array, voting strategy

1. Introduction

The problem of fitting lines to a given set of data points is a well-known scientific and engineering task. In computer vision, for example, a digitised image may contain a number of discrete "1" pixels lying on a "0" background, and the goal is the detection and parameter extraction of a number of straight lines that fit groups of collinear "1" pixels. A number of standard methods exist for such problems [1]. Ordinary or total least squares methods, for example, seek to minimize the sum of squared vertical, horizontal, or normal distances of all points to the desired line. However, there are two major limitations in all least squares approaches. One is sensitivity to outliers. The other is their inability to handle cases there is more than one underlying line, especially crossing lines, in an image.

Another now classical method of finding line parameters to fit a given data points set is Hough transform [2], [3], [4] which is a special case of the Radon transform [5]. In this approach, first a special transformation is applied to all points and then a two-dimensional search is accomplished to find the maxima in the transform plane. The line parameters are r, the normal distance of the desired line from the origin, and q, the angle that the normal to the line makes with positive x axis; r and q are also the coordinates in the Hough transform output plane. Each data point (x,y) is mapped under the Hough transform to the curve (sinusoid) $r = x \cos q + y \sin q$ in the r - q plane. This equation also represents the line in the x - y plane that has a distance r to the origin and the normal to which makes an angle q with the x axis (Fig. 1). Therefore, all points in the x - y plane located on line $r_0 = x \cos q_0 + y \sin q_0$ are mapped to curves in the r - q plane that all pass through the point (r_0, q_0).

To fit a straight line to a set of data points, both r and q axes have to be quantized and hence a two-dimensional accumulator array must be constructed in the r - q plane. The Hough transform equation is applied to each point in the data set and the contents of all the cells in the transform plane that the corresponding curve passes through are incremented. Then, a search is made to locate the "maximal" points in the r-q plane.

The Hough transform method is capable of handling a fairly high amount of noise contamination; however, it has certain drawbacks that may drastically limit its use. It is the heavy burden of computational complexity and massive storage requirement.

To overcome these drawbacks, a new technique of Hough transform, "probabilistic Hough transform (PHT)", has appeared in recent years. The PHT approaches [6], [7], [8], [9], [10] although differing in many respects, share a common feature: each set of n feature points in the image, where n equals the number of parameters in Hough space, is mapped into just one cell in the parameter space. This principle is known as many-to-one mapping [11]. In this way the data points vote for the more probable candidates as opposed to all possible cells in the parameter space. Votes accumulate in the areas of the transform plane associated with the higher probabilities of patterns.

A new approach, introduced by authors, also uses a many-to-one mapping and the estimation of line parameters is accomplished in two phases. First the direction (normal parameter q) of the line is estimated and slope of the line is calculated. In second phase, an estimate of intercept of the line is obtained.

The organisation of the rest of the paper is as follows. In Section 2, the model of data sets contaminated by impulsive and gaussian noise is introduced. In Section 3, the proposed new HT method for line fitting is presented and detailed description of this method is provided. Experimental results are presented in the Section 4. This Section includes the comparison between least squares method and proposed new HT method. Section 5 summarises the results and offers some concluding remarks.

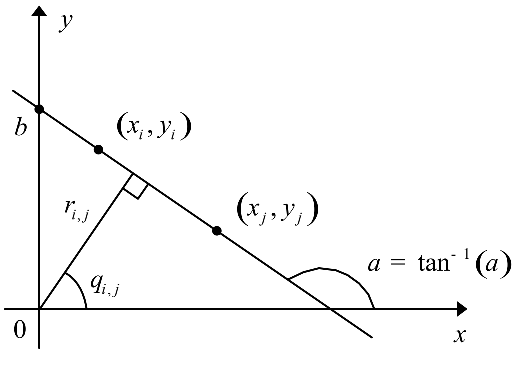


Fig. 1 Relationship between (x, y), (r, q) and (a, b)

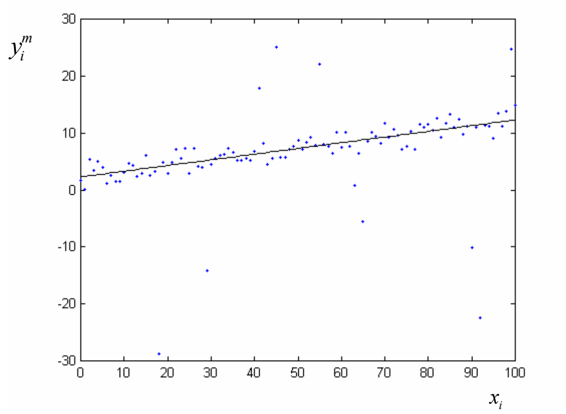


Fig. 2 Example of modelling data set for $I_{2,1.5,10}$

2. Modelling of data set

Consider some physical process, which can be described by linear form with parameters (a,b)

$$y = ax + b \tag{1}$$

Next consider a data set y^m, which is obtained from measurement of the process. In ideal case, without presence of some kind of noise, the measured values can be expressed by $y_i^m = ax_i + b$; $i = 1,2,3,\ldots$. In real cases we must consider the effect of noise due to errors involved by measuring equipment or human operator, respectively. In our experiments we model the effect of noise (gaussian and impulsive) by additive quantity $I_{\sigma,h,n}$:

$$y_i^m = ax_i + b + I_{\sigma,h,n} \ , \tag{2}$$

where

$$I_{\sigma,h,n} = \begin{cases} g(0,\sigma) \\ \\ g(0,n\sigma) & \text{if } \ g(0,\sigma) \geq h\sigma \end{cases} \tag{3}$$

where $g(0, \sigma)$ and $g(0, n\sigma)$ are normal random numbers with zero mean and standard deviation σ or $n\sigma$, respectively. h and n are threshold and multiplicative parameters for impulsive noise. If $g(0, \sigma) < h\sigma$, the value of $g(0, \sigma)$ represents gaussian noise with parameters $(0, \sigma)$. However, if $g(0, \sigma) \geq h\sigma$, a new normal random number with parameters $(0, n\sigma)$ is generated representing impulsive noise. An example of such data set is shown in Fig. 2.

3. Proposed new method for line fitting

The earliest and most classical PHT algorithm is the randomized Hough transform (RHT) [10], which is based on the fact that a single parameter point can be determined uniquely with a pair, triple or generally n feature points from the data set, depending on the complexity of the curves to be detected. For example, in the case of line detection, each point in parameter space can be expressed with two points from set of data points. Given set of data points

$$D = \{d_i = (x_i, y_i) \mid i = 1,2,3,\ldots\} \tag{4}$$

such point pairs (d_i, d_j) are selected randomly, where $d_i = (x_i, y_i)$ and $d_j = (x_j, y_j)$, the parameter point (a, b) is solved from the curve equation, i.e.,

$$\begin{aligned} y_i &= ax_i + b \\ y_j &= ax_j + b \end{aligned} \tag{5}$$

The cell $H(a, b)$ is accumulated in the accumulator space. This random selection of data point pair is called "random sampling". The RHT is run long enough to detect a global maximum in the accumulator space. The parameter point (a, b) of the global maximum describes the parameters of the detected curve, which can then be removed from the set to start the algorithm again with the remaining data points. The algorithm of the RHT is

1. Create the set D of all data points.
2. Select a point pair (d_i, d_j) randomly from the set D.
3. If the points do not satisfy the predefined distance criterion, which is set by the user, go to Step 2; otherwise continue to Step 4.
4. Solve the parameter space point (a, b) using the curve equation with the points (d_i, d_j).
5. Accumulate the cell $H(a, b)$ in the accumulator space. If the $H(a, b)$ is equal to the threshold T, parameters a and b describe the parameters of the detected curve; otherwise continue to Step 2.

The proposed approach is inspired by RHT algorithm, but estimation of line parameters is accomplished in two phases. First the direction (normal parameter q) of the line is estimated and slope of the line is calculated. In second phase, an estimate of intercept of the line is obtained. It follows that instead two-dimensional accumulator array, a two one-dimensional accumulator arrays is used. The amount of storage used by this method is much less than that used in the above HT implementations. The computational cost resulting from two-dimensional search for finding the maximum point is also reduced on two one-dimensional searching processes.

The following are the explanations of each step of the proposed algorithm.

Step 1: Select start point. Given a set of N measured data pairs $\{(x_i, y_i^m) \mid i = 1, 2, \ldots, N\}$. Each data pair we can consider as a single data point in $\mathbf{R}^2$. One such point is picked out from the set as the starting one according to the sequence of the elements in the set. When this point has been processed, it will be removed from the set and next time the next point will be selected as the start point.

Step 2: Select each remaining data point. After the start point has been selected, each remaining points will be selected in turn. If the start point is the mth data element in the data set, the $(m+d)$th, $(m+d+1)$th, ... , until the Nth element is chosen one by one. d is the distance criterion similar those in RHT [10] and effect of the criterion will be shown further. The selected point and the start point would jointly create a pair of points, which would determine just one straight line.

Step 3: Calculate θ parameter. Straight line crossing the particular pair of data points $\{(x_i, y_i^m), (x_j, y_j^m)\}$ where i is the subscript of the start point, and j satisfies $i < j \le N$) is to be detected, (Fig. 1) and the corresponding Hough parameters will be calculated. Suppose the coordinates of the start data point are (x_i, y_i^m), and the coordinates of another one are (x_j, y_j^m). From the standard HT we have:

$$\begin{aligned} r_{i,j} &= x_i \cos q_{i,j} + y_i^m \sin q_{i,j} \\ r_{i,j} &= x_j \cos q_{i,j} + y_j^m \sin q_{i,j} \end{aligned} \tag{6}$$

where θ and r are defined in Section 1. Then

$$\theta_{i,j} = \tan^{-1}\left(\frac{x_j - x_i}{y_i^m - y_j^m}\right) \tag{7}$$

Because only parameter θ is needed in this step, parameter r or b, respectively, will be calculated in further step.

Step 4: Accumulate parameter space $H(\theta)$. A one dimensional accumulator array $H(\theta)$ is created, and each different $\theta_{i,j}$ defined by (7) will be vote for appropriate cell in this accumulator array. A classical voting strategy known from standard HT is not usable in our task because data where K is user defined value. Effect of this voting strategy in comparison with classical one is depicted in Fig. 3. Above mentioned distance criterion d has influence at distribution of votes in accumulator array. If d is greater than 1, the votes are distributed in closer neighbourhood of valid peak than for d =1. It comes to this, that for d=1 more insignificant votes are generated than for d>1. Effect of the distance criterion is shown in Fig. 4.

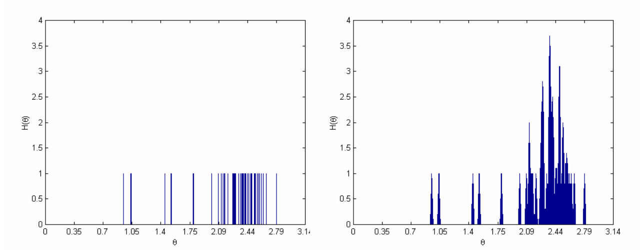
Step 5. Detect peak in $H(\theta)$ and calculate parameter a. After above voting procedure has been finished, the accounts in the accumulator array $H(q)$ is detected. If one cell's value is larger than each other, $H(q_{max}) = \max\{H(q)\}$ the q_{max} is found as a best estimation of parameter q. A slope-intercept parameter a of the investigated line we have easily obtain from (8) (Fig. 1).

$$a_{est} = \tan\left(q_{\max} + p/2\right) \tag{8}$$

Step 6. Accumulate parameter space $H(b)$. Again a one dimensional accumulator array $H(b)$ is created. Each value b_i defined by

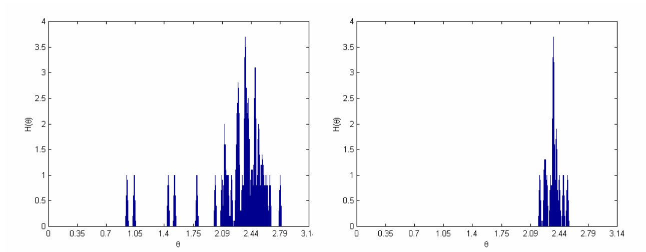
$$b_i = y_i^m - a_{est} x_i \tag{9}$$

where a_{est} is value computed in previous step, will be vote for appropriate cell in the parameter space and again is used above introduced voting strategy : for b_i defined by (9) increment the cell $H(b_{i\pm 1})$ by $(1-l/L)$, $l = 0, 1, 2, \ldots, L$.



(a) (b)

Fig. 3 Comparison of two voting strategies: (a) classical one and (b) introduced by authors.



(a) (b)

Fig. 4 Effect of distance criterion: (a) without distance criterion d=1, (b) with distance criterion d=5, for N=10 data points and $I_{1,2,5}$.

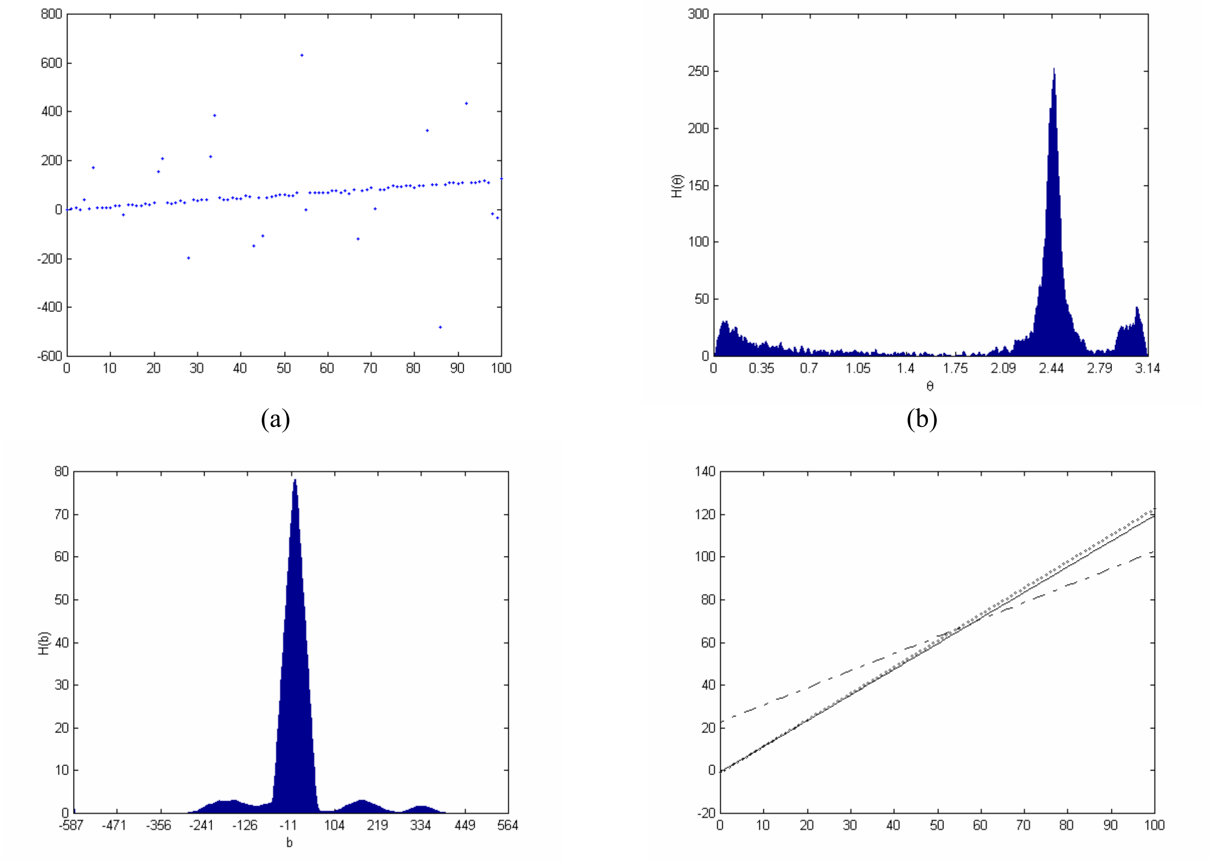
Step 7. Detect peak in $H(b)$ and estimate parameter b. Parameter space $H(b)$ is again searched for a maximum value and parameter b is estimated by following manner:

$$b_{est} = b_i \mid H(b_i) = \max_b \{H(b)\} \tag{10}$$

Step 8. End.

4. Experimental results

The performance of the proposed HT method was tested using model of data sequence (2) in comparison with standard least squares method. Before doing experiments, we test our method with several combinations of the parameters d, K, L and the best combination is presented as the test results. In first experiment we apply both methods on data sequence with bipolar impulsive noise:



(a) (b) (c) (d)

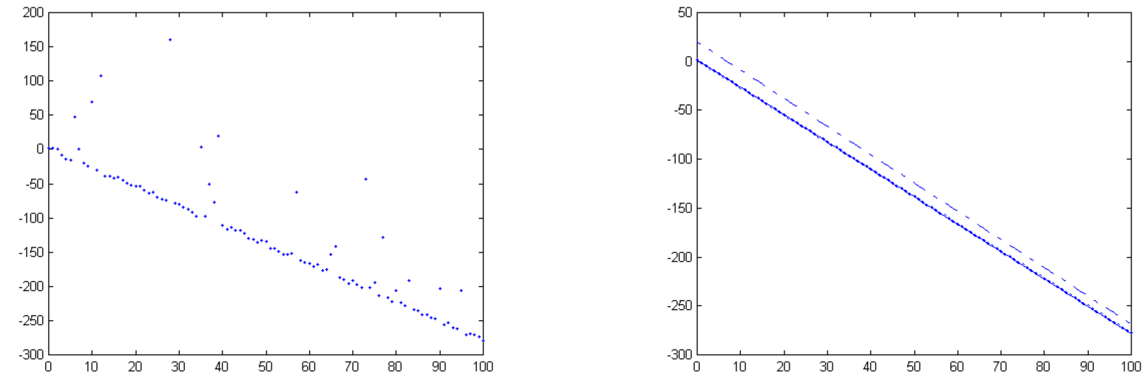
Fig. 5 An example of data sequence and its extracted results. (a) Data sequence $y_i^m = 1.2i - 0.5 + I_{5,1.5,50}$, i=1, 2, ..., 100. (b) Accumulator array *H(θ)* with peak at θ =2.4609 rad $\Rightarrow$ *a* = tan(2.4609 + π/2) = 1.2349 . (c) Accumulator array *H(b)* with peak at *b*=0.9020. (d) ___ ideal straight line $y_i = 1.2i - 0.5$, ⋯ - line estimated by proposed HT method , -.-.- - line estimated by least squares method.

	least squares method				proposed HT method			
	a	b	Δa	Δb	a	b	Δa	Δb
$I_{1,\,1.5,\,10}$	1.193	-0.4258	0.007	0.0742	1.196	-0.5094	0.004	0.0094
$I_{2,\,1.5,\,20}$	1.2379	-1.5289	0.0379	1.0289	1.2088	-0.5044	0.0088	0.0044
$I_{3,\,1.5,\,30}$	1.515	-15.1336	0.315	14.6336	1.2131	-0.6911	0.0131	0.1911
$I_{4,\,1.5,\,40}$	1.3002	-9.3020	0.1002	8.802	1.1960	-0.4549	0.004	0.0451
$I_{5,\,1.5,\,50}$	0.7984	22.9110	0.4016	23.411	1.2349	-0.902	0.0349	0.402

Tab. 1 The comparison between least squares method and proposed HT method (experiment 1)

	least squares method				proposed HT method			
	a	b	Δa	Δb	a	b	Δa	Δb
$I_{1,\,1.5,\,10}$	-2.7927	2.1804	0.0073	0.4804	-2.7929	1.6804	0.0071	0.0196
$I_{2,\,1.5,\,20}$	-2.8046	6.3777	0.0046	4.6777	-2.7929	1.8354	0.0071	0.1354
$I_{3,\,1.5,\,30}$	-2.8872	20.1850	0.0872	18.485	-2.7929	1.3891	0.0071	0.3109

Tab. 2 The comparison between least squares method and proposed HT method (experiment 2)



(a) (b)

Fig. 6 An example of data sequence with unipolar impulsive noise and its extracted results. (a) Data sequence $y_i^m = -2.8i + 1.7 + |I_{3,1.5,30}|$, i=1, 2, ..., 100. (b) ___ - ideal straight line $y_i = -2.8i + 1.7$, $\cdots$ - line estimated by proposed HT method , -.-.- - line estimated by least squares method.

$$y_i^m = 1.2i - 0.5 + I_{\sigma,h,n} \ , \tag{11}$$

where a=1.2, b=-0.5 and data sequence has length N=100. Fig. 5 illustrate an example for $I_{2,1.5,20}$. The comparison between least squares method and proposed HT method is shown in Table 1.

In second experiment we examine data sequence with unipolar impulsive noise:

$$y_i^m = -2.8i + 1.7 + |I_{\sigma,h,n}| \ , \tag{12}$$

where a=-2.8, b=1.7 and the sequence has length N = 100 again. An example of such data and achieved results is shown in Fig. 6. Tab. 2 presents estimated parameters using least squares method and proposed HT method.

5. Conclusions

In this paper, a new approach to line fitting problem based on the Hough transform is presented. To overcome the drawbacks of the Standard Hough transform, the 2D voting process was divided into two 1D substages for each parameter. To handle a scattered data points, the new voting strategy inspired by Fuzzy HT was introduced. The potential benefits arising from the application of the proposed HT method has been illustrated by examples. Through the experiments with data sets contaminated by both bipolar and unipolar impulses, the performance of the proposed method is evaluated in comparison with standard least squares method. The proposed Hough transform method was found to be robust in the presence of impulsive and Gaussian noise and the results obtained using this approach are favourable when compared with those from traditional least squares based approaches.

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