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Proceedings of the 15th European Conference on Antennas and Propagation

eISBN: 978-88-31299-02-2

DOI: https://doi.org/10.23919/EuCAP51087.2021.9411446

Pulsed EM Field Scattering From a Narrow Superconducting Strip: A Solution Based on the Marching-On-In-Time Cagniard-DeHoop Method

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Dedicated to the memory of Professor Hans Blok

***Abstract*—Pulsed electromagnetic (EM) scattering from a relatively narrow superconducting strip is analyzed with the aid of the EM reciprocity theorem and the Cagniard-DeHoop (CdH) technique. The analysis yields a stable convolution-type equation that is solved using the marching-on-in-time (MOT) technique for coefficients representing the time-domain (TD) electric current induced in the strip. Illustrative numerical examples are validated with the help of the CdH method of moments (CdH-MoM).**

***Index Terms*—computational electromagnetics, time-domain analysis, time-domain integral equation technique, Cagniard-DeHoop technique, transient scattering, superconductivity.**

I. Introduction

The CdH technique [1]–[5] is a joint transform method that was originally developed to explain seismic data. Since the CdH technique is capable of solving a large class of canonical EM problems directly in TD, it has been found useful for benchmarking purely numerical techniques, both in accuracy and in speed of computation. More recently, a fundamentally new TD integral-equation technique, referred to as the CdH-MoM, has been put forward [6], [7], thereby demonstrating the applicability of the CdH technique in computational electromagnetics (see also [8]). An illustrative application of the CdH-MoM is also the subject of the present contribution, where the EM plane-wave induced TD electric current in a relatively narrow, planar superconducting strip is analyzed. For a thorough discussion of applications of superconductors in antenna and microwave engineering, the reader is referred to [9], [10].

II. Problem Definition and its Formulation

We shall analyze the TD response of a superconducting planar strip that occupies $\{-w/2 < x < w/2, -\delta/2 < z < \delta/2\}$, where $w > 0$ denotes the strip's width and $\delta > 0$ its (vanishing) thickness. The strip is located in the homogeneous, isotropic and loss-free background medium described by (real-valued, scalar and positive) parameters ϵ_0 and μ_0 with the corresponding EM wave speed $c_0 = (\epsilon_0\mu_0)^{-1/2} > 0$ and wave admittance $Y_0 = 1/c_0\mu_0$. The time coordinate is $\{t \in \mathbb{R}; t > 0\}$, $\mathrm{H}(t)$ denotes the Heaviside unit-step function and $\delta(t)$ is the Dirac-delta distribution. The strip is irradiated by a E-polarized, uniform impulsive EM plane wave, $E_y^{\mathrm{i}}(x,z,t) = e^{\mathrm{i}}(t - p_0 x + \gamma_0 z)$, where $e^{\mathrm{i}}(t)$ denotes its (causal) signature and $p_0 = \sin(\theta)/c_0$ with $\gamma_0 = \cos(\theta)/c_0$ are the wave slowness parameters in the x- and z-direction, respectively (see Fig. 1). The EM properties of the superconducting strip

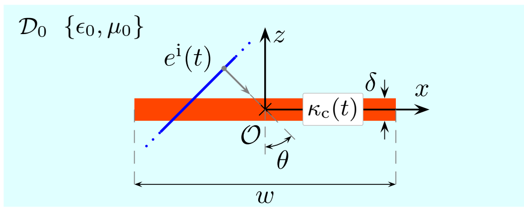

Fig. 1. A narrow superconducting strip irradiated by the impulsive EM plane wave.

are described by its two-fluid conduction relaxation function (see [10, Sec. 1.2] and [11, Sec. 19.5])

$$\kappa_{\mathrm{c}}(t) = \sigma_n (T/T_{\mathrm{c}})^4 \delta(t) + \left[1 - (T/T_{\mathrm{c}})^4\right] \Lambda^{-1} \mathrm{H}(t) \tag{1}$$

where σ_n denotes the normal-state conductivity at the critical temperature T_{c}, $\Lambda = \mu_0 \lambda^2$ is the London parameter, in which λ represents the penetration depth at temperature $T = 0$ and, finally, T/T_{c} is the reduced temperature. It is assumed that the strip shows a high contrast with respect to the embedding such that the equivalent TD layer conductance $G^{\mathrm{L}}(t) = \delta\kappa_{\mathrm{c}}(t)$ is of order $O(1)$. Then, the following TD cross-layer conditions apply

$$\lim_{z\downarrow\delta/2} E_y(x,z,t) - \lim_{z\uparrow-\delta/2} E_y(x,z,t) = 0 \tag{2}$$

$$\lim_{z\downarrow\delta/2} H_x(x,z,t) - \lim_{z\uparrow-\delta/2} H_x(x,z,t) = \partial J_y^{\mathrm{s}}(x,t) \tag{3}$$

as $\delta \downarrow 0$, for all $\{-w/2 < x < w/2\}$ and $t > 0$, where

$$\partial J_y^{\mathrm{s}}(x,t) = G^{\mathrm{L}}(t) *_t E_y(x,0,t) \tag{4}$$

has the meaning of contrast sheet electric current density [12, Eq. (12)] and $*_t$ denotes the time-convolution operator.

Employing the TD cross-layer conditions, the EM reciprocity theorem of the time-convolution type [11, Sec. 28.2] is next applied to interrelate the (causal) scattered EM field state (further denoted by superscript $^{\mathrm{s}}$) with the (causal) testing field state (further denoted by superscript $^{\mathrm{T}}$). In this way, we arrive at

$$\int_{x=-w/2}^{w/2} E_y^{\mathrm{T}}(x,0,t) *_t \partial J_y^{\mathrm{s}}(x,t)\mathrm{d}x = \int_{x=-w/2}^{w/2} E_y^{\mathrm{s}}(x,0,t) *_t \partial J_y^{\mathrm{T}}(x,t)\mathrm{d}x \tag{5}$$

The TD reciprocity relation (5) with Eqs. (4) and (1) is further solved for the induced electric-current response induced in the superconducting strip. The TD solution presented in this contribution is limited by the assumption that the strip's width is relatively small with respect to the spatial support of the exciting plane-wave signature $e^{\mathrm{i}}(t)$.

III. Problem Solution

The EM scattering problem formulated in the previous section is further solved via the CdH technique that employs the one-sided time Laplace transformation

$$\hat{E}_y(x,z,s) = \int_{t=0}^{\infty} \exp(-st) E_y(x,z,t)\mathrm{d}t \tag{6}$$

with $\{s \in \mathbb{R}; s > 0\}$ thus relying on Lerch's uniqueness theorem [13, Appendix]. The temporal transformation is further combined with the wave slowness representation

$$\hat{E}_y(x,z,s) = (s/2\pi\mathrm{i}) \int_{p=-\mathrm{i}\infty}^{\mathrm{i}\infty} \exp(-spx)\tilde{E}_y(p,z,s)\mathrm{d}p \tag{7}$$

that entails $\partial_x \to -sp$. Assuming the uniform spatial distribution of the induced current density and introducing Eqs. (6) and (7) in (5), the resulting (transform-domain) reciprocity relation can be readily cast into the following form

$$\left[\hat{Z}^{\mathrm{ext}}(s) + 1/w\hat{G}^{\mathrm{L}}(s)\right]\, \hat{I}^{\mathrm{s}}(s) = \hat{e}^{\mathrm{i}}(s)\,\mathrm{i}_0(sp_0 w/2) \tag{8}$$

where $\mathrm{i}_0(x)$ denotes the modified spherical Bessel function of the first kind and

$$\hat{Z}^{\mathrm{ext}}(s) = \frac{s\mu_0}{2\pi\mathrm{i}} \int_{p=-\mathrm{i}\infty}^{\mathrm{i}\infty} \mathrm{i}_0^2(spw/2)\frac{\mathrm{d}p}{2\gamma_0(p)} \tag{9}$$

has the meaning of external impedance [14], where $\gamma_0(p) = (1/c_0^2 - p^2)^{1/2}$ with $\mathrm{Re}(\gamma_0) \geq 0$.

To solve the resulting relation (8) iteratively via the MOT approach, the unknown electric current is expanded in

$$\hat{I}^{\mathrm{s}}(s) \simeq \sum_{k=1}^{M} i_k \hat{\Lambda}_k(s) \tag{10}$$

where i_k denotes the unknown coefficients (in A) and the TD original of the triangular function is defined by

$$\Lambda_k(t) = \begin{cases} 1 + (t - t_k)/\Delta t & \text{for } t \in [t_{k-1}, t_k] \\ 1 - (t - t_k)/\Delta t & \text{for } t \in [t_k, t_{k+1}] \end{cases} \tag{11}$$

along the discretized time axis $\{t_k = k\Delta t, \Delta t > 0, k = 1, 2, \cdots, M\} \subset \{t \in \mathbb{R}; t > 0\}$. Substituting Eq. (10) the s-domain reciprocity-based relation (8) and transforming the result to the TD, we obtain a convolution-type equation that can be solved for the current coefficients using the MOT technique. This leads to

$$i_m = c_0 \Delta t Y_0 e^{\mathrm{i}}(t_m)/\Phi(\Delta t) - \Phi^{-1}(\Delta t) \sum_{k=1}^{m-1} i_k \times \left[\Phi(t_m - t_{k-1}) - 2\Phi(t_m - t_k) + \Phi(t_m - t_{k+1})\right] \tag{12}$$

for all $m = \{1, \ldots, M\}$, where

$$\Phi(t) = \left[\Upsilon(w,t) - 2\Upsilon(0,t) + \Upsilon(-w,t)\right]/w^2 + (\Lambda_1/\mu_0 w\delta)\left[1 - \exp(-t/\sigma_1\Lambda_1)\right]\mathrm{H}(t) \tag{13}$$

and $\sigma_1 = \sigma_n(T/T_{\mathrm{c}})^4$, $\Lambda_1 = \Lambda/[1 - (T/T_{\mathrm{c}})^4]$ (see Eq. (1)), and the TD function $\Upsilon(x,t)$ follows upon applying the CdH technique as

$$\Upsilon(x,t) = \frac{1}{2\pi}\left\{\frac{x^2}{2}\cosh^{-1}\left(\frac{c_0 t}{|x|}\right) + \frac{c_0 t x}{2}\left(\frac{c_0^2 t^2}{x^2} - 1\right)^{1/2} - c_0 t|x|\tan^{-1}\left[\left(\frac{c_0^2 t^2}{x^2} - 1\right)^{1/2}\right]\right\}\mathrm{H}(t - |x|/c_0) + \frac{c_0 t x}{2}\mathrm{H}(x)\mathrm{H}(t) \tag{14}$$

for all $x \in \mathbb{R}$ and $t > 0$.

IV. Numerical Example

In this section, the MOT solution (12) is validated with the aid of the CdH-MoM technique [6]. To that end, we shall analyze the TD response of a YBCO (= Yttrium Barium Copper Oxide) superconducting planar strip of width $w = 10\,\mu\mathrm{m}$ and thickness $\delta = 0.5\,\mu\mathrm{m}$. Its conduction relaxation function is described via parameters taken from [15]: $T = 77\,\mathrm{K}$, $T_{\mathrm{c}} = 92.5\,\mathrm{K}$, $\sigma_n = 1.7 \cdot 10^6\,\mathrm{S/m}$ and $\lambda = 0.3\,\mu\mathrm{m}$. The plane wave is defined by its bipolar triangular signature

$$e^{\mathrm{i}}(t) = (2e_{\mathrm{m}}/t_{\mathrm{w}})\Big[t\,\mathrm{H}(t) - 2\big(t - t_{\mathrm{w}}/2\big)\mathrm{H}\big(t - t_{\mathrm{w}}/2\big) + 2\big(t - 3t_{\mathrm{w}}/2\big)\mathrm{H}\big(t - 3t_{\mathrm{w}}/2\big) - (t - 2t_{\mathrm{w}})\,\mathrm{H}(t - 2t_{\mathrm{w}})\Big] \tag{15}$$

with $e_{\mathrm{m}} = 1.0\,\mathrm{V/m}$, $c_0 t_{\mathrm{w}} = 100\,w$ and $\theta = 0$. Figure 3 shows the electric-current density induced in the center of the superconducting strip. As can be seen, the results calculated via the

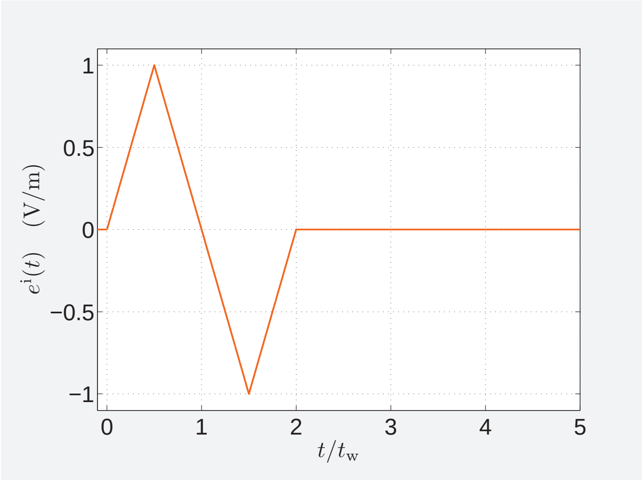

Fig. 2. Plane wave pulse shape.

MOT procedure agree well with the ones achieved using the CdH-MoM assuming the piecewise linear spatial distribution over the strip divided into 2 nodes. Minor discrepancies can be attributed to the difference in the spatial basis functions.

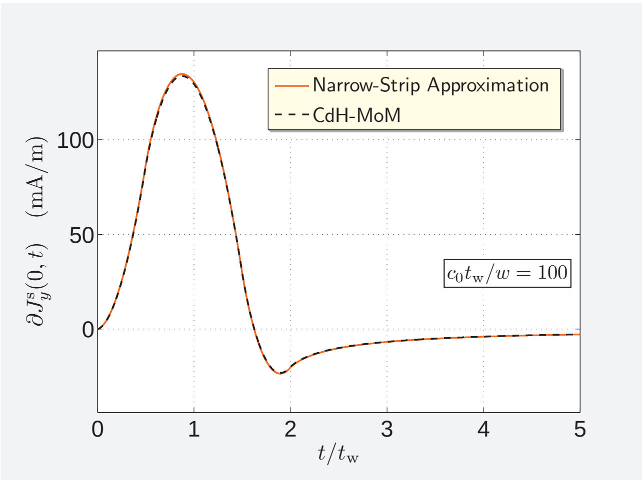

Fig. 3. Pulse shape of the electric-current density induced at the center of the superconducting strip.

V. Conclusion

Combining the time-convolution EM reciprocity theorem with the CdH method, a novel TD integral-equation technique for analyzing the TD plane-wave EM scattering from a relatively narrow superconducting planar strip was introduced. The validity of the proposed computational methodology was conclusively demonstrated with the aid of the CdH-MoM [6].

Acknowledgment

The research reported in this paper was financed by the Czech Science Foundation under Grant No. 20-01090S.

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