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# Probabilistic design of fibre concrete structures

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**Abstract**. Advanced computer simulation is recently well-established methodology for evaluation of resistance of concrete engineering structures. The nonlinear finite element analysis enables to realistically predict structural damage, peak load, failure, post-peak response, development of cracks in concrete, yielding of reinforcement, concrete crushing or shear failure. The nonlinear material models can cover various types of concrete and reinforced concrete: ordinary concrete, plain or reinforced, without or with prestressing, fibre concrete, (ultra) high performance concrete, lightweight concrete, etc. Advanced material models taking into account fibre concrete properties such as shape of tensile softening branch, high toughness and ductility are described in the paper. Since the variability of the fibre concrete material properties is rather high, the probabilistic analysis seems to be the most appropriate format for structural design and evaluation of structural performance, reliability and safety. The presented combination of the nonlinear analysis with advanced probabilistic methods allows evaluation of structural safety characterized by failure probability or by reliability index respectively. Authors offer a methodology and computer tools for realistic safety assessment of concrete structures; the utilized approach is based on randomization of the nonlinear finite element analysis of the structural model. Uncertainty of the material properties or their randomness obtained from material tests are accounted in the random distribution. Furthermore, degradation of the reinforced concrete materials such as carbonation of concrete, corrosion of reinforcement, etc. can be accounted in order to analyze life-cycle structural performance and to enable prediction of the structural reliability and safety in time development. The results can serve as a rational basis for design of fibre concrete engineering structures based on advanced nonlinear computer analysis. The presented methodology is illustrated on results from two probabilistic studies with different types of concrete structures related to practical applications and made from various materials (with the parameters obtained from real material tests).

## 1. Introduction

The probabilistic approach represents the most accurate method for the safety assessment of civil engineering structures. The accuracy of this approach is much higher if the nonlinear structural analysis is used as a limit state function. The numerical simulation resembles a real testing of structures by considering a representative group of samples, which can be statistically analyzed for the assessment of safety. This concept is supported by the new ***fib*** Model Code 2010 [1] where rational safety assessment approach is presented, which reflects new developments in safety formats based on probabilistic methods. In the Chapter 4 on *Principles of structural design* the probabilistic safety format is introduced as a general and rational basis of safety evaluation. In addition to the partial factor format (which remains as the main safety format for most practical cases) a global resistance format is recommended for nonlinear analysis.

In case of fibre concrete structures the design codes are generally not available or not sufficient. Therefore, the computer simulation based on advanced nonlinear finite element analysis has a big potential in design of these structures. The numerical results can be introduced into a suitable engineering safety concept, from which the fully probabilistic analysis is the ultimate tool for design and safety assessment. It is superior to simplified methods because it provides information on the variability of resistance. However, it is computationally demanding and requires good information about random properties of input variables. Therefore, it is applied in special cases, where consequences of failure substantiate the increased effort.

The probabilistic analysis is performed here using software SARA, which integrates ATENA and FReET program tools. The variability of basic properties is described by distribution functions and its parameters (mean, standard deviation, etc.). Probabilistic analysis of the resistance is performed by numerical method such as Latin hypercube sampling (LHS) method. Resulting set of resistance values is approximated by a probability distribution function (PDF) of global resistance, and describes the random properties of the resistance. Finally, for a required reliability index $\beta$, or failure probability $P_f$, a value of the design resistance $R_d$ shall be calculated.

## 2. Probabilistic nonlinear assessment of engineering structures

Probabilistic analysis is an efficient tool for safety assessment of civil engineering structures, in particular of concrete or fibre concrete structures. In the probabilistic nonlinear approach the structural resistance $R_d$ is calculated by means of the probabilistic nonlinear analysis. The classical statistical and reliability approach is to consider material parameters as random variables with prescribed distribution function. The stochastic response requires repeated analyses of the structure with these random input parameters, which reflects randomness and uncertainties in the input values (see e.g. [2]). In this approach, the resistance function *r(r)* is represented by nonlinear structural analysis and loading function *s(s)* is represented by action model. Safety can be evaluated by the reliability index $\beta$, or alternatively by failure probability $P_f$ taking into account all uncertainties due to random variation of the input values – material properties, dimensions, loading, and other.

Probabilistic analysis based on the nonlinear numerical simulation includes following steps:

- *Numerical model based on the nonlinear finite element analysis.* This model describes the resistance function *r(r)* and performs a deterministic analysis of resistance for given set of input variables.
- *Randomization of input variables* (material properties, dimensions, boundary conditions, etc.). This can also include some effects of actions, which are not in the action function *s(s)* (for example pre-stressing, dead load etc.). Random properties are defined by random distribution type and its parameters (mean standard deviation, etc.). They describe the uncertainties due to statistical variation of resistance properties.
- *Probabilistic analysis of resistance and action.* This can be performed by stratified method of Monte Carlo-type of sampling, such as LHS sampling method. Results of this analysis provide random parameters of resistance and actions, such as mean, standard deviation, etc. and the type of distribution function for resistance.
- *Evaluation of safety* using reliability index $\beta$ or failure probability $P_f$ .
- Probabilistic analysis can be also used for *determination of design value of resistance function r(r)* expressed as $R_d$. Such analysis involves repeatedly the first three steps above, and $R_d$ is determined for required reliability $\beta$ or failure probability $P_f$.

In order to make the application of the probabilistic nonlinear analysis user-friendly, special software tool has been developed by the authors and their co-workers. The resulting software SARA (Structural Analysis and Reliability Assessment) integrates software tools ATENA and FReET. It is equipped with a user-friendly shell called *SARA Studio*, which leads the user interactively through the modelling and randomization process of the solved problem as described above. All features (or just selected ones) of the involved programs including modeling of deterioration/degradation phenomena can be utilized also in the reliability analysis and performance-based assessment of concrete structures. For this purpose, the interconnectivity between ATENA Engineering as well as ATENA Science (i.e. ATENA input file), with the probabilistic modules was achieved. The program control and data exchange is organized by an efficient and user friendly shell interface (Figure 1).

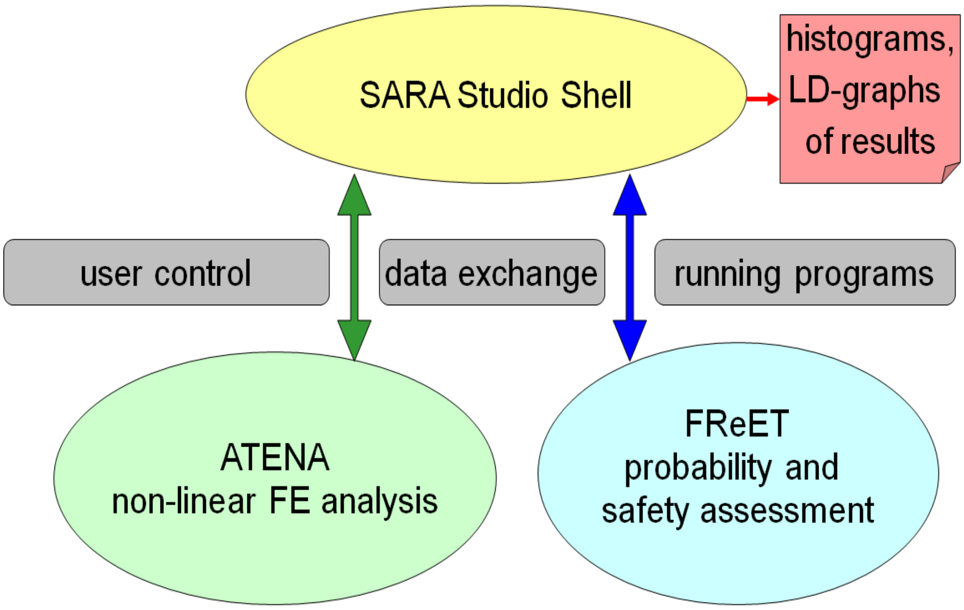


**Figure 1.** Structure of the SARA system.

## 3. Material damage modeling

The damage and failure of fiber concrete structures is analyzed in program ATENA [3] for nonlinear analysis of concrete structures. ATENA is capable of a realistic simulation of concrete behavior in the entire loading range with ductile as well as brittle failure modes as shown for instance in [4]. The program was developed for realistic simulation of reinforced concrete structures. It is based on the finite element method with nonlinear material models, and utilized for analysis of beams and girders, plates and shells, bridges, tunnels, dams, composite structures, strengthenings, structural details, fastenings, timber and masonry structures, and also structures made of fibre concrete, e.g. [5], [6].

The ATENA software consists of calculating core ensuring the nonlinear numerical analysis, and a user-friendly graphical interface for an efficient communication between end-user and program core. The numerical core covers the finite element technology, nonlinear material models and nonlinear solution.

*The nonlinear material models* are based on the orthotropic damage theory and special concrete-related theory of plasticity. As one of the main features the nonlinear fracture mechanics is employed for concrete cracking in tension. Based on the fracture energy approach the tensile cracks are modeled as smeared material damage which enables utilization of the continuum mechanics even for the damaged material. Objectivity of the solution (independency on the finite element mesh) is ensured using crack band method. The material law for concrete exhibits softening after reaching the tensile strength. The behavior of concrete in compression is defined by special theory of plasticity (three-parameter model), with non-associated plastic flow rule and softening. This material model for concrete can successfully reproduce also other important effect, such as volume change under plastic compression or compressive confinement. The constitutive model is described in more detail e.g. in [7]. Nonlinear solution is performed incrementally with equilibrium iterations in each load step.

The native graphical user-interface in ATENA Engineering supports all the specifics of reinforced concrete, e.g. input of discrete reinforcing bars, or evaluation of crack patterns in the damaged structural model. ATENA Engineering is perfectly suitable for static analysis of concrete structures, obtaining their load-displacement response and resistance, crack pattern including crack widths and identification of the failure mode. It can be used for structure optimization, assessment of retrofitting or reinforcement detailing.

In order to extend ATENA potential and features, the recent development combines the calculating core with AtenaStudio runtime and post-processing environment, and a powerful third-party program GiD for pre-processing. The resulting product ATENA Science covers broad range of structural and material behavior in time. It enables to model geometrically complicated shapes and it is suitable for analysis of complex structural problems, such as:

- dynamic implicit analysis
- dynamic eigenvalue analysis
- static stress analysis
- creep analysis
- transport of heat and fluids
- fire analysis

The coupling of the above effects can be often achieved through simultaneous solution of various constitutive models. Thus, dynamic analysis can capture nonlinear material response due to cracking, etc. In the eigenvalue analysis vibration frequencies reflect the stiffness changes due to material damage. In creep analysis, the cracking of concrete and redistribution of stress due to plastic deformations is reflected. In fire analysis material response is strongly dependent on changing temperature fields.

The connectivity to the stochastic analysis and the corresponding SARA Studio is recently available for all levels of the ATENA software – ATENA Engineering (2D and 3D) as well as ATENA Science.

### *3.1. Modeling of fibre concrete*

Typical procedure of modeling a fibre concrete structure by nonlinear finite element analysis consists of several steps. The first one is the estimation of input material parameters based on mixture, contents and type of fibres, etc. The next step is modification of initial input parameters by inverse analysis of material tests, mainly three-point or four-point bending test, to better describe the fibre concrete behavior. The third and last step consist of analysis of the whole structural element using determined material parameters.

The tensile response of fibre concrete material differs from normal concrete not only in the values like tensile strength and especially fracture energy, but also in the shape of tensile softening branch. The original exponential function valid for normal concrete can be used as a first approach, but in most cases a more realistic form of the tensile constitutive law should be employed. Therefore, special material models at macroscopic level are needed for modeling of the fibre concrete material in the numerical simulation of fibre concrete structures as described e.g. in [8].

The most sophisticated and most general model of the fibre concrete material represents an extension to the basic fracture-plastic constitutive law [7] called *3D NLC2 User model*. It describes the tensile behavior according to the material response measured in tests point-wise in terms of the stress-strain relationship. The first part of the diagram is the usual stress-strain constitutive law. After exceeding the localization strain $\varepsilon_{loc}$ the material law assumed for the characteristic crack band width $L_{ch}$ is adjusted to the actual crack band width $L_t$ (Figure 2(a)). The characteristic crack band width (characteristic length) is the size (length) for which the defined material law is valid.

The compressive behavior of fibre concrete material model should be also adjusted in order to represent the realistic response. The higher material ductility should be reflected in the compressive material law. Therefore, similar procedure (with eventually different characteristic length) is used for the compression part of the compressive stress-strain law of the mentioned material model shown in Figure (b).

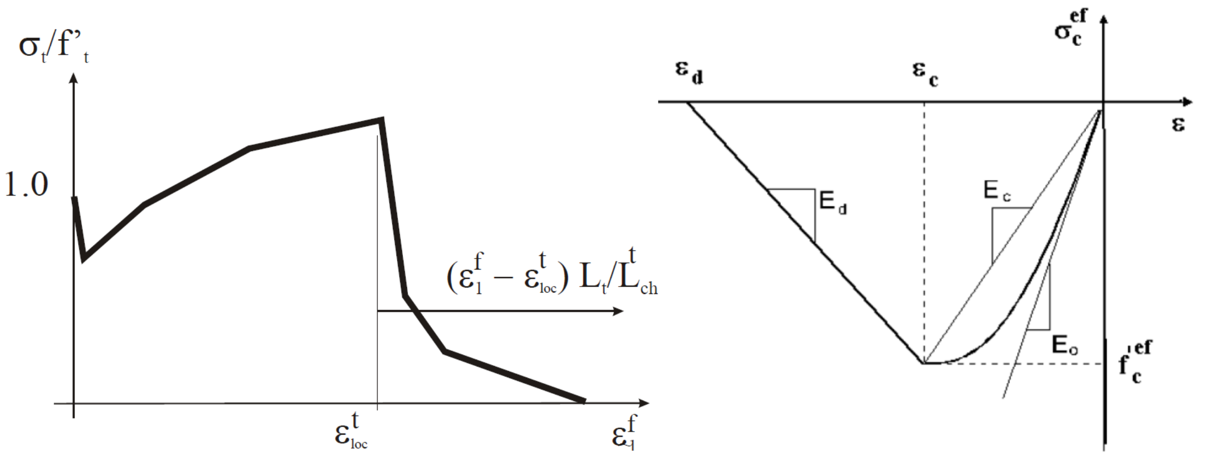

**Figure 2.** (a) User defined tensile behavior (b) Compressive stress-strain law

The softening law in compression is linearly descending and the end point of the softening curve is defined by plastic displacement $w_d$ (corresponding to $\varepsilon_d$ in Figure (b)). By increasing material parameter $w_d$ the contribution of the fibres to the compressive behavior of concrete is considered. Another important compressive parameter for modeling of fibre concrete is reduction of compressive strength due to cracks which says how the strength is reduced while the material is subjected to lateral tension.

It should be noted that the resulting material parameters are appropriate coefficients of the adjusted material model. They don´t necessarily represent the real material properties which would correspond to the material values as described by the property name in the material model.

### *3.2. Identification of material parameters*

The appropriate material parameters such as fracture energy, tensile and compressive strength, and modulus of elasticity of the structural material, are generally not well known, but crucial for a successful computer simulation of the structural response. For identification of these parameters from experimental results an inverse method based on artificial neural networks is applied [9], [10], [11], [12]. Background of the inverse analysis is finite element method model which is used for numerical simulation of fracture test; the model was created in ATENA software.

Subject of identification are the basic three fracture mechanical parameters of the concrete model: modulus of elasticity, tensile strength, fracture energy. These material model parameters are considered as random variables described by probability distribution. The rectangular distribution was chosen as the lower and upper limits represent the bounded range of physical existence. The variables are then simulated randomly based on the small-sample simulation Latin Hypercube Sampling. Multiple calculation of deterministic computational model using random realizations of material model parameters is performed, and statistical set of the virtual structural response is obtained. Random realizations and the corresponding responses from the computational model serve as the basis for the training of the neural network. After the training the neural network is ready to solve the main task: To provide the material parameters for which the numerical simulation will result in the best agreement with the provided experimental data. This task is performed by means of the simulation of the neural network using measured response as an input. It results in a set of identified optimal material input parameters. The last step of the procedure is results verification – calculation of computational model using identified parameters and comparison with the measured data.

Depending on the sample size of the statistical set, the basic statistical characteristics (mean values and standard deviations) of material parameters can be determined using two approaches:

1. "one by one" approach – the parameters of each specimen are identified separately and the final statistics are calculated from the set of all values for each parameter
2. "direct" approach – in the case of a larger statistical set it is more efficient not to identify each specimen one by one but to identify the whole statistical set for all specimens together based on the random response of fracture tests [13]

### *3.3. Material degradation*

Material degradation can be covered by deterministic or stochastic assessment of degradation effects such as concrete carbonation, deterioration, depasivation and reinforcement corrosion. As an extension to the probabilistic software FReET the program FReET-D (FReET Deterioration/Degradation Module, see e.g. [14]) was developed in order to support the life-cycle and performance-based assessment and design of reinforced concrete structures. It represents deterioration module for statistical, sensitivity and reliability assessment of degradation effects in concrete structures in the SARA system. Several recognized models for concrete carbonation and deterioration as well as reinforcement corrosion are included. This deterioration module provides:

- modeling of degradation phenomena in concrete structures
- assessment of service life
- assessment of reliability measure

The results (stochastic or deterministic) can be directly used as inputs in ATENA or SARA programs to account for material deterioration and degradation and to perform life-cycle analysis of concrete engineering structures or structural parts.

## **4. Stochastic analysis**

The stochastic analysis is based on repeated analysis of the prepared model with randomly generated properties. Since each of these samples represent a demanding nonlinear finite element analysis, the number of samples should be kept on moderate level. In the same time the applied methodology should be sufficiently accurate and representative.

The probabilistic software FReET [15] has been developed for stochastic and probabilistic analysis of computationally intensive problems such as nonlinear finite analysis. Stratified simulation technique LHS is used to keep the number of required simulations at an acceptable level. This technique can be used for both random variables and random fields levels. Statistical correlation is imposed by the stochastic optimization technique called simulated annealing. Sensitivity analysis of the input parameters to resulting values is based on nonparametric rank-order correlation coefficients. Procedure can be briefly outlined:

- random input parameters are generated according to their PDF using LHS sampling
- samples are reordered by the Simulated Annealing approach in order to match the required correlation matrix as closely as possible
- generated realizations of random parameters are used as inputs for the analyzed function (computational model)
- solution is performed many times and the results (structural response) are saved.
- simulation process the resulting set of structural responses is statistically evaluated

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Main results from the stochastic analysis are:

- estimates of the mean value
- variance
- coefficient of skewness and kurtosis
- empirical cumulative probability density function estimated by an empirical histogram of structural response

This basic statistical assessment is visualized through histograms. This is followed by reliability analysis based on several approximation techniques:

- estimate of reliability by the Cornell safety index $\beta$
- curve fitting approach applied to the computed empirical histogram of response variables
- estimate of probability of failure based on the ratio of failed trials to the total number of simulations

State-of-the-art probabilistic algorithms are implemented to compute the probabilistic response and reliability. The main features of the FReET software are:

(a) stochastic model (inputs)
- direct connectivity to the nonlinear analysis input data
- friendly Graphical User Environment (GUE)
- 30 probability distribution functions (PDF), mostly 2-parametric, some 3-parametric, two 4-parametric (Beta PDF and normal PDF with Weibullian left tail)
- unified description of random variables optionally by statistical moments or parameters or a combination of moments and parameters
- PDF calculator
- statistical correlation (also weighting option)
- categories and comparative values for PDFs
- basic random variables visualization, including statistical correlation in both Cartesian and parallel coordinates

(b) probabilistic techniques
- Crude Monte Carlo simulation
- Latin Hypercube Sampling (3 alternatives)
- First Order Reliability Method (FORM)
- Curve fitting
- Simulated Annealing
- Bayesian updating

(c) response/limit state function
- numerical form directly connected to the results of nonlinear FE analysis
- multiple response functions assessed in same simulation run

## 5. Design and safety of fibre concrete structures
In design of engineering structures sufficient safety must be assured. Comparison of the safety formats by several methods (partial safety factors (PSF), ECOV method, EN 1992-2, see [16], [5]) has been performed for two fibre concrete structures including the probabilistic nonlinear analysis using SARA software system. The results are collected in Table 1.

The first example was a four-point bending beam (Figure 3) made from steel fibre concrete. Model has been created in ATENA 2D (Figure 3(b)). The mean material parameters were tested, calibrated by inverse analysis according to Section 3.2 (Figure 3(c)), the model was loaded until localized failure (Figure 3(d)), and consequently various safety approaches were used to calculate the corresponding design resistances.

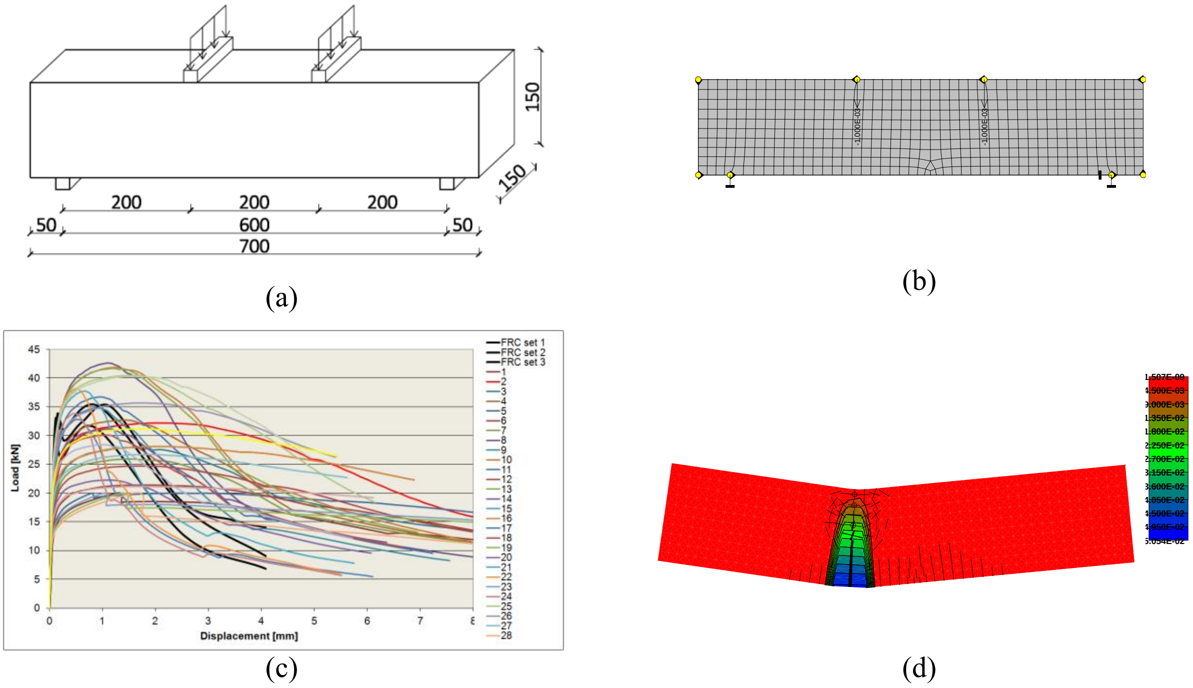


**Figure 3.** Fibre concrete four-point bending beam.

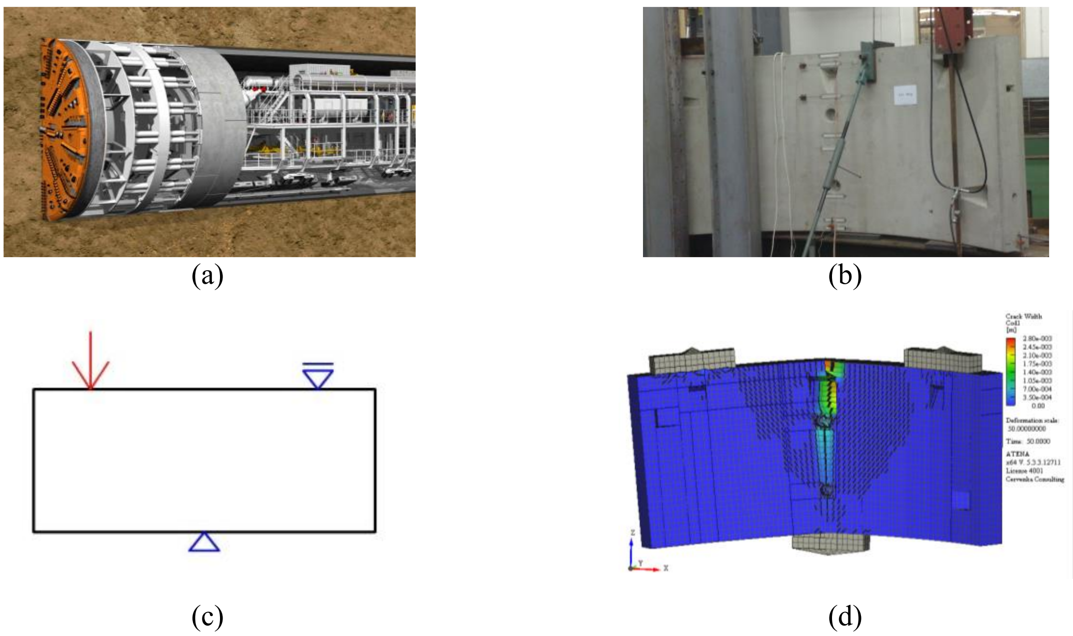


**Figure 4.** Tunnel tubing made of fibre concrete.

The second example represents a tunnel lining segment (Figure 4) installed by TBM (tunnel boring machine, Figure 4(a)) during tunnel excavation. Laboratory tests of the fibre concrete segments have been performed in Klokner Institute of CTU in order to check their resistance under various loading conditions (Figures 3(b, c)). The numerical model was first validated by experimental data for a loading scenario simulating the action of the TBM machine during the installation and assembly of segments (Figure 4(d)). After that the design resistance was calculated using various safety formats (see Table 1).

**Table 1.** Safety format results for fibre concrete structures

| | $R_d/R_d^{PSF}$ | | | |
|---|---|---|---|---|
| | *PSF* | *Probabilistic* | *ECOV* | *EN 1992-2* |
| FC bending beam | 1 | 1.16 | 1.23 | 0.97 |
| FC tubing | 1 | 1.22 | 1.27 | 1.00 |
| Average | 1 | 1.19 | 1.25 | 0.99 |

Note: $R_d$ is normalized with respect to PSF

## 6. Conclusions

- Authors offer a methodology and user-friendly computer tools for probabilistic design of fibre concrete structures. The utilized approach is based on randomization of the nonlinear finite element analysis of the structural finite element model.
- The input material parameters are represented as random variables or fields. They can be identified from material tests by neural network technology. The input parameters can be reduced in a rational way due to stochastic degradation and deterioration modelling. According to the user request this reduction can be applied in deterministic or stochastic way.
- The results from the nonlinear probabilistic analysis can serve as a rational basis for (performance-based) design of fibre concrete structures.
- The presented methodology is illustrated on results from selected probabilistic studies with different types of fibre concrete structures. The probabilistic analysis resulted here in higher resistance values (by ~20%) with regard to the PSF and to the EN 1992-2 (which is slightly more conservative even regarding to PSF). The ECOV method gives slightly higher resistance values than the probabilistic approach (by ~25% with regard to PSF); however, the presented results do not include model uncertainties which would reduce the calculated structural resistance.
- Due to high variability of fibre concrete material properties it can be recommended to utilize preferably the probabilistic methods, where the actual material variability can be accounted for the evaluation of structural performance, safety and reliability. The probabilistic nonlinear safety analysis is a demanding, but the most suitable method for design and performance assessment of fibre concrete structures.

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