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Güvenilir Yüksek Frekanslı Kesirli Dereceden Kapasitörler ve Pasif Devre Modellleri

Reliable High-Frequency Fabricated Fractional-Order Capacitors and Their Passive Circuit Models

Aslihan Kartci*†, Norbert Herencsar†, and Khaled Nabil Salama*

*Computer, Electrical and Mathematical Sciences and Engineering Division, King Abdullah University of Science and Technology, Thuwal 23955–6900, Saudi Arabia

†Department of Telecommunications, Brno University of Technology, 616 00 Brno, Czech Republic Email: {aslihan.kartci; khaled.salama}@kaust.edu.sa; herencsn@feec.vutbr.cz

Özetçe—Bu çalışmada, -0.74, -0.79 ve -0.91 sırasıyla üç farklı tipte kesirli dereceden kapasitör (FOC) üretilmiş ve empedans özellikleri analiz edilmiştir. Kullanılan kapasitörler, 10 MHz - 100 MHz frekans aralığında sabit faz açısı gibi mükemmel özelliklere sahiptir. Empedans verileri standart EIA-48 uyumlu bileşen değerleri kullanılarak Foster-I ve Foster-II'nin ikinci dereceden pasif devre modeli yapıları ile modellenmiş ve elde edilen sonuçlar, göreceli büyüklük ve faz hatası olarak değerlendirilmiştir. Seri, paralel ve karışık olarak birbirine bağlı FOC'lerin detaylı olarak ölçümleri yapılmış ve sonuçlar kullanılarak eş değer faz ve pseudo-kapasitansı üzerindeki etkisi de gösterilmiştir.

Anahtar Kelimeler—devre bağlantıları; Foster-I; Foster-II; kesirli dereceden kapasitör; seri bağlama; paralel bağlama.

Abstract—The impedance characteristics of three different type of fractional-order capacitors (FOCs) with an order of -0.74, -0.79, and -0.91 are analyzed. The used devices have excellent feature such as constant phase angle in the frequency range 10 MHz - 100 MHz. Their impedance data is fitted with second-order passive electrical model structures of Foster-I and Foster-II using standard EIA-48 compliant component values and obtained results are evaluated i.e. relative magnitude and phase error. The effect on phase and pseudo-capacitance using a detailed experimental study of series-, parallel-, and interconnected FOCs is also shown.

Keywords—circuit network connections; Foster-I; Foster-II; fractional-order capacitor; series connection; parallel connection.

I. INTRODUCTION

With advancements in theory of fractional calculus and also with widespread engineering application of fractional-order dynamics, analog implementation of fractional dynamics has received considerable attention [1]. One of them is the fabrication of fractional-order capacitors (FOCs) [2]. A single FOC is a fabricated passive two terminal fractional-order devices benefiting from the lossy nature of the dielectric materials. Fabrication of FOCs allows us to make direct and easy implementation of fractional-order systems. Therefore,

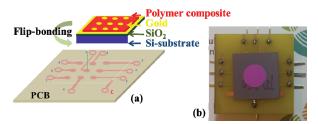


Fig. 1. (a) Illustration and (b) photograph of a PCB compatible FOC [6].

while improving the performance and increasing the variability of FOCs, their stability and accuracy becomes important [3]. This can be simply tested in circuit network connections [4]. It is particularly important because a FOC possess both the real and imaginary parts of the impedance $Z(s) = \frac{1}{2} \sum_{i=1}^{n} \frac{1}{2} \sum$

$$\omega^{\alpha} \left[\cos \left(\frac{\alpha \pi}{2} \right) + j \sin \left(\frac{\alpha \pi}{2} \right) \right] / C_{\alpha}$$
. Hence, it is clear that, if a

configuration using capacitors is required, small errors may accumulate the metrics of the individual components. In this study, impedance characteristics of three types of fabricated FOCs presenting three different orders are described. Selected pins from each device is evaluated in the frequency range of $10 \, \mathrm{MHz} - 100 \, \mathrm{MHz}$ to show the pureness of devices. Later, the devices are used in circuit network connections such as series-, parallel-, and inter-connected FOCs. A vector network analyzer is used to collect the experimental data, hence the measurement setup differs from [4]–[6]. These measurements further strengthen the reliability of these devices. Finally, the impedance characteristics of the selected FOCs are fitted to passive RC models. The relative magnitude and phase angle error are evaluated.

II. FABRICATED FOCS

The Poly(vinylidene fluoride)-P(VDF), Poly (vinylidene fluoride trifluoroethylene cholorfluroethylene)-P(VDF-TrFE-CFE), and multiwall carbon nanotube (MWCNT) [6], [7] based ferroelectric polymer blends are used as filler, instead of pure dielectrics in conventional capacitor, in the fabrication of FOCs

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TABLE I. MEASUREMENT RESULTS OF FABRICATED FRACTIONAL-ORDER CAPACITORS.

Capacitor No.	Fabrication Technology	Device Pin No.	$ \mathbf{Z} [\Omega]$	Order α* [−]	Pseudo-Capacitance* [Farad·sec ^{a-1}]	PAD in (10-100) MHz [°]
1	MWCNT	1	247.63	-0.75	3.63 n	± 2.80
2	MWCNT	2	216.17	-0.74	4.83 n	± 2.25
3	MWCNT	3	224.26	-0.74	4.72 n	± 2.33
4	MWCNT	4	188.50	-0.73	6.58 n	± 1.43
5	P(VDF-TrFE-CFE)	1	483.32	-0.81	0.50 n	± 2.89
6	P(VDF-TrFE-CFE)	2	250.77	-0.78	1.89 n	± 2.41
7	P(VDF-TrFE-CFE)	3	273.87	-0.80	1.22 n	± 2.43
8	P(VDF-TrFE-CFE)	4	280.83	-0.79	1.36 n	± 2.82
9	P(VDF-TrFE-CFE)	5	301.70	-0.79	1.43 n	± 3.22
10	P(VDF-TrFE-CFE)	6	240.57	-0.76	3.08 n	± 3.16
11	P(VDF)	1	584.51	-0.90	63.09 p	± 0.02
12	P(VDF)	2	592.63	-0.91	50.56 p	± 0.07
13	P(VDF)	3	511.32	-0.91	62.85 p	± 0.01
14	P(VDF)	4	687.60	-0.91	42.38 p	± 0.16
15	P(VDF)	5	647.50	-0.91	55.44 p	± 1.33
16	P(VDF)	6	742.18	-0.92	34.27 p	± 1.11

Note: * (a) $f_c = 30$ MHz; PAD – maximum difference between a measured phase and a target phase.

obtaining three different phase angle. Their structures are similar to double-layer capacitors as shown in Fig. 1. 200 nm gold (Au) is sputtered on SiO_2/Si substrate representing the bottom electrode while the circular top electrode is sputtered on the polymer composite using a 3 mm diameter shadow mask. Finally, nine individual FOCs are fabricated on a 2 cm \times 2 cm sample area and the common electrode is shared with each capacitor. Then, the device is flip bonded on a PCB to be easily used in any kind of circuit measurements. The detailed fabrication procedure was mentioned in [7]. The presented FOCs show better performance in terms of fabrication cost and dynamic range of constant phase angle compared to FOCs from already existing devices [7].

III. ELECTRICAL CHARACTERIZATION OF FOCS

In this section, the electrical characterization of the fabricated devices and their circuit network connections are studied. Results are based on general approach previously presented in [4], [6]. However, the main aim of this paper is to show the reliability of the devices at high frequencies by using an ENA Series Network Analyzer E5071C. Considering an input impedance 50 Ω of the SMA connector, the frequency response is measured by defining the following impedance $Z = 50[(1 + S_{11})/(1 - S_{11})]$ in the frequency range of 10 MHz – 100 MHz with 201 logarithmically spaced points. Measurement results of each single device at center frequency $f_c = 30 \text{ MHz}$ are noted in Table I. MWCNT-polymer, P(VDF-TrFE-CFE)-, and P(VDF) composite devices retaining the permittivity of the mixture have phase responses of approximately -66.60° , -71.2° , and -81.90° ($\alpha \rightarrow -0.74$, -0.79, and -0.91), respectively and shown in Fig. 2(a). Their magnitude values are about $\{219.14, 305.17, 627.61\}$ Ω and their frequency responses are shown in Fig. 2(c) from which pseudo-capacitance values in Fig. 2(b) are calculated as $\{3.43 \text{ n}, 950.54 \text{ p}, 46.97 \text{ p}\}$ Farad·sec $^{\alpha-1}$. To estimate the

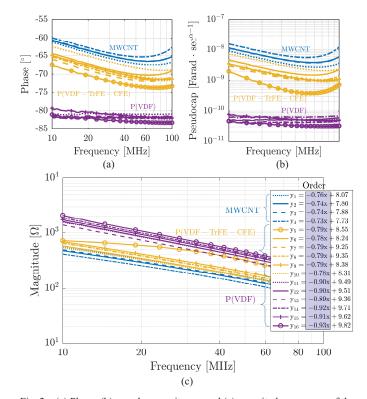


Fig. 2. (a) Phase, (b) pseudo-capacitance, and (c) magnitude responses of the tested pins.

equivalent order (or phase), the measured magnitude data are fitted to the function $\log |Z| = \alpha \log f + \log (2\pi) C_{\alpha}$ using the linear least squares (LLS) method. The obtained equivalent equations from fitting the magnitude are equal to measurement samples that are provided inside Fig. 2(c). As a result, the order of single MWCNT-polymer, P(VDF-TrFE-CFE)-, and

TABLE II. EVALUATION OF MEASURED SERIES-, PARALLEL, AND INTER-CONNECTED FOCS.

Connection	Fabrication Technology (Capacitor No.)	$ Z [\Omega]$	Order a* [-]	Pseudo-Capacitance [F·sec ^{α-1}]	PAD in (10-100) MHz [°]
Parallel	MWCNT (1 2 3 4)	54.39	-0.69	35.85 n	± 3.74
Parallel	P(VDF-TrFE-CFE) (5 6 7 8)	73.16	-0.78	4.80 n	± 0.26
Parallel	P(VDF) (11 12 13 14)	153.61	-0.90	232.17 p	± 0.46
Series	MWCNT $(1+2+3+4)$	519.98	-0.74	1.45 n	± 2.09
Series	P(VDF-TrFE-CFE) (5 + 6 + 9 + 10)	775.67	-0.85	119.21 p	± 3.97
Series	P(VDF) (13 + 14 + 15 + 16)	1.24 k	-0.93	16.24 p	± 0.60
Inter-Conn.	MWCNT, P(VDF-TrFE-CFE), P(VDF) (5 12) + 1 +9	113.40	-0.79	2.56 n	± 0.94* (10–60) MHz

Note: * $@f_c = 30 \text{ MHz}$; PAD – maximum difference between a measured phase and a target phase.

P(VDF) composite FOCs is evidently respond to their equivalent orders that are found to be -0.74, -0.79, and -0.91, respectively.

In case of their circuit network connections as series and parallel of four identical FOCs, the equivalent magnitude, phase, and pseudo-capacitance responses are noted in Table II and obtained measurement results are shown in Fig. 3 and Fig. 4. It is evident that the phase angle remains identical to initial single FOCs phase value and the only change is in the magnitude response, which reflects the pseudo-capacitance C_{α} . In general, as in the classic well known capacitor theory also works with identical-order FOCs. In parallel connection, the magnitude is the one-quarter of individual FOC, while in series connection it is four times greater than the single FOC. For instance, when four MWCNT devices are connected in parallel and series, their equivalent magnitude and pseudo-capacitance are 54.39 Ω and 35.85 nF·sec^{-0.31}, 519.98 Ω and 1.45 nF·sec^{-0.31}, respectively.

Here, it is also worth to show flexibility and degree of freedom of fabricated FOCs and how to obtain any order of FOCs via random connection of devices. Therefore, their interconnections are also studied. Considering the network inside Fig. 5, its equivalent impedance is given as:

$$Z = \frac{1}{s^{\alpha_5} C_{\alpha_5} + s^{\alpha_{12}} C_{\alpha_{12}}} + \frac{1}{s^{\alpha_1} C_{\alpha_1}} + \frac{1}{s^{\alpha_9} C_{\alpha_9}}.$$
 (1)

The measured phase, magnitude, and pseudo-capacitance responses of the inter-connected FOCs are shown in Fig. 5 and the detailed comparison of results @ 30 MHz are given in Table II. It is clear that small deviation in phase effects on pseudo-capacitance significantly due to the power of order.

IV. EQUIVALENT RC MODELS OF FABRICATED FOCS

The equivalent passive models of the fabricated devices, with selecting from each material one pin example, are constructed with second-order Foster-I and Foster-II type of RC networks [8]. Standard EIA-48 compliant component values are used to fit the data given in Table I. Obtained values are listed in Table III together with evaluation of the networks by means of relative magnitude (%) and phase error (°).

Considering the passive component values, the minimum total resistance is obtained with Foster-I structure of $C_{\alpha 1}$ device while the minimum total capacitance is with Foster-II structure

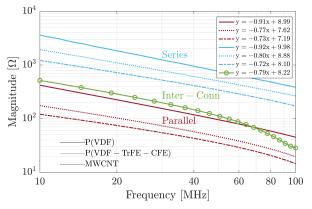


Fig. 3. Magnitude response of the circuit network connections.

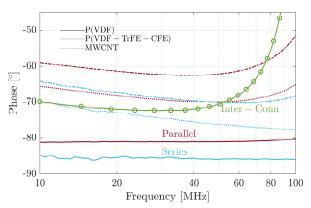


Fig. 4. Phase response of the circuit network connections.

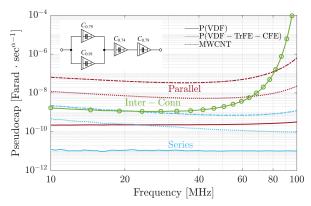


Fig. 5. Pseudo-capacitance of the circuit network connections and schematic of the inter-connected FOCs as an inset.

TABLE III. LIST OF STANDARD EIA-48 COMPLIANT COMPONENT VALUES USED IN RC STRUCTURES FOR FITTING AND EVALUATION OF MODELS.

		Foster-I		Foster-II						
Elements		$Z(s) = R_0 + \sum_{k=1}^{2} \frac{R_k}{sR_kC_k + 1}$		$Y(s) \longrightarrow R_0 \qquad R_1 \qquad R_2$ $C_1 \longrightarrow C_2$ $Y(s) = \frac{1}{R_0} + \sum_{k=1}^{2} \frac{sC_k}{sR_kC_k + 1}$						
	MWCNT ($C_{\alpha 1}$)	P(VDF-TrFE-CFE) ($C_{\alpha 5}$)	$P(VDF)(C_{\alpha 14})$	MWCNT ($C_{\alpha 1}$)	P(VDF-TrFE-CFE) ($C_{\alpha 5}$)	$P(VDF)(C_{\alpha 14})$				
$R_0\left(\Omega\right)$	22	27	15	1.78 k	100 k	20 k				
$R_1(\Omega)/C_1(F)$	1.6 k / 33 p	4.3 k / 20 p	18.7 k / 10 p	953 / 10 p	1.5 k / 12 p	18 / 8.25 p				
$R_2\left(\Omega\right)/C_2\left(\mathrm{F}\right)$	115 / 43 p	270 / 30 p	91 / 47 p	27 / 17.8 p	28.7 / 11 p	3.65 k / 1.62 p				
	Relative magnitude error (%)									
Max	6.04	43.46	8.41	4.78	50.54	7.85				
Mean	2.42	14.94	2.71	1.55	14.10	2.36				
Median	2.22	15.20	1.88	1.57	12.75	1.25				
Std. Dev.	1.50	8.86	2.26	0.90	10.57	2.20				
	Phase angle error (Deg.)									
Max	0.65	1.38	0.31	0.92	1.54	0.25				
Mean	0.42	0.91	0.15	0.50	0.76	0.08				
Median	0.48	1.00	0.16	0.51	0.82	0.08				
Std. Dev.	0.19	0.38	0.08	0.26	0.35	0.06				

of $C_{\alpha 14}$. On the other hand, the percentage of relative magnitude error is evaluated with its maximum, mean, median, and standard deviation. The best performance is shown by $C_{\alpha 1}$ with less than 4.78% error. Considering the small number of passive elements in modelling and one decade of frequency range, the phase angle error is not greater than 1.54 degrees, which is super precise and match well with the fabricated fractional-order devices. This is particularly important to find the correct model and analyze the materials electrical characteristics. The equivalent passive model of devices and their circuit network connections are correlated with dielectric properties in different microstructures. Therefore, their equivalent models can give us the understanding of electrical characteristics of each material. In other words, mimicking the nature of the real device further may help us to understand the mystery of the materials. Besides, circuit designers can use these models in simulation programs and see the real performance of the full system such as filters, oscillators.

V. CONCLUSION

The polymer composite identical-order FOC connections, their impedance responses, and equivalent passive models were studied. It was also shown that C_{α} match well with classical circuit theory expected by application designers. The standard EIA-48 compliant component values are used to obtain their passive models. These passive models are correlated with dielectric properties in different microstructures. Therefore, the FOC characteristics with connection of different dielectric materials is the key point. Since the order of fabricated FOCs is limited till now, the non-existing orders might be obtained with their inter-connections as shown. This study is the very first

step to achieve understanding of natural behavior of FOCs and move towards the computational part.

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