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PARAMETER ESTIMATION FOR DYNAMIC MODEL OF THE FINANCIAL SYSTEM

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Abstract

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Economy can be considered a large, open system which is influenced by fluctuations, both internal and external. Based on non-linear dynamics theory, the dynamic models of a financial system try to provide a new perspective by explaining the complicated behaviour of the system not as a result of external influences or random behaviour, but as a result of the behaviour and trends of the system's internal structures. The present article analyses a chaotic financial system from the point of view of determining the time delay of the model variables – the interest rate, investment demand, and price index. The theory is briefly explained in the first chapters of the paper and serves as a basis for formulating the relations. This article aims to determine the appropriate length of time delay variables in a dynamic model of the financial system in order to express the real economic situation and respect the effect of the history of factors under consideration. The determination of the delay length is carried out for the time series representing Euro area. The methodology for the determination of the time delay is illustrated by a concrete example.

Keywords: financial system, dynamic system, time delay, investment demand, interest rate, price index

INTRODUCTION

One of the natural trends in most fields of science is to study various model situations which subsequently enable us to analyse the simulated processes, more accurately specify the conditions under which they function, make practical conclusions from the findings, find an optimal solution etc. If an economist wishes to assess the impact of certain real steps in an economic system, he creates an economic model and uses historical facts to verify his theory. Therefore, it is very often necessary for them to get involved in the modelling of complicated economic phenomena and systems, analysis and verification of these models, prediction, and optimal decision-making.

Dynamic models based on the theory of non-linear dynamics are applied to cases where relations cannot be effectively studied by traditional analytical methods which are based on linearity, stability and a stable equilibrium point. The theory of non-linear system dynamics has developed along with the development of computer technologies and with the possibilities this development has brought. This theory is capable of showing what the relation of the whole to a change of its individual parts is, and what the differences between the whole and its parts are. More precisely, chaotic motion in dynamic systems is specified by the so-called chaos theory. This theory provides tools in dynamic systems which allow explaining complicated behaviour of the system as a result of the behaviour and trend of its internal structures. At the same time, this theory can be used in many areas of economics, it can be applied, for instance to economic cycle, economic growth, or the relation between microeconomic and macroeconomic structures.

The functioning of a financial system, which is part of an economic system, may be explained on the basis of a non-linear economic model. However, when we investigate certain economic quantities as functions of time, we must take account of the fact that a quantity may also depend on its preceding values or on the preceding values of other quantities.

This article analyses a chaotic financial system from the point of view of determining the time delay of the model variables – the interest rate, investment demand, and price index. The theory is briefly explained in the first chapters and serves as a basis for formulating other relations. The purpose of the paper is to set up a dynamic non-linear model of a financial system which would express a real economic situation and respect the effect of the history of the factors under consideration. Determination of the delay length is carried out for the time series representing Euro area. To determine the time delay, a statistics method will be used, which captures the nature of the relation between the time series. The methodology for the determination of the time delay is illustrated by a concrete example.

Literature Review

Efforts by economists and mathematicians to apply models of dynamic systems to economics have a long history. In economics, state variables may be represented by such quantities as production, consumption, investment, etc. The modelling of phenomena which are based on economic reality and described by statistical data, is facilitated in particular by methods from various fields of mathematics such as differential calculus (Aluf, 2012; Balasubramaniam, 2014; Faria, 2013), statistics (David, 2013), (Plaček, 2013), linear and dynamic programming (Hassan, 2011), optimization (Subagyo, 2014; Zaarour, 2014), etc. The current methods for investigation these systems are focused primarily on the identification of states in which the system behaviour is predictable, and states in which the system shows signs of deterministic chaos.

The study of non-linear dynamic systems was extended especially by mathematicians such as Ljapunov, Pontryagin, or Andronov. Their work was later continued by Smale in the USA, Peixoto in Brasil, and Kolmogorov, Arnold and Sinai in the Soviet Union. In 1975, a new type of motion in dynamic systems was discovered which is currently referred to as "chaotic motion". This new type of motion was described as unstable or fluctuating with a large number of periods, but not as quasi-periodic. Non-linear systems are characterized by the random appearance of structural changes with random motion such as bifurcations. Since 1989, when the first treatise of chaos control was published by Huber, chaos control has attracted great attention due to its potential application to physics, chemical reactors, control theory, biological networks, artificial neural networks, telecommunications etc. In the last few decades, much effort has been devoted to the theory of chaos control, primarily in the areas of unstable equilibrium points and unstable periodic solutions (Hubler, 1989). The methods which have been developed are in particular suitable for the case of chaos suppression in various chaotic systems (Wu, 2010; He, 2013; Chen, 2014).

In the areas of finance or social economics the internal structures are often non-linear and the mutual relations are very complicated. This is why studies investigating the effects of internal structural characteristics of such a system represent the system as a system of differential equations with a possible chaotic behaviour. Chaos in a financial system was first demonstrated in 2001 (Ma, 2001a; Ma, 2001b). Later on, in 2007, a new attractor was proposed for a modified chaotic finance system (Cai, 2007). In 2009, a hyper chaotic finance system was proposed from the modified chaotic finance system (Ding, 2012). Chen (2014) and Yu (2012) presented a 4D chaotic finance system, over which they achieved control by means of linear feedback and speed feedback controllers..

Non-Linear Dynamic Finance System

A finance system can be understood as a set of markets, institutions, laws, regulations and techniques on the basis of which all types of financial transactions are carried out. Models of dynamic finance systems may contain certain trends indicating the significance of sensitivity of the parameters to initial values from which the systems subsequently evolve towards structural changes. These may cause fluctuations over a long period of time. One must realize, however, that occurrence of these properties may not necessarily be caused by economic activity. Similar properties may then pass into mechanisms which are able to cause a change in the structure of the economy, leading to a state in which chaotic behaviour occurs (Ma, 2001a; Ma, 2001b).

In the publications (Ma, 2001a; Ma, 2001b) the key part of the financial model was simplified for simplification reasons. It was decided, based on detailed analysis and experiments, that the model will comprise the following variables: x for the interest rate, y for the investment demand, and z for the price index. An important property of these variables is their sensitivity to a change of information known in the economy. Since the effect of the sensitivity of these variables to a change of information in time is precisely the problem being investigated, the changes of these variables in time are defined as the state variables: $\dot{x} = \mathrm{d}x/\mathrm{d}t$, $\dot{y} = \mathrm{d}y/\mathrm{d}t$, $\dot{z} = \mathrm{d}z/\mathrm{d}t$.

As a result, the model of the financial system is represented by three differential equations:

$$\dot{x} = z(t) + (y(t) - \mathrm{a})x(t),$$

$$\dot{y} = 1 - by(t) - x^2(t), \qquad (1)$$

$$\dot{z} = -x(t) - cz(t),$$

where a stands for household savings, b for the cost of investment, and c for the demand elasticity of commercial markets. All these three model parameters are assumed to be positive.

According to (Ma, 2001a; Ma, 2001b) a dynamic financial system represented by a system of ordinary differential equations should comprise chaotic behaviour. This assertion has been investigated on the basis of numerical experiments, performed with this system, and has been confirmed by other publications (Ding, 2009; Chen, 2014).

In Chen (2008) and Son (2011), the system (2) was modified by adding delayed feedbacks into the system. This modification is based on the finding made in Holyst (2000) and Holyst (2001) that chaotic behaviour of a microeconomic model can be stabilized, if we use the Pyragas delayed feedback control. The DCF (delayed continuous feedback) method proposed in (Pyragas, 1995) can be applied to experimental systems.

If we consider the effect of the past, adding feedback to the system, we obtain a new system:

$$\dot{x} = z(t) + (y(t) - a)x(t) + k_1(z(t) + (y(t) - a)x(t) - (z(t - \tau_3) + (y(t - \tau_2) - a)x(t - \tau_1))),$$

$$\dot{y} = 1 - by(t) - x^2(t) + k_2(1 - by(t) - x^2(t) - (1 - by(t - \tau_2) - x^2(t - \tau_1))), \quad (2)$$

$$\dot{z} = -x(t) - cz(t) + k_3(-x(t) - cz(t) - (-x(t - \tau_1) - cz(t - \tau_3))),$$

where k_1, k_2, k_3 is strength of the feedback and τ_1, τ_2, τ_3 is length of the time delay.

In contemporary literature on the solvability of systems of differential equations with delayed arguments, one can find a number of results which can be applied to problems from economic practice. A general theory of the solution of the above-mentioned problem as well as related problems can be found, for instance, in (Půža, 2010; Půža, 2011; Půža, 2012).

In the present article, however, we wish to discuss the possibility of determining the length of the time delay in the model for real data.

Correlation Analysis

To determine a suitable length of the delay between the variables, we have used correlation analysis, in particular the sample correlation coefficient. Correlation analysis is one of the important tools used in analysing relations between measured variables. By using the sample correlation coefficients r, we are mainly trying to determine whether the corresponding correlation coefficient is non-zero, which would amount to the statistical proving (at a selected significance level) of a relation between the corresponding variables. The strength of the linear relation between the variables $X, \ldots, Y$ is measured by the Pearson correlation coefficient. Stochastic independence of the components X, Y of a normally distributed vector is equivalent to their non-correlation. The sample correlation coefficient r is then defined as the sample covariance divided by the product of sample standard deviations, provided these are positive. Equivalently, can be expressed by the formula

$$r = \frac{S_{XY}}{S_X S_Y},\ S_x^2 > 0,\ S_y^2 > 0, \quad (3)$$

and, subsequently, significance is tested by the two-sided zero-hypothesis test

$$H_0: \rho = 0,\ dependence,\ |t| \le t_{!-\frac{\alpha}{2}}(n-2),$$

$$H_0:\ \rho \ne 0,\ independence |t| > t_{!-\frac{\alpha}{2}}(n-2),$$

where the test statistics has the form

$$t = \frac{r}{\sqrt{1-r^2}}\sqrt{(n-2)},$$

n being the sample size (Anděl, 1993).

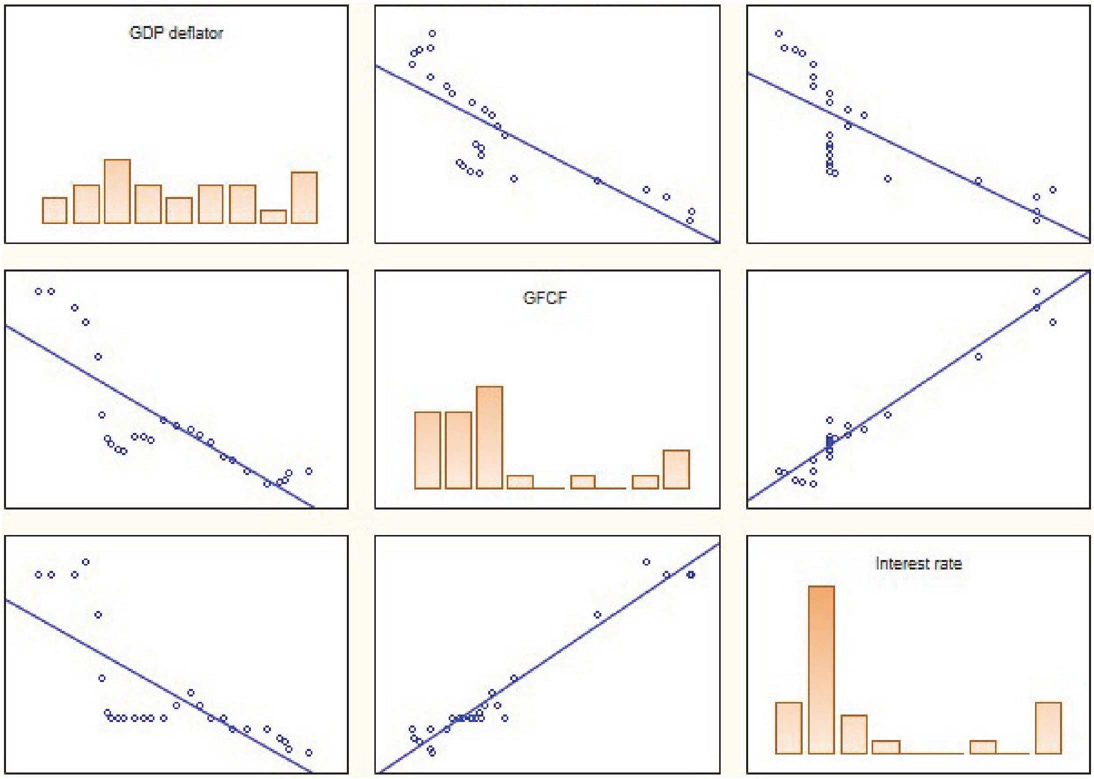
If normal distribution is not confirmed, then to find an applicable distribution of the test criterion under small values of n, the sample correlation coefficient needs to be transformed. This can be done by using the Fischer transform to the Z quantity. As a test criterion, the U quantity can be used, which has an approximately standard normal distribution.

Computations

The computations of the correlation coefficients have been done by the Statistica program. Some of the modules of this program are specific, others are very universal, and the greatest advantage of this program is its user-friendliness.

There were analysed the data obtained from http://www.tradingeconomics.com/. There were analysed quarterly time series of 27 values for the period from fourth quarter 2007 till first quarter 2014 (quarterly data were used because of the finding the delay and its impact on the development of the model and therefore the most suitable period was selected so that it has enough observation to verify dependencies and is therefore a sufficiently long period of time), representing the quarterly value of the GDP deflator, interest rate (as an indicator of financial market developments – Central Bank interest rates – this for the Bank based market model) and GFCF (Gross fixed capital formation) of Euro area. Correlation analysis was performed for the situation considering a delay of 1–4 quarters. From the frequency histograms can be concluded that the selected data are not normally distributed, primarily the time series of GFCF and interested rate.

After the graphical analysis there was performed the Kolmogorov-Smirnov test. We tested the null hypothesis H0 that the data come from a normal distribution at a significance level $\alpha = 0.05$. The test at a significance level of 5% did not reject only one variable, the GDP deflator. Only for this variable could be the assumption of a normal distribution,



1: *Frequency histograms and dot chart matrix*

with values GFCF and interest rate assumption, we must reject the normal distribution. It was therefore necessary to use data transformation in order to continue the correlation analysis.

Correlation analysis confirmed that the addition of delayed feedback to the model of financial system is absolutely justified. However simplifying assumption in the length of the delay, which does not change for each variable in relation to others, must be rejected. We can assume that this simplification based on subsequent method for solution of systems of differential equations with delay by using methods that do not allow such a system to deal with different delays. Given the current familiarity with differential equations with delayed or, more generally, deviated argument, or so-called functional differential equations, we have the possibility to apply the method of solving a significantly more general mathematical model, which may provide us with a wider range of results, including a comparison of the impact various parameters might have. Therefore, it is possible to generalize and write the model as follows:

$$\begin{aligned}
\dot{x} &= z(t) + (y(t) - a)x(t) + k_1(z(t) + (y(t) - a)x(t) - (z(t - \tau_3) + \\
&\quad + (y(t - \tau_2) - a)x(t - \tau_1))), \\
\dot{y} &= 1 - by(t) - x^2(t) + k_2(1 - by(t) - x^2(t) - (1 - by(t - \tau_4) - \\
&\quad - x^2(t - \tau_2))), \\
\dot{z} &= -x(t) - cz(t) + k_3(-x(t) - cz(t) - (-x(t - \tau_3) - cz(t - \tau_5))),
\end{aligned} \tag{4}$$

where k_1, k_2, k_3 is the strength of the feedback and τ_1, τ_2, τ_3, τ_4, τ_5 the length of time delay. The strongest association was established for Euro area for the length of time delay $\tau_1 = 1$, $\tau_2 = 4$, $\tau_3 = 1$, $\tau_4 = 2$, $\tau_5 = 1$.

CONCLUSION

When modelling complex economic problems, we are often faced with the fact that relations of particular quantities are variable in time. One way of incorporating the dynamics of processes in a model is to consider time a continuous quantity and to describe dynamical models by means of differential equations. When specifying a model's structure, the dynamic character may be taken into account by incorporating delayed impact of both exogenous and endogenous variables. Authors presents financial system model, which is expressed as system of three delayed differential equations. This article analyses this system from the point of view of determining the time delay of the model variables, represented by the time series representing Euro area.

Authors in this article confirmed that the addition of delayed feedback to the model of financial system is absolutely justified and we can use model, which is closer to real economic situation, due to respecting the influence of history.

The model discussed in this paper can be expanded in the future. One option is generalization of the model, which would allow us to work even with non-constant delay, etc.

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